# EE 341 - Exam 1

**September 22, 2004**

Name: ______________________________

Closed book. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work. You may use one page of notes, the handouts on time-domain solutions to differential and difference equations, and a calculator.

1. Here are some questions about periodic signals.

   (a) Consider the continuous-time signal $x(t) = \cos(\frac{4}{5}t)$. Is this signal periodic? if so, what is the period of $x(t)$?

   $$T = \frac{2\pi}{\omega} = \frac{2\pi}{4/5} = \frac{5\pi}{2}$$

   Periodic, $T = \frac{5\pi}{2}$

   (b) Consider the continuous-time signal $x(t) = \cos(\frac{4}{5}t) - 2\sin(\frac{3}{4}\pi t)$. Is this signal periodic? If so, what is the period

   $$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4/5} = \frac{5\pi}{2} \qquad T_2 = \frac{2\pi}{\frac{3}{4}\pi} = \frac{8}{3} \qquad \frac{T_1}{T_2} = \frac{\frac{5\pi}{2}}{\frac{8}{3}} = \frac{15\pi}{16}$$

   $\frac{T_1}{T_2}$ is irrational, so not periodic

   (c) Consider the discrete-time signal $x[n] = \cos(\frac{4}{5}\pi n) - 2\sin(\frac{3}{4}\pi n)$. Is this signal periodic? If so, what is the period of $x[n]$?

   $$N_1 = \frac{2\pi}{\omega_1}k = \frac{2\pi}{\frac{4}{5}\pi}k = \frac{5}{2}k = 5$$ (k=2 is smallest integer which makes $N_1$ an integer)

   $$N_2 = \frac{2\pi}{\omega_2}k = \frac{2\pi}{\frac{3}{4}\pi}k = \frac{8}{3}k = 8$$

   $$\frac{N_1}{N_2} = \frac{q}{r} = \frac{5}{8} \qquad N = rN_1 = qN_2 = 8 \cdot 5 = 40$$

   Periodic, $N = 40$

2. Consider the systems described by the following differential or difference equations. For each one, determine if it is linear, time-invariant, causal or memoryless. Give reasons for your answers.

(a)

$$y(t) = x(t-2) + x(2-t)$$

Memoryless: No  Causal: No  Linear: Yes  Time Invariant: No

$y[0] = x[-2] + x[2]$  $y[0]$ depends on input at times other than 0 $\Rightarrow$ memory

$y(0)$ depends on future input $x(2) \Rightarrow$ not causal

$y_1(t) = x_1(t-2) + x_2(2-t)$  $y_2(t) = x_2(t-2) + x_2(2-t)$  $\tilde{y}(t) = \tilde{x}(t-2) + \tilde{x}(2-t)$

Let $\tilde{x}(t) = a_1 x_1(t) + a_2 x_2(t)$.  $\tilde{y}(t) = [a_1 x_1(t-2) + a_2 x_2(t-2)] + [a_1 x_1(2-t) + a_2 x_2(2-t)]$

$= a_1[x_1(t-2) + x_1(2-t)] + a_2[x_2(t-2) + x_2(2-t)]$

$= a_1 y_1(t) + a_2 y_2(t) \Rightarrow$ Linear

$y(t) = x(t-2) + x(2-t)$

$\tilde{y}(t) = \tilde{x}(t-2) + \tilde{x}(2-t)$

Let $\tilde{x}(t) = x(t-t_0)$

$\tilde{y}(t) = x(t-2-t_0) + x(2-t-t_0)$

$y(t-t_0) = x((t-t_0)-2) + x(2-(t-t_0))$

$= x(t-t_0-2) + x(2-t+t_0)$

$y(t-t_0) \neq \tilde{y}(t) \Rightarrow$ not time invariant

(b)

$$y(t) = (\cos(3t))x(t)$$

Memoryless: Yes  Causal: Yes  Linear: Yes  Time Invariant: No

$y(t)$ depends only on current time $t \Rightarrow$ memoryless

memoryless $\Rightarrow$ Causal

$y_1(t) = \cos(3t)\, x(t)$  $y_2(t) = \cos(3t)\, x_2(t)$  $\tilde{y}(t) = \cos(3t)\, \tilde{x}(t)$

Let $\tilde{x}(t) = a_1 x_1(t) + a_2 x_2(t)$  $\tilde{y}(t) = \cos(3t)[a_1 x_1(t) + a_2 x_2(t)]$

$= a_1 \cos(3t)\, x_1(t) + a_2 \cos(3t)\, x_2(t)$

$= a_1 y_1(t) + a_2 y_2(t) \Rightarrow$ Linear

$y(t) = \cos(3t)\, x(t)$

$\tilde{y}(t) = \cos(3t)\, \tilde{x}(t)$

Let $\tilde{x}(t) = x(t-t_0)$

$\tilde{y}(t) = \cos(3t)\, x(t-t_0)$

$y(t-t_0) = \cos(3(t-t_0))\, x(t-t_0)$

$y(t-t_0) \neq \tilde{y}(t) \Rightarrow$ not time invariant

(c)

$$y(t) = \begin{cases} 0, & t < 0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$$

Memoryless: No Causal: Yes Linear: Yes Time Invariant: No

$y(0) = x(0) + x(-2)$ $y(0)$ depends on previous input $x(-2)$, so has memory

$y(t)$ does not depend on future inputs, so Causal

Can rewrite as $y(t) = u(t)\,x(t) + u(t)\,x(t-2)$

$y_1(t) = u(t)\,x_1(t) + u(t)\,x_1(t-2)$ $y_2(t) = u(t)\,x_2(t) + u(t)\,x_2(t-2)$

$\tilde{y}(t) = u(t)\,\tilde{x}(t) + u(t)\,\tilde{x}(t-2)$ Let $\tilde{x}(t) = a_1 x_1(t) + a_2 x_2(t)$

$\tilde{y}(t) = u(t)\,[a_1 x_1(t) + a_2 x_2(t)] + u(t)\,[a_1 x_1(t-2) + a_2 x_2(t-2)]$

$= a_1 u(t)\,(x_1(t) + x_1(t-2)) + a_2 u(t)\,(x_2(t) + x_2(t-2)) = a_1 y_1(t) + a_2 y_2(t) \Rightarrow$ Linear

Put in a signal at $t<0$, output is 0. Put in same signal at $t>0$, output not zero $\Rightarrow$ not time invariant

(d)

$$y[n] + 2y[n-1] = x[n] + nx(n-3)$$

Memoryless: No Causal: Yes Linear: Yes Time Invariant: No

$y[0]$ depends on $x(-3)$, so has memory

$y(0)$ does not depend on future values of input, so is causal

Is of form $y(n) + \sum_{k=1}^{\tilde{N}} a_k\, y(n-k) = \sum_{k=0}^{M} b_k\, x(n-k)$

Coefficients do not depend on $x(n)$ or $y(n)$, so linear

Coefficents do depend on time $n$, so not time invariant

3. Consider the system described by the difference equation

$$y[n] + y[n-1] + \frac{1}{4}y]n-2] = 2x[n] + 3x[n-1].$$

The system starts with the initial conditions $y[-1] = -4$, and $y[-2] = 4$. The input to the system is $x[n] = \delta[n]$.

(a) Write the characteristic polynomial for the system.

$\lambda^2 + \lambda + \frac{1}{4} = 0$

$(\lambda + \frac{1}{2})^2 = 0$

$\lambda_1 = -\frac{1}{2}, \; \lambda_2 = -\frac{1}{2}$

(b) Write the general form of $y[n]$ for $n \geq 0$.

Because $\lambda_1 = \lambda_2$:

$y_h[n] = A_1 \lambda_1^n + A_2 n \lambda_2^n = A_1(-\frac{1}{2})^n + A_2 n(-\frac{1}{2})^n$

$y_p[n] = k\,\delta[n]$ Same form as input

$y[n] = A_1(-\frac{1}{2})^n + A_2 n(-\frac{1}{2})^n + k\,\delta[n]$

(c) Find $y[0]$, $y[1]$, and $y[2]$. $y[n] = 2x[n] + 3x[n-1] - y[n-1] - \frac{1}{4}y[n-2]$

$y[0] = 2\delta[0] + 3\delta[n-1] - y[-1] - \frac{1}{4}y[-2] = 2\cdot 1 + 3\cdot 0 - (-4) - \frac{1}{4}(4) = 5$

$y[1] = 2\delta[1] + 3\delta[0] - y[0] - \frac{1}{4}y[-1] = 2\cdot 0 + 3\cdot 1 - 5 - \frac{1}{4}(-4) = -1$

$y[2] = 2\delta[2] + 3\delta[1] - y[1] - \frac{1}{4}y[0] = 2\cdot 0 + 3\cdot 0 - (-1) - \frac{1}{4}(5) = -\frac{1}{4}$

(d) Write a set of equations which will allow you to find the unknowns in Part (b). Note: You do not have to solve these equations.

$y[0] = A_1(-\frac{1}{2})^0 + A_2\cdot 0\cdot(-\frac{1}{2})^0 + k\delta[0] = A_1 + k = 5$

$y[1] = A_1(-\frac{1}{2})^1 + A_2\cdot 1\cdot(-\frac{1}{2})^1 + k\delta[1] = -\frac{1}{2}A_1 - \frac{1}{2}A_2 = -1$

$y[2] = A_1(-\frac{1}{2})^2 + A_2\cdot 2\cdot(-\frac{1}{2})^2 + k\delta[2] = \frac{1}{4}A_1 + \frac{1}{2}A_2 = -\frac{1}{4}$

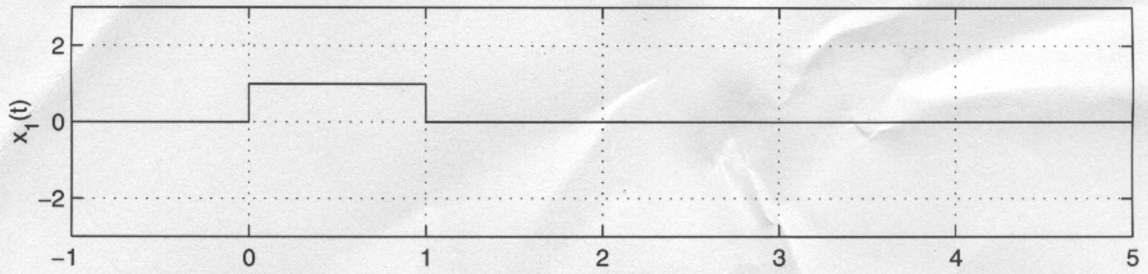
Solution (you don't need to solve):

$A_1 = 5$ $\quad y[n] = 5(-\frac{1}{2})^n - 3n(-\frac{1}{2})^n$ for $n \geq 0$

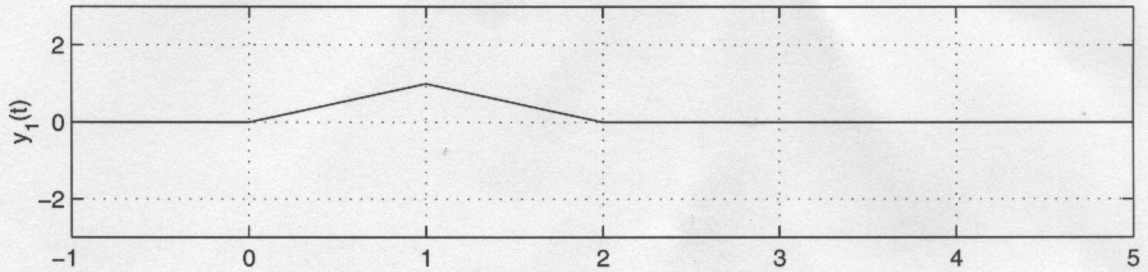
$A_2 = -3$

$k = 0$

4. Consider a linear time-invariant system. When the input to the system is $x_1(t)$ shown in Figure (a) below, the output is $y_1(t)$ shown in Figure (b).

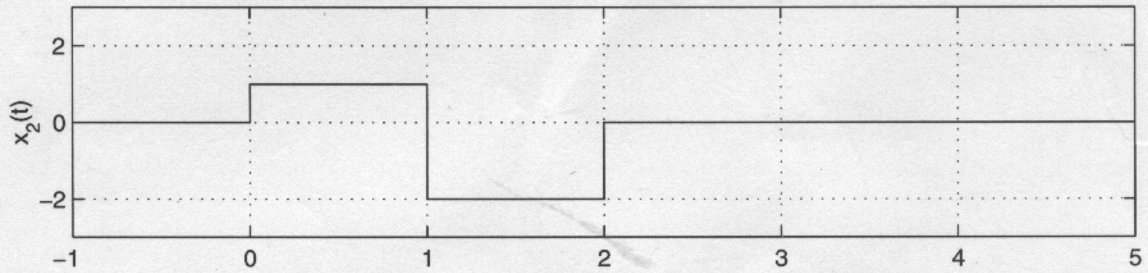


(a)

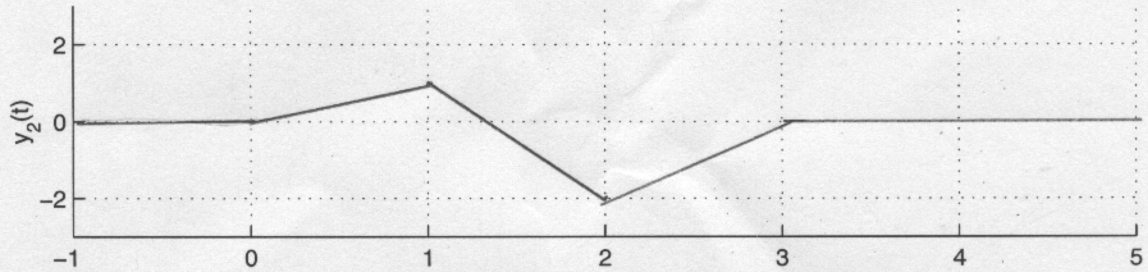


(b)

Determine and sketch carefully the response of the system to the input $x_2(t)$ shown in Figure (c). You can use Figure (d) for your sketch. Explain your answer. Note: This is easy if you consider the properies of linear, time-invariant systems.



(c)

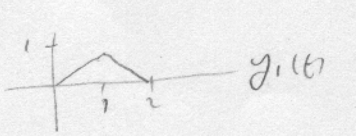


(d)

$x_1(t) = u(t) - u(t-1)$

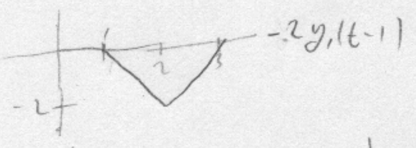
$x_2(t) = u(t) - 3u(t-1) + 2u(t-2) = [u(t) - u(t-1)] - 2[u(t-1) - u(t-2)]$

$= x_1(t) - 2x_1(t-1)$




Because system is linear and time invariant

$y_2(t) = y_1(t) - 2y_1(t-1)$






Add these two together