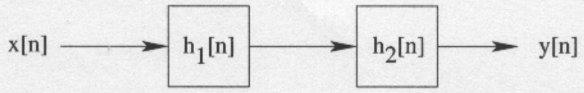
EE 341 - Exam 2

October 27, 2004

Name: ______________________________

Closed book. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work. You may use one page of notes, the handouts on time-domain solutions to differential and difference equations, and a calculator.

1. Consider the cascade interconnection of two LTI systems shown below:



The impulse response of system 1 is

$$h_1[n] = u[n] - u[n-2]$$

The impulse response of system 2 is

$$h_2[n] = \frac{1}{2}\delta[n] - \delta[n-1] + \delta[n-2] - \frac{1}{2}\delta[n-3]$$

Find the overall impulse response of the system; i.e., find $y[n]$ when $x[n] = \delta[n]$.

$h_1[n] = \delta[n] + \delta[n-1]$

Two easy ways to do:

1) $h[n] = h_1[n] * h_2[n]$ $\quad h_1[n] = \{1, 1\}$

$h_2[n] = \{\frac{1}{2} \ -1 \ 1 \ -\frac{1}{2}\}$

Flip $h_1[n]$ and slide

$h[0]$:
$$\begin{array}{cccc} \frac{1}{2} & -1 & 1 & -\frac{1}{2} \\ 1 & 1 & & \\ \hline \frac{1}{2} & & & \end{array} = \frac{1}{2}$$

$h[1]$:
$$\begin{array}{cccc} \frac{1}{2} & -1 & 1 & -\frac{1}{2} \\ 1 & 1 & & \\ \hline \frac{1}{2} & -1 & & \end{array} = -\frac{1}{2}$$

$h[2]$:
$$\begin{array}{cccc} \frac{1}{2} & -1 & 1 & -\frac{1}{2} \\ & 1 & 1 & \\ \hline & -1 & 1 & \end{array} = 0$$

$h[3]$:
$$\begin{array}{cccc} \frac{1}{2} & -1 & 1 & -\frac{1}{2} \\ & & 1 & 1 \\ \hline & & 1 & -\frac{1}{2} \end{array} = \frac{1}{2}$$

$h[4]$ $\quad \frac{1}{2} \quad -1 \quad 1 \quad -\frac{1}{2}$

$\qquad\qquad\qquad 1 \quad 1$

$\qquad\qquad -1/2 \quad = -1/2$

$$h[n] = \{ \tfrac{1}{2} \quad -\tfrac{1}{2} \quad 0 \quad \tfrac{1}{2} \quad -\tfrac{1}{2} \} = \tfrac{1}{2}\delta[0] - \tfrac{1}{2}\delta[1] + \tfrac{1}{2}\delta[3] - \tfrac{1}{2}\delta[4]$$

2) Input to system 1 is $\delta[n]$, output is $\delta[n] + \delta[n-1]$

Input to system 2 is $\delta[n] + \delta[n-1]$, output is $h_2[n] + h_2[n-1]$

$$h[n] = h_2[n] + h_2[n-1]$$

$$= \tfrac{1}{2}\delta[n] - \delta[n-1] + \delta[n-2] - \tfrac{1}{2}\delta[n-3]$$

$$+ \quad \tfrac{1}{2}\delta[n-1] - \delta(n-2) + \delta(n-3) - \tfrac{1}{2}\delta[n-4]$$

$$= \tfrac{1}{2}\delta[n] - \tfrac{1}{2}\delta[n-1] + 0\,\delta[n-2] + \tfrac{1}{2}\delta[n-3] - \tfrac{1}{2}\delta[n-4]$$

$$= \{ \tfrac{1}{2} \quad -\tfrac{1}{2} \quad 0 \quad \tfrac{1}{2} \quad -\tfrac{1}{2} \}$$

2. Consider the signal

$$x(t) = 3\cos(t) + 5\sin(5t - \frac{\pi}{6}) - 2\cos(8t - \frac{\pi}{3})$$

(a) What is the period of $x(t)$

Period of $\cos(t)$ is 2π; Period of $\sin(5t - \frac{\pi}{6})$ is $\frac{2\pi}{5}$.
Period of $\cos(8t - \pi/3)$ is $\frac{2\pi}{8}$
Fundamental period is 2π

(b) What is the fundamental frequency of $x(t)$

$$\omega_0 = \frac{2\pi}{T} = 1$$

(c) Write the exponential Fourier series for $x(t)$.

$$x(t) = 3\,\frac{e^{jt} + e^{-jt}}{2} + 5\,\frac{e^{j(5t-\frac{\pi}{6})} - e^{-j(5t-\frac{\pi}{6})}}{2j} - 2\,\frac{e^{j(8t-\frac{\pi}{3})} + e^{-j(8t-\frac{\pi}{3})}}{2}$$

$$= \frac{3}{2}e^{jt} + \frac{3}{2}e^{-jt} - j\frac{5}{2}e^{j(5t-\frac{\pi}{6})} + j\frac{5}{2}e^{-j(5t-\frac{\pi}{6})} - e^{j(8t-\frac{\pi}{3})} - e^{-j(8t-\frac{\pi}{3})}$$

$$\boxed{x(t) = -e^{j\frac{\pi}{3}}e^{-j8t} + j\frac{5}{2}e^{j\frac{\pi}{6}}e^{-j5t} + \frac{3}{2}e^{-jt} + \frac{3}{2}e^{jt} - j\frac{5}{2}e^{-j\frac{\pi}{6}}e^{j5t} - e^{-j\frac{\pi}{3}}e^{j8t}}$$

$C_{-8} = -e^{j\frac{\pi}{3}} = e^{-j\frac{2\pi}{3}}$ $\quad C_{-5} = j\frac{5}{2}e^{j\frac{\pi}{6}} = \frac{5}{2}e^{j\frac{2\pi}{3}}$

$C_{-1} = \frac{3}{2}$ $\quad C_1 = \frac{3}{2}$ $\quad C_5 = -j\frac{5}{2}e^{-j\frac{\pi}{6}} = \frac{5}{2}e^{-j\frac{2\pi}{3}}$

$C_8 = -e^{-j\frac{\pi}{3}} = e^{j\frac{2\pi}{3}}$ $\quad$ All other $C_k = 0$

3. Compute the Fourier transforms of the following signals. Note: You should not have to do any integrals.

   (a) $x(t) = t\, e^{-2t} \cos(4\, t)\, u(t)$

$$e^{-2t} u(t) \Rightarrow \frac{1}{j\omega + 2}$$

$$\cos(4t)\, e^{-2t} u(t) \Rightarrow \frac{1/2}{j(\omega+4)+2} + \frac{1/2}{j(\omega-4)+2}$$

$$t \cos(4t)\, e^{-t} u(t) \Rightarrow j \frac{d}{d\omega}\left(\quad \right) = \frac{-\frac{1}{2} j}{(j(\omega+4)+2)^2} + \frac{-\frac{1}{2} j}{(j(\omega-4)+2)^2}$$

   (b)

$$x(t) = \begin{cases} 1 + \cos(\pi\, t) & |t| < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = P_2(t)\,(1 + \cos(\pi t))$$

$$= P_2(t) + P_2(t) \cos(\pi t)$$

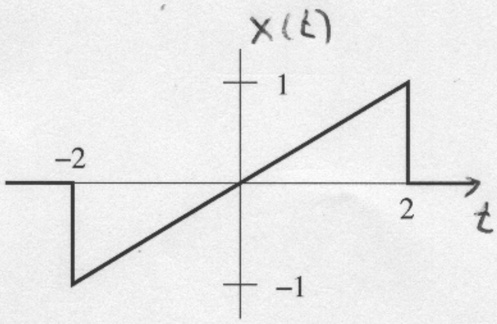
$$P_2(t) \Rightarrow 2 \operatorname{sinc}\left(\frac{2\omega}{2\pi}\right) = 2 \operatorname{sinc}\left(\frac{\omega}{\pi}\right)$$

$$P_2(t) \cos(\pi t) \Rightarrow \frac{1}{2}\left(2 \operatorname{sinc}\left(\frac{\omega+\pi}{\pi}\right) + 2 \operatorname{sinc}\left(\frac{\omega-\pi}{\pi}\right)\right)$$

$$x(t) \Rightarrow 2 \operatorname{sinc}\left(\frac{\omega}{\pi}\right) + \operatorname{sinc}\left(\frac{\omega+\pi}{\pi}\right) + \operatorname{sinc}\left(\frac{\omega-\pi}{\pi}\right)$$

4. Below are some continuous-time signals. Answer the questions about the Fourier transforms of the signals. Be sure to explain your answers.

(a)



i. Is $X(\omega)$ real? X(t) is odd. X(w) for an odd signal is imaginary. ∴ X(w) is not real

ii. Is $X(\omega)$ imaginary?

From (i), X(w) is imaginary

iii. Is $\int_{-\infty}^{\infty} X(\omega)\, dw$ equal to zero?

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w) e^{jwt} dw \Rightarrow x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(w)\, dw \Rightarrow \int_{-\infty}^{\infty} X(w)\, dw = 2\pi x(0)$

$\int_{-\infty}^{\infty} X(w)\, dw = 2\pi x(0) = 0$

iv. Is $X(\omega)$ periodic?

$X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$ For X(w) to be periodic, x(t) must have δ fnc. x(t) does not have δ, so X(w) not periodic

(b) $x(t) = \delta(t-2)$

i. Is $X(\omega)$ real?

$X(w) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt = \int_{-\infty}^{\infty} \delta(t-2) e^{-jwt} dt = e^{-jw2}$

$= \cos(2w) + j\sin(2w)$. X(w) is complex - neither real or imaginary

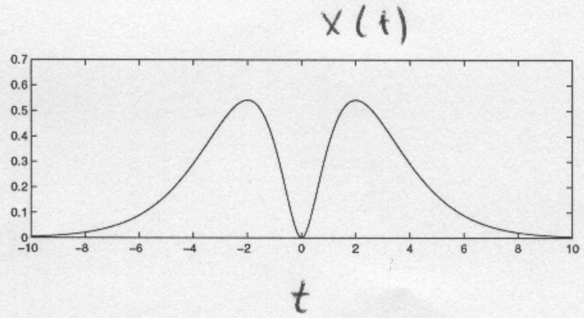
ii. Is $X(\omega)$ imaginary?

iii. Is $\int_{-\infty}^{\infty} X(\omega)\, dw$ equal to zero?

$\int_{-\infty}^{\infty} X(w)\, dw = 2\pi x(0) = 0$

iv. Is $X(\omega)$ periodic? Yes - with a period of π

(c)



i. Is $X(\omega)$ real?

$x(t)$ even $\Rightarrow$ $X(\omega)$ real, so yes

ii. Is $X(\omega)$ imaginary?

From (i), no

iii. Is $\int_{-\infty}^{\infty} X(\omega)\, dw$ equal to zero?

$$\int_{-\infty}^{\infty} X(\omega)\, d\omega = 2\pi\, x(0) = 0$$

iv. Is $X(\omega)$ periodic?

No - $x(t)$ does not have a δ function