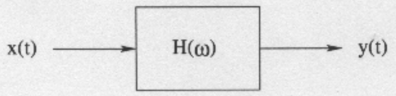
EE 341 - Exam 3

November 22, 2004

Name: Solutions

Closed book. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work. You may use one page of notes and a calculator.

1. Consider the following system:



The frequency response of the filter is

$$H(\omega) = 1 - e^{-j\omega/2}$$

The input $x(t)$ is:

$$x(t) = 5 + 2\cos(\pi t) + 3\sin(2\pi t)$$

(a) Find $X(\omega)$, the Fourier transform of $x(t)$.

$1 \Leftrightarrow 2\pi\delta(\omega)$ $\cos(\omega_0 t) \Leftrightarrow \pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$

$\sin(\omega_0 t) \Leftrightarrow j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$

$X(\omega) = 10\pi\delta(\omega) + 2\pi\delta(\omega+\pi) + 2\pi\delta(\omega-\pi) + j3\pi\delta(\omega+2\pi) - j3\pi\delta(\omega-2\pi)$

(b) Find $Y(\omega)$. $Y(\omega) = H(\omega)X(\omega) = (1-e^{-j\omega/2}) \times$

$(1-e^{-j\omega/2})\delta(\omega) = (1-e^{0})\delta(\omega) = 0$

$(1-e^{-j\omega/2})\delta(\omega+\pi) = (1-e^{+j\pi/2})\delta(\omega+\pi) = (1-j)\delta(\omega+\pi)$

Similarly, $(1-e^{-j\omega/2})\delta(\omega-\pi) = (1+j)\delta(\omega-\pi)$, $(1-e^{-j\omega/2})\delta(\omega+2\pi) = 2\delta(\omega+2\pi)$

$(1-e^{-j\omega/2})\delta(\omega-2\pi) = 2\delta(\omega-2\pi)$

(c) Find $y(t)$.

$$\boxed{Y(\omega) = 2\pi(1-j)\delta(\omega+\pi) + 2\pi(1+j)\delta(\omega-\pi) + j6\pi\delta(\omega+2\pi) - j6\pi\delta(\omega-2\pi)}$$

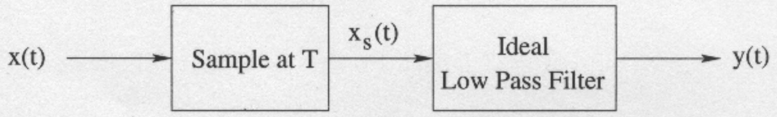
$x(t) = \cos(\omega_0 t) \Rightarrow y(t) = |H(\omega_0)|\cos(\omega_0 t + \measuredangle H(\omega_0))$

5: $\omega_0 = 0$, $H(0) = 0$ $2\cos(\pi t)$: $\omega_0 = \pi$, $H(\pi) = 1+j$, $|H(\pi)| = \sqrt{2}$ $\measuredangle H(\pi) = \frac{\pi}{4}$

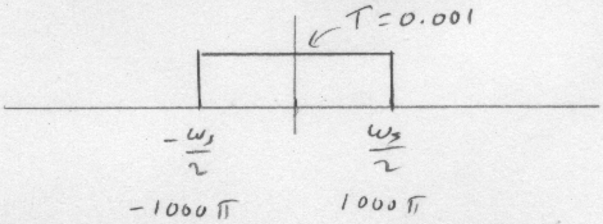
$3\sin(\pi t)$: $\omega_0 = 2\pi$ $H(2\pi) = 2$

$y(t) = 2\sqrt{2}\cos(\pi t + \frac{\pi}{4}) + 6\sin(2\pi t)$

2. Consider the following system:



(a) Let $T = 0.001$ sec. Sketch the frequency response $|H(\omega)|$ of the low pass filter which will make it an ideal reconstruction filter.



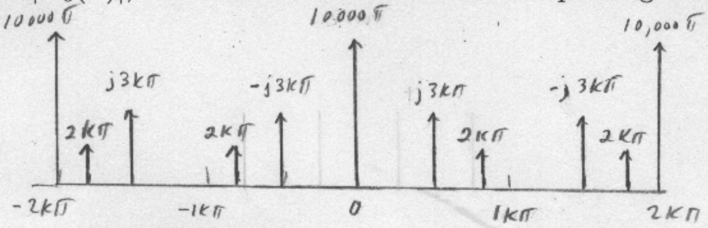
$T = 0.001$

$\omega_s = \frac{2\pi}{T} = 2000\pi$

(b) Let $x(t) = 5 + 2\cos(800\pi t) + 3\sin(1500\pi t)$. Sketch $|X(\omega)|$, the magnitude of the Fourier transform of $x(t)$.

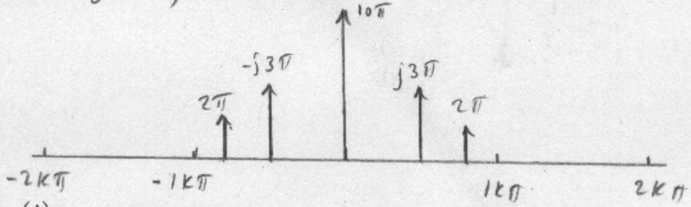
$X(\omega) = 10\pi\,\delta(\omega) + 2\pi\,\delta(\omega+800\pi) + 2\pi\,\delta(\omega-800\pi) + j3\pi\,\delta(\omega+1500\pi) - j3\pi\,\delta(\omega-1500\pi)$

(c) Sketch $|X_s(\omega)|$, the Fourier transform of the sampled signal $x_s(t)$.



(d) Sketch $|Y(\omega)|$, the magnitude of the Fourier transform of $y(t)$.

Keep everything between -1000π and 1000π, multiply amplitude by 0.001



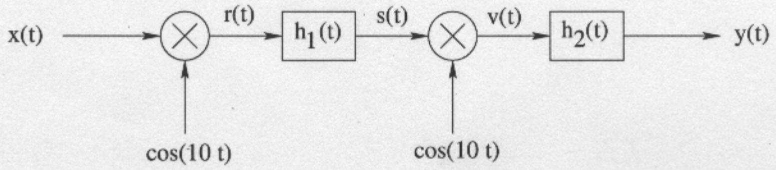
(e) Find $y(t)$.

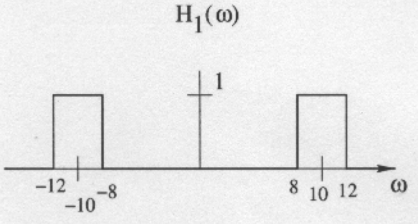
$Y(\omega) = 10\pi\,\delta(\omega) + 2\pi\,\delta(\omega+800\pi) + 2\pi\,\delta(\omega-800\pi) - j3\pi\,\delta(\omega+500\pi) + j3\pi\,\delta(\omega-500\pi)$

$= 5(2\pi\delta(\omega)) + 2[\pi(\delta(\omega+800\pi) + \delta(\omega-800\pi))] - 3[j\pi(\delta(\omega+500\pi) - \delta(\omega-500\pi))]$

$y(t) = 5 + 2\cos(800\pi t) - 3\sin(500\pi t)$

3. Consider the follwing system:





The input to the system is

$$x(t) = \text{sinc}\left(\frac{3t}{\pi}\right)$$

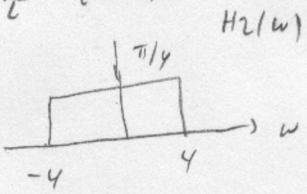
The impulse response $h_2(t)$ is

$$h_2(t) = \text{sinc}\left(\frac{4t}{\pi}\right)$$

(a) Find $H_2(\omega)$, the Fourier transform of $h_2(t)$

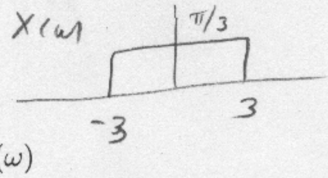
$\tau \,\text{sinc}\left(\frac{\tau t}{2\pi}\right) \Leftrightarrow 2\pi\, p_\tau(\omega)$ $\quad$ $\text{sinc}\left(\frac{\tau t}{2\pi}\right) \Leftrightarrow \frac{2\pi}{\tau}\, p_\tau(\omega)$

$h_2(t) = \text{sinc}\left(\frac{8t}{2\pi}\right) \Leftrightarrow H_2(\omega) = \frac{2\pi}{8}\, p_8(\omega) = \frac{\pi}{4}\, p_8(\omega)$



(b) Find $X(\omega)$, the Fourier transform of $x(t)$

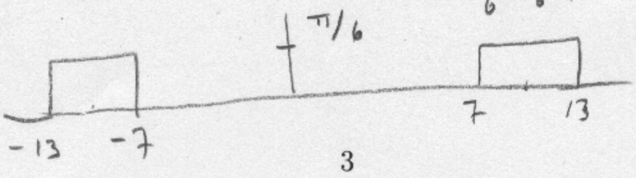
$x(t) = \text{sinc}\left(\frac{6t}{2\pi}\right) \Leftrightarrow X(\omega) = \frac{2\pi}{6}\, p_6(\omega) = \frac{\pi}{3}\, p_6(\omega)$



(c) Sketch $R(\omega)$

$r(t) = \cos(10t)\, x(t)$ $\quad$ $R(\omega) = \frac{1}{2} X(\omega+10) + \frac{1}{2} X(\omega-10)$

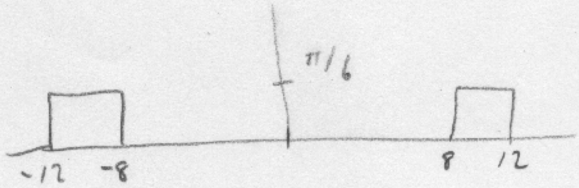
$= \frac{\pi}{6} p_6(\omega+10) + \frac{\pi}{6} p_6(\omega-10)$



(d) Sketch $S(\omega)$

$$S(\omega) = H_1(\omega)\, R(\omega) = \frac{\pi}{6} P_4(\omega+10)\, P_6(\omega+10) + \frac{\pi}{6} P_4(\omega-10)\, P_6(\omega-10)$$

$$= \frac{\pi}{6} P_4(\omega+10) + \frac{\pi}{6} P_4(\omega-10)$$



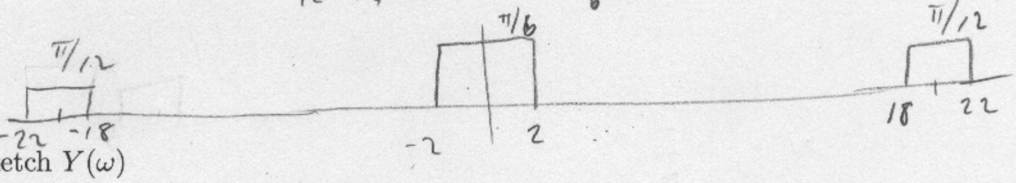
(e) Sketch $V(\omega)$

$$v(t) = \cos(10t)\, s(t)$$

$$V(\omega) = \frac{1}{2} S(\omega+10) + \frac{1}{2} S(\omega-10)$$

$$= \frac{\pi}{12} P_4(\omega+20) + \frac{\pi}{12} P_4(\omega) + \frac{\pi}{12} P_4(\omega) + \frac{\pi}{12} P_4(\omega-20)$$

$$= \frac{\pi}{12} P_4(\omega+20) + \frac{\pi}{6} P_4(\omega) + \frac{\pi}{12} P_4(\omega-20)$$

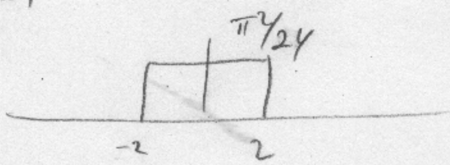


(f) Sketch $Y(\omega)$

$$Y(\omega) = H_2(\omega)\, V(\omega)$$

$$= \frac{\pi}{4} P_8(\omega) \left[\frac{\pi}{12} P_4(\omega+20) + \frac{\pi}{6} P_4(\omega) + \frac{\pi}{12} P_4(\omega-20)\right)$$

$$= \frac{\pi^2}{24} P_4(\omega)$$



(g) Find $y(t)$.

$$2\pi\, P_\tau(\omega) \Leftrightarrow \tau\, \text{sinc}\left(\frac{\tau t}{2\pi}\right)$$

$$\frac{\pi^2}{24} P_4(\omega) = \frac{\pi}{48}\left(2\pi\, P_4(\omega)\right) \Leftrightarrow \frac{\pi}{48}\left(4\, \text{sinc}\left(\frac{4t}{2\pi}\right)\right) = \frac{\pi}{12} \text{sinc}\left(\frac{2t}{\pi}\right)$$

$$y(t) = \frac{\pi}{9} \text{sinc}\left(\frac{2t}{\pi}\right)$$