Problem 3.1 Part (e)

$$y[n+2] + 1/2y[n+1] + 1/4y[n] = x[n+1] - x[n]$$
$$y[n] + 1/2y[n-1] + 1/4y[n-2] = x[n-1] - x[n-2]$$

$$\lambda^2 + (1/2)\lambda + (1/4) = 0$$
$$\lambda = -\left(\frac{1}{4}\right) \pm \left(\frac{j\sqrt{3}}{4}\right)$$

$$
\begin{aligned}
\lambda &= |\lambda|e^{j\angle\lambda} \\
\lambda_1 &= 0.5e^{j2\pi/3} \\
\lambda_2 &= 0.5e^{-j2\pi/3}
\end{aligned}
$$

$$
\begin{aligned}
h_h[n] &= (A_1\lambda_1^n + A_2\lambda_2^n)u[n] \\
h_p[n] &= K\delta(n)
\end{aligned}
$$

$$
\begin{aligned}
h[n] &= h_h[n] + h_p[n] \\
&= (A_1\lambda_1^n + A_2\lambda_2^n)u[n] + K\delta(n)
\end{aligned}
$$

$$
\begin{array}{llllll}
h[0] &= A_1 & +A_2 & +K &= 0 \\
h[1] &= A_1 0.5e^{j2\pi/3} & +A_2 0.5e^{-j2\pi/3} & &= 1 \\
h[2] &= A_1 0.25e^{j4\pi/3} & +A_2 0.25e^{-j4\pi/3} & &= -1.5
\end{array}
$$

From MATLAB:

$$
\begin{aligned}
A_1 &= 2 - j2.31 &= 3.05e^{-j0.857} \\
A_2 &= 2 + j2.31 &= 3.05e^{j0.857} \\
K &= -4
\end{aligned}
$$

$$
\begin{aligned}
h[n] &= \left(3.05e^{-j0.857}(0.5)^n e^{j2\pi n/3} + 3.05e^{j0.857}(0.5)^n e^{-j2\pi n/3}\right)u[n] - 4\delta(n) \\
&= \left(3.05(0.5)^n \left(e^{j(2\pi n/3 - 0.875)} + e^{-(j2\pi n/3 - 0.875)}\right)\right)u[n] - 4\delta(n) \\
&= 6.10(0.5)^n cos(2\pi n/3 - 0.875)u[n] - 4\delta(n)
\end{aligned}
$$