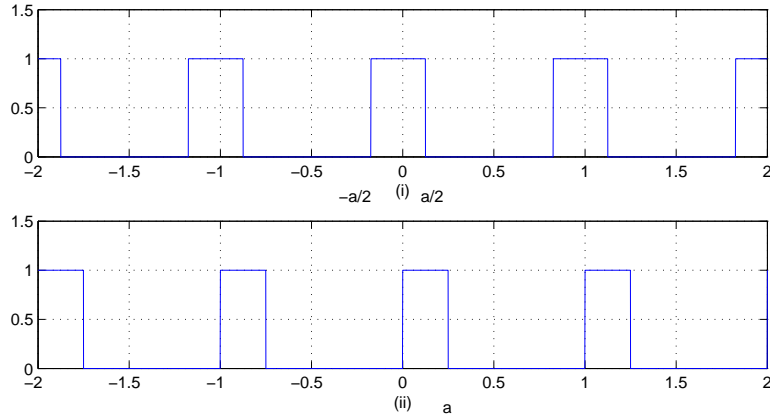


EE 341

Effect of phase on Fourier series

Consider the two plots below:



The two signals differ only in a time shift – i.e., they differ only in phase.

The Fourier coefficients for Figure (i) (for $c_k \neq 0$) are:

$$c_k = \frac{1}{\pi k} \sin(\pi k a / T)$$

The Fourier coefficients for Figure (ii) (for $c_k \neq 0$) are:

$$c_k = \frac{j}{2\pi k} \left(e^{-j2\pi k a / T} - 1 \right)$$

We can do some algebra on this:

$$\begin{aligned} c_k &= \frac{j}{2\pi k} \left(e^{-j2\pi k a / T} - 1 \right) \\ &= \frac{j}{2\pi k} e^{-j\pi k a / T} \left(e^{-j\pi k a / T} - e^{j\pi k a / T} \right) \\ &= \frac{1}{2\pi k} e^{-j\pi k a / T} \left(\frac{e^{j\pi k a / T} - e^{-j\pi k a / T}}{2j} \right) \\ &= \frac{1}{\pi k} e^{-j\pi k a / T} \sin(\pi k a / T) \end{aligned}$$

Thus the two series are the same except for a phase difference of

$$e^{-j\pi k a / T}$$