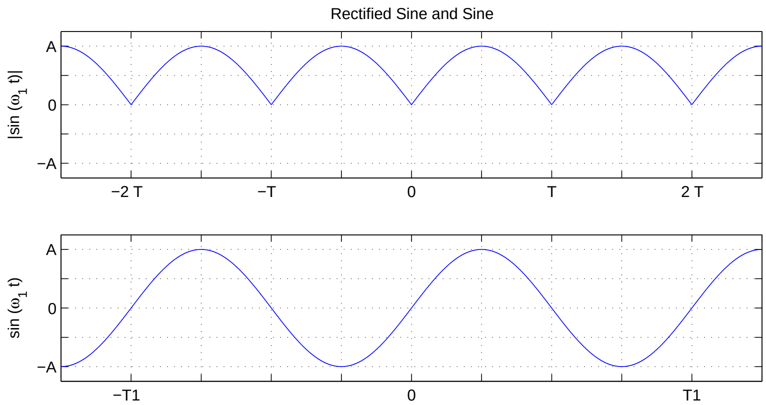
EE 341

Fourier Series for Rectified Sine Wave

Consider the signal

$$x(t) = A|\sin(\omega_1 t)|$$



The period of the sinusoid (inside the absolute value symbols) is $T_1 = 2\pi/\omega_1$. The period of the rectified sinusoid is one half of this, or $T = T_1/2 = \pi/\omega_1$. Therefore,

$$\omega_1 = \frac{2\pi}{T_1} = \frac{\pi}{T} = \frac{\omega_o}{2}$$

Thus we can write

$$x(t) = A\left|\sin\left(\frac{\omega_o t}{2}\right)\right|$$

The Fourier series coefficients are:

$$\begin{aligned} c_k^x &= \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega_o t}\, dt \\ &= \frac{1}{T}\int_0^T A\left|\sin\left(\frac{\omega_o t}{2}\right)\right| e^{-jk\omega_o t}\, dt \end{aligned}$$

From 0 to T, $|sin(\omega_o t/2)| = sin(\omega_o t/2)$. Also, replace ω_o by $2\pi/T$:

$$c_k^x = \frac{1}{T}\int_0^T A\sin\left(\frac{\pi t}{T}\right) e^{-j2\pi kt/T}\, dt$$

$$
\begin{aligned}
c_k^x &= \frac{A}{T}\int_0^T \left(\frac{e^{j\pi t/T}-e^{-j\pi t/T}}{j2}\right) e^{-j2\pi kt/T}\,dt \\
&= \frac{A}{j2T}\int_0^T \left(e^{j\pi t(1-2k)/T}-e^{-j\pi t(1+2k)/T}\right)\,dt \\
&= \frac{A}{j2T}\left[\frac{e^{j\pi t(1-2k)/T}}{j\pi(1-2k)/T}-\frac{e^{-j\pi t(1+2k)/T}}{j\pi(1+2k)/T}\right]_0^T \\
&= -\frac{A}{2T}\left[\frac{e^{j\pi(1-2k)}}{\pi(1-2k)/T}+\frac{e^{-j\pi(1+2k)}}{\pi(1+2k)/T}-\frac{1}{\pi(1-2k)/T}-\frac{1}{\pi(1+2k)/T}\right]
\end{aligned}
$$

But $e^{j\pi(1-2k)} = e^{j\pi}\,e^{-j2\pi k}$. $e^{j\pi} = -1$ and e$^{-j2\pi k} = 1$, so $e^{j\pi(1-2k)} = -1$. Similarly, $e^{-j\pi(1+2k)} = -1$, so:

$$
\begin{aligned}
c_k^x &= -\frac{A}{2T}\left[\frac{-1}{\pi(1-2k)/T}+\frac{-1}{\pi(1+2k)/T}-\frac{1}{\pi(1-2k)/T}-\frac{1}{\pi(1+2k)/T}\right] \\
&= -\frac{A}{2}\left[\frac{-2}{\pi(1-2k)}-\frac{2}{\pi(1+2k)}\right] \\
&= \frac{A}{\pi}\left[\frac{1}{1-2k}+\frac{1}{1+2k}\right] \\
&= \frac{2A}{\pi}\left(\frac{1}{1-4k^2}\right)
\end{aligned}
$$

For $k = 0$

$$
\begin{aligned}
c_0^x &= \frac{1}{T}\int_0^T A\sin\left(\frac{\pi\, t}{T}\right)dt \\
&= \frac{A}{T}\left[\frac{-\cos(\pi t/T)}{\pi/T}\right]_0^T \\
&= \frac{2A}{\pi}
\end{aligned}
$$

Note: Because the signal is even, you could also find c_k^x by:

$$c_k^x = \frac{2}{T}\int_0^{T/2} x(t)\ \cos(k\omega_o t)\ dt$$

For an odd signal,

$$c_k^x = -\frac{j2}{T}\int_0^{T/2} x(t)\ \sin(k\omega_o t)\ dt$$