EE 341 - Exam 1

September 28, 2005

Name: Solutions

Closed book. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work. You may use one page of notes and a calculator.

1. Determine which of the following signals are periodic. If the signal is periodic, determine the period.

(a) The continuous-time signal $x(t) = \cos(10\pi t) - \cos(20t)$.

$\omega_1 = 10\pi \quad T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{10\pi} = \frac{1}{5}$

$\omega_2 = 20 \quad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{20} = \frac{\pi}{10} \qquad \frac{T_1}{T_2} = \frac{\frac{1}{5}}{\frac{\pi}{20}} = \frac{4}{\pi}$ Not rational
Not periodic

(b) The continuous-time signal $x(t) = \cos(2t) - \sqrt{2}\cos(2t - \pi/4)$.

$\omega_1 = 2 \quad T_1 = \frac{2\pi}{\omega_1} = \pi$

$\omega_2 = 2 \quad T_2 = \frac{2\pi}{\omega_2} = \pi \qquad \frac{T_1}{T_2} = \frac{r}{q} = 1 \qquad T = qT_1 = 1 \cdot \pi = \pi$

Periodic, $T = \pi$ sec

(c) The discrete-time signal $x[n] = 4 - 3\sin(12\pi n) + 2\cos(28\pi n)$.

$\Omega_1 = 12\pi \quad N_1 = \frac{2\pi k}{\Omega_1} = \frac{2\pi k}{12\pi} = \frac{k}{6} = 1$ (k=6 is smallest integer to make N_1 an integer)

$\Omega_2 = 28\pi \quad N_2 = \frac{2\pi k}{\Omega_2} = \frac{2\pi k}{28\pi} = \frac{k}{14} = 1$

$\frac{N_1}{N_2} = \frac{r}{q} = 1 \Rightarrow N = qN_1 = 1 \cdot 1 = 1$

Periodic, $N = 1$ rad/sample

2. Consider the system $y(t) = x(\alpha t)$. Answer the following questions. Justify your answers with brief explanations.

   (a) For what values of α is the system linear? $y_1(t) = x_1(\alpha t)$ $y_2(t) = x_2(\alpha t)$

   Let $\tilde{x}(t) = a_1 x_1(t) + a_2 x_2(t)$ $\tilde{y}(t) = \tilde{x}(\alpha t) = a_1 x_1(\alpha t) + a_2 x_2(\alpha t)$

   $= a_1 y_1(t) + a_2 y_2(t)$ True for all α

   Linear for any α

   (b) For what values of α is the system time invariant?

   Let $\tilde{x}(t) = x(t - t_0)$ $\tilde{y}(t) = \tilde{x}(\alpha t) = x(\alpha t - t_0)$

   $y(t-t_0) = x(\alpha(t-t_0))$ Time invariant if $\alpha(t-t_0) = \alpha t - t_0$

   Need $\alpha = 1$ for this to be true

   (c) For what values of α is the system causal?

   $y(t) = x(\alpha t)$ Causal if $t \geq \alpha t$

   Positive t: $t \geq \alpha t \Rightarrow 1 \geq \alpha$ Negative t: $t \geq \alpha t \Rightarrow 1 \leq \alpha$

   The only α for which both are true is $\alpha = 1$

   (d) For what values of α is the system memoryless?

   $y(t) = x(\alpha t)$ Memoryless if $t = \alpha t \Rightarrow \alpha = 1$

3. Consider the system $y(t) = x(t - \alpha)$. Answer the following questions. Justify your answers with brief explanations.

   (a) For what values of α is the system linear? $y_1(t) = x_1(t-\alpha)$ $y_2(t) = x_2(t-\alpha)$

   Let $\tilde{x}(t) = a_1 x_1(t) + a_2 x_2(t)$ $\tilde{y}(t) = \tilde{x}(t-\alpha) = a_1 x_1(t-\alpha) + a_2 x_2(t-\alpha)$

   $= a_1 y_1(t) + a_2 y_2(t)$ True for all α

   Linear for any α

   (b) For what values of α is the system time invariant?

   Let $\tilde{x}(t) = x(t - t_0)$ $\tilde{y}(t) = \tilde{x}(t-\alpha) = x(t - \alpha - t_0)$

   $y(t-t_0) = x(t - t_0 - \alpha) = x(t - \alpha - t_0) = \tilde{y}(t)$ True for all α

   Time invarient for all α

   (c) For what values of α is the system causal?

   $y(t) = x(t-\alpha)$ Causal if $t \geq t - \alpha$ $0 \geq -\alpha$ $\alpha \geq 0$

   (d) For what values of α is the system memoryless?

   $y(t) = x(t-\alpha)$ Memoryless if $t = t - \alpha$ $0 = \alpha$

4. The impulse response of a linear time-invariant continuous-time system is $h(t) = e^{-t}u(t)$.

(a) Find the step response of the system.

$$g(t) = \int_{-\infty}^{t} h(\lambda)\, d\lambda = \int_{-\infty}^{t} e^{-\lambda} u(\lambda)\, d\lambda \qquad g(t) = 0 \text{ for } t < 0$$

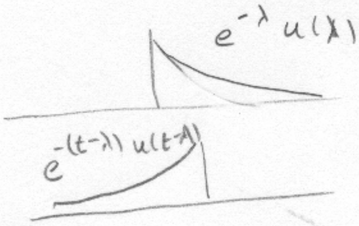
$$g(t) = -e^{-\lambda}\Big|_0^t = -e^{-t} - (-1) = 1 - e^{-t} \quad \text{for } t \geq 0$$

$$g(t) = (1 - e^{-t})\, u(t)$$

(b) Find the response of the system to the input $x(t) = e^{-t}u(t)$.

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} e^{-\lambda} u(\lambda)\, e^{-(t-\lambda)} u(t-\lambda)\, d\lambda$$



No overlap for $t < 0$ ∴ $y(0) = 0$ for $t < 0$

Integrate from 0 to t for $t \geq 0$

$$y(t) = \int_0^t e^{-\lambda} e^{-(t-\lambda)}\, d\lambda = \int_0^t e^{-t}\, d\lambda = e^{-t}\int_0^t d\lambda = te^{-t}$$

$$y(t) = te^{-t}u(t)$$

5. Below are the impulse responses $h[n]$ and the input $x[n]$ to two linear time-invariant discrete-time systems. Find the outputs $y[n]$ of the systems.

(a) $h[n] = \{\overset{\Downarrow}{1}, 2, 3\}$, $\quad x[n] = \{-2, \overset{\Downarrow}{1}, 3\}$ $\qquad y(n) = h(n) * x(n) = \sum h(k)\, x(n-k)$

$n=-1$:

$$\begin{array}{rrrrl} & 1 & 2 & 3 & \leftarrow h(k) \\ 3 & 1 & -2 & & \leftarrow x(-1-k) \\ \hline & -2 & & & \end{array}$$

$n=0$:

$$\begin{array}{rrr} 1 & 2 & 3 \\ 3\ \ 1 & -2 & \\ \hline 1 & -4 & \end{array} = -3$$

$n=1$:

$$\begin{array}{rrr} 1 & 2 & 3 \\ 3 & 1 & -2 \\ \hline 3 & 2 & -6 \end{array} = -1$$

$n=2$:

$$\begin{array}{rrrr} 1 & 2 & 3 & \\ & 3 & 1 & -2 \\ \hline & 6 & 3 & \end{array} = 9$$

$n=3$:

$$\begin{array}{rrrrr} 1 & 2 & 3 & & \\ & & 3 & 1 & -2 \\ \hline & & 9 & & \end{array}$$

$y(n) = \{-2, \overset{\downarrow}{-3}, -1, 9, 9\}$

(b) $h[n] = \delta[n] - \delta[n-1]$, $\quad x[n] = (-1/2)^n u[n]$

$$y(n) = h(n) * x(n) = [\delta(n) - \delta(n-1)] * x(n)$$

$$= \delta(n) * x(n) - \delta(n-1) * x(n)$$

$$= x(n) - x(n-1)$$

$$= \left(-\tfrac{1}{2}\right)^n u(n) - \left(-\tfrac{1}{2}\right)^{n-1} u(n-1)$$

For $n = 0$: $\quad y(0) = \left(-\tfrac{1}{2}\right)^0 u(0) - \left(-\tfrac{1}{2}\right)^{-1} u(-1) = 1$

For $n \geq 1$: $\quad y(n) = \left(-\tfrac{1}{2}\right)^n u(n) - \left(-\tfrac{1}{2}\right)^{n-1} u(n-1)$

$$= \left(-\tfrac{1}{2}\right)^n - \left(-\tfrac{1}{2}\right)^{n-1} = \left(-\tfrac{1}{2}\right)^n + 2\left(-\tfrac{1}{2}\right)^n = 3\left(-\tfrac{1}{2}\right)^n$$

$y[n] = 3\left(-\tfrac{1}{2}\right)^n u(n) - 2\delta(n)$ $\qquad$ gives the correct value for $n=0$ and $n \geq 1$