# EE 341 - Exam 3

October 21, 2005

Name: Solutions

Closed book. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work. You may use one page of notes and a calculator.

1. Find the discrete time Fourier transform of the follwing signal:

$$x[n] = (0.5)^n \cos(0.5\pi n)u[n]$$

$$0.5^n u(n) \leftrightarrow \frac{1}{1-0.5e^{-j\Omega}}$$

$$0.5^n \cos(0.5\pi n)u(n) \leftrightarrow \frac{1}{2}\left[\frac{1}{1-0.5e^{-j(\Omega+0.5\pi)}} + \frac{1}{1-0.5e^{-j(\Omega-0.5\pi)}}\right]$$

2. Consider a system with the frequency response

$$H(\Omega) = 2\cos(\Omega/2)e^{-j\Omega/2}$$

Find the output ~~$y(t)$~~ $y(n)$ of the system when the input is

$$x[n] = 1 + \cos(\pi n/2) + 2\cos(\pi n)$$

$1: \Omega = 0 \quad H(0) = 2\cos(0)e^0 = 2$

$\cos(\frac{\pi}{2}n): \Omega = \frac{\pi}{2} \quad H(\frac{\pi}{2}) = 2\cos(\frac{\pi}{4})e^{-j\pi/4} = \sqrt{2}e^{-j\frac{\pi}{4}}$

$\cos(\pi n): \Omega = \pi \quad H(\pi) = 2\cos(\frac{\pi}{2})e^{-j\pi/2} = 0$

$y(n) = H(0)(1) + |H(\frac{\pi}{2})|\cos(\frac{\pi}{2}n + \angle H(\frac{\pi}{2})) + 2|H(\pi)|\cos(\pi n + \angle H(\pi))$

$= 2 + \sqrt{2}\cos(\frac{\pi}{2}n - \frac{\pi}{4})$

3. Find the initial and final values for each of the following signals. If the initial value or final value does not exist, explain why.

(a) $X(s) = \frac{(s-2)^2}{s+1}$

$$x(0) = \lim_{s \to \infty} \frac{s(s+2)^2}{s+1} \to \infty$$

$$x(\infty) = \lim_{s \to 0} \frac{s(s+2)^2}{s+1} = 0$$

Final value exists - pole at $-1$

(b) $X(s) = \frac{s^2}{(s+2)(s^2+2s+2)}$

$$x(0) = \lim_{s \to \infty} \frac{s^3}{(s+2)(s^2+2s+2)} = 1$$

$$x(\infty) = \lim_{s \to 0} \frac{s^3}{(s+2)(s^2+2s+2)} = 0$$

(c) $X(s) = \frac{2(s+1)}{s(s^2+1)^2}$

$$x(0) = \lim_{s \to \infty} \frac{2\cancel{s}(s+1)}{\cancel{s}(s^2+1)^2} = 0$$

$x(\infty)$ does not exist: $X(s) = \frac{2(s+1)}{s(s+j)^2(s-j)^2}$

Double poles at $\pm j$ means

$x(t) = r_1 t \cos(t) u(t)$ blows up

4. Consider a system described by the differential equation

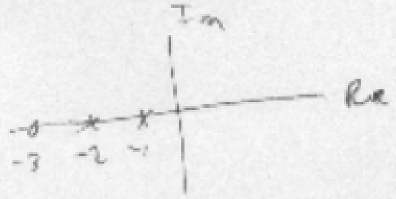
$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2\frac{dx(t)}{dt} + 6x(t)$$

(a) Find the transfer function $H(s)$ of the system.

$$s^2 Y(s) + 3s Y(s) + 2 Y(s) = 2s X(s) + 6 X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2(s+3)}{s^2+3s+2} = \frac{2(s+3)}{(s+1)(s+2)}$$

(b) Sketch the pole-zero diagram of the system.




(c) Find the impulse response $h(t)$ of the system.

$$H(s) = \frac{4}{s+1} + \frac{-2}{s+2} \qquad h(t) = 4e^{-t}u(t) - 2e^{-2t}u(t)$$

(d) Find the output of the system when the the input to the system is $x(t) = u(t)$ and the initial conditions are $y(0) = 0$ and $\dot{y}(0) = 1$

$$s^2 Y(s) - y(0^-)s - \dot{y}(0^-) + 3(sY(s) - y(0^-)) + 2Y(s) = 2sX(s) + 6X(s)$$

$$s^2 Y(s) + 3sY(s) + 2Y(s) = 1 + 2sX(s) + 6X(s)$$

$$X(s) = \frac{1}{s}$$

$$(s^2+3s+2)Y(s) = 1 + \frac{2s}{s} + \frac{6}{s} = \frac{3s+6}{s} = \frac{3(s+2)}{s}$$

$$Y(s) = \frac{3(s+1)}{s(s^2+3s+2)} = \frac{3(s+2)}{s(s+1)(s+2)} = \frac{3}{s(s+1)} = \frac{3}{s} + \frac{-3}{s+1}$$

$$y(t) = 3u(t) - 3e^{-t}u(t)$$

5. Consider the system described by the Laplace transform

$$H(s) = \frac{(s-16)^2}{(s+2)^2(s+0.1)(s^2-s2+10)}$$

(a) What is the form of the step response of the system? (You do not have to take the inverse Laplace trasform – to write down the form of the response, you do not have to give the exact amplitude and phase of each term.)

$$G(s) = \frac{(s+16)^2}{(s+2)^2(s+0.1)(s-1-j3)(s-1+j3)s}$$

$$g(t) = r_1 e^{-2t}u(t) + r_2 t e^{-2t}u(t) + r_3 e^{-0.1t} + 2|r_4| e^{-t}\cos(3t + \angle r_4)$$
$$+ r_5 u(t)$$

(b) What is the final value of the step response of the system?

$$G(s) = \frac{(s+16)^2}{s(s+2)^2(s+0.1)(s^2+2s+10)}$$

$$g(\infty) = \lim_{s \to 0} \frac{(s+16)^2}{(s+2)^2(s+0.1)(s^2+2s+10)} = \frac{16^2}{2^2(0.1)(10)} = \frac{256}{4} = 64$$

(c) About how long will it take the system to get close to the final value? (Close means $x(t)$ will be about 90% of the final value. You do not need to give an exact value — give an approximate value and justify it.)

Takes about 3 time constants of slowest term

Time costant of $e^{-0.1t}u(t) = e^{-t/10}u(t)$ is 10 sec.

System takes about 30 sec to get close to the final value