EE 341 - Homework 4

Due September 21, 2005

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots.

1. Find the convolution $y[n] = x[n] * h[n]$ for each of the following signal pairs. Use an arrow to indicate $n = 0$. Do the convolution by hand, and verify with the `conv` function of MATLAB.
   (a) $x[n] = \{\overset{\Downarrow}{1}, 2, 0, 1\}$ $h[n] = \{\overset{\Downarrow}{2}, 2, 3\}$
   (b) $x[n] = \{3, 2, \overset{\Downarrow}{1}, 1, 2\}$ $h[n] = \{4, \overset{\Downarrow}{2}, 3, 2\}$
   (c) $x[n] = \{\overset{\Downarrow}{0}, 0, 0, 3, 1, 2\}$ $h[n] = \{4, \overset{\Downarrow}{2}, 3, 2\}$
2. Let $x[n] = h[n] = \{\overset{\Downarrow}{3}, 4, 2, 1\}$.
   (a) Find $x[n] * h[n]$
   (b) Find $x[n] * h[-n]$
   (c) Use the properties of convolution to find $x[n-1] * h[n+4]$
3. Consider a system which computes the average value of the current and past two inputs.
   (a) Write a difference equation for the system.
   (b) Use the difference equation to find the response of the system to the input $x[n] = \{\overset{\Downarrow}{1}, 2, 3, 4, 5\}$
   (c) Find the impulse response of the system.
   (d) Find $x[n] * h[n]$, and show the answer is the same as for part (b).
4. A linear time-invariant discrete-time system has an impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n]$. Find the output to the system when the input is $x[n] = \left(\frac{1}{3}\right)^n u[n]$.
5. Problem 3.16 (d) (e).
6. The impulse response of an LTI CT system is $h(t) = 2e^{-t}u(t) - \delta(t)$.
   (a) Find the output $y(t)$ of the system when the input is $x(t) = u(t)$.
   (b) Find the output $y(t)$ of the system when the input is $x(t) = e^{-t}u(t)$.
7. Problem 3.26

EE 341 HW #4

1\. (a) $x(n) = [\underset{\uparrow}{1}\ 2\ 0\ 1]$ $h(n) = [\underset{\uparrow}{2}\ 2\ 3]$

$y(n) = h(n) * x(n)$

y(0):

		1	2	0	1
3	2	2			
0	0	2	0	0	0

= 2

y(1):

	1	2	0	1
3	2	2		
0	2	4	0	1

= 6

y(2):

	1	2	0	1
	3	2	2	
0	3	4	0	0

= 7

y(3):

1	2	0	1
	3	2	2
0	6	0	2

= 8

Continue on: $y(n) = [\underset{\uparrow}{2}, 6, 7, 8, 2, 3]$

MATLAB: x = (1, 2, 0, 1); h = (2, 2, 3); y = conv(x, h)

gives same results

(b) $x(n) = \{3 \quad 2 \quad \underset{\uparrow}{1} \quad 1 \quad 2\}$ $\qquad h(n) = \{4, \underset{\uparrow}{2}, 3, 2\}$

$y(n) = x(n) * h(n)$

$y(0)$:

$$\begin{array}{ccccc} 3 & 2 & \overset{\downarrow}{1} & 1 & 2 \\ 2 & 3 & 2 & 4 & \\ \hline 6 & 6 & 2 & 4 & 0 \end{array} \quad = 18$$

$y(-1)$:

$$\begin{array}{cccccc} & 3 & 2 & 1 & 1 & 2 \\ 2 & 3 & 2 & 4 & & \\ \hline & 9 & 4 & 4 & & \end{array} \quad = 17$$

$y(-3)$:

$$\begin{array}{cccccccc} & & & 3 & 2 & 1 & 1 & 2 \\ 2 & 3 & 2 & 4 & & & & \\ \hline & & & 12 & & & & \end{array} \quad = 12$$

Smallest index is -3; when $h(-n)$ is shifted more than this, there is no overlap

Continuing on: $y(n) = \{12, 14, 17, \underset{\uparrow}{18}, 17, 9, 8, 4\}$

(c) $x(n) = \{0, 0, 0, 3, 1, 2\}$ $\qquad h(n) = \{4, 2, 3, 2\}$

$y(n) = h(n) * x(n)$

$y(0)$:

$$\begin{array}{cccccccc} & & & 0 & 0 & 0 & 3 & 1 \; 2 \\ 2 & 3 & 2 & 4 & & & & \\ \hline & & & 0 & 0 & & & \end{array} = 0$$

$y(2)$:

$$\begin{array}{cccccc} 0 & 0 & 0 & 3 & 1 & 2 \\ 2 & 3 & 2 & 4 & & \\ \hline & & & 12 & & \end{array} = 12$$

$y(3)$:

$$\begin{array}{cccccc} 0 & 0 & 0 & 3 & 1 & 2 \\ & 2 & 3 & 2 & 4 & \\ \hline & & & 6 & 4 & \end{array} = 10$$

Continuing on: $y(n) = \{0, 0, 12, 10, 19, 13, 8, 4\}$

MATLAB gives same answer

2. (a) $x(n) = h(n) = \{3, 4, 2, 1\}$

$y_a(n) = h(n) * x(n)$

$y_a(0)$:

$$\begin{array}{ccccccc} & & & 3 & 4 & 2 & 1 \\ 1 & 2 & 4 & 3 & & & \\ \hline & & & 9 & & & \end{array} = 9$$

Continuing on: $y_a(n) = \{9, 24, 28, 22, 12, 4, 1\}$

(b) $h(-n) = \{1 \;\; 2 \;\; 4 \;\; 3\}$

$y_b(n) = x(n) * h(-n)$

$y_b(0)$:

$$\begin{array}{cccc} 3 & 4 & 2 & 1 \\ 3 & 4 & 2 & 1 \\ \hline 9 & 16 & 4 & 1 \end{array} = 30$$

$$y_b(-3): \quad \begin{array}{cccccccc} & & & 3 & 4 & 2 & 1 \\ 3 & 4 & 2 & 1 & & & \\ \hline & & & 3 & & & \end{array} \neq 0$$

Continuing on: $y_b(n) = \{3, 10, 22, \underset{\uparrow}{30}, 22, 10, 3\}$

(c) $y_c(n) = x(n-1) * h(n+4) = y_a(n+3)$

$$= \{9, 24, 28, \underset{\uparrow}{22}, 12, 4, 1\}$$

3\. (a) $y(n) = \frac{1}{3}\left(x(n) + x(n-1) + x(n-2)\right)$

$y(0) = \frac{1}{3}(x(0) + x(-1) + x(-2)) = \frac{1}{3}(1) = \frac{1}{3}$

$y(1) = \frac{1}{3}(1+2) = 1$

$y(2) = \frac{1}{3}(1+2+3) = 2$

$y(3) = \frac{1}{3}(2+3+4) = 3$

$\vdots$

$y(n) = \{\frac{1}{3}, 1, 2, 3, 4, 3, 5/3\}$

(b) $h(n)$: output when $x(n) = \delta(n)$

$h(0) = \frac{1}{3}(\delta(0) + \delta(-1) + \delta(-2)) = 1/3$

$h(1) = \frac{1}{3}(\delta(1) + \delta(0) + \delta(-1)) = 1/3$

$h(2) = \frac{1}{3}(\delta(2) + \delta(1) + \delta(0)) = 1/3$

$h(n) = 0$ for all other n

$h(n) = \{\underset{\uparrow}{\frac{1}{3}}, \frac{1}{3}, \frac{1}{3}\}$

(c) $y(n) = x(n) * h(n)$

$y(0)$:

$$\begin{array}{cccccccc} & & & 1 & 2 & 3 & 4 & 5 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & & & & \\ \hline & & & \frac{1}{3} & & & & \end{array} = \frac{1}{3}$$

$y(1)$:

$$\begin{array}{ccccccc} & & 1 & 2 & 3 & 4 & 5 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & & & \\ \hline & & \frac{1}{3} & \frac{2}{3} & & & \end{array} = 1$$

Continuing on: $y(n) = \{ \frac{1}{3}, 1, 2, 3, 4, \frac{5}{3} \}$

Same as (a)

4. $h(n) = \left(\frac{1}{2}\right)^n u(n) \qquad x(n) = \left(\frac{1}{3}\right)^n u(n)$

$$y(n) = h(n) * x(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u(k) \left(\frac{1}{3}\right)^{n-k} u(n-k)$$

For $n < 0$, $y(n) = 0$

For $n \geq 0$, sum starts at $k=0$ (because of $u(k)$)
and ends at $n-k=0 \Rightarrow n=k$ (because of $u(n-k)$)

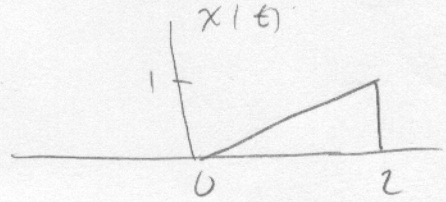
$$y(n) = \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \left(\frac{1}{3}\right)^{n-k} = \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k \left(\frac{1}{3}\right)^{-k} = \left(\frac{1}{3}\right)^n \sum_{k=0}^{n} \left(\frac{3}{2}\right)^k$$

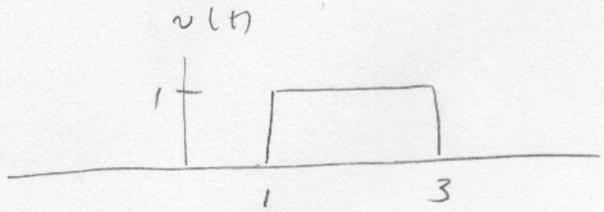
$$= \left(\frac{1}{3}\right)^n \frac{1 - \left(\frac{3}{2}\right)^{n+1}}{1 - 3/2} = \frac{\left(\frac{1}{3}\right)^n - \left(\frac{1}{3}\right)^n \left(\frac{3}{2}\right)^n \left(\frac{3}{2}\right)}{-1/2}$$

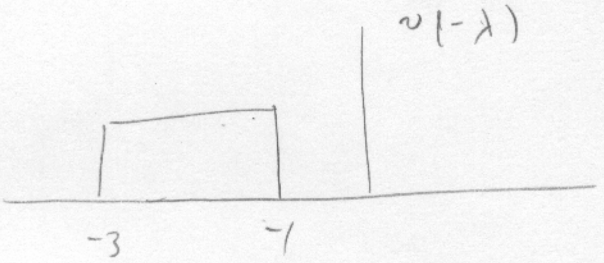
$$= \frac{\left(\frac{1}{3}\right)^n - \frac{3}{2}\left(\frac{1}{2}\right)^n}{-1/2} = 3\left(\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n$$

5. Problem 3.16

(d)

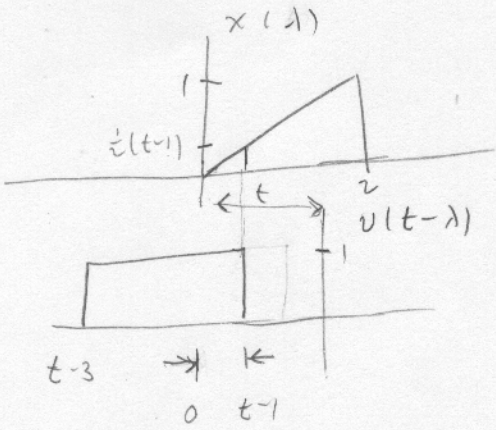






No output until $t = 1$.

For $1 < t < 3$



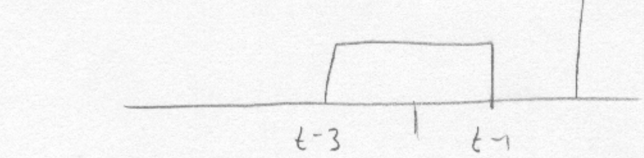
overlap from $\lambda = 0$ to $\lambda = t-1$

Area: $\frac{1}{2}(\text{base})(\text{height})$

$\frac{1}{2}(t-1)\left[\frac{1}{2}(t-1)\right]$

$\frac{1}{4}(t-1)^2$

For $3 < t < 5$



overlap from $t-3$ to 2

Area of rectangle:

$[2-(t-3)]\left(\frac{1}{2}(t-3)\right)$

$= \frac{1}{2}(5-t)(t-3)$

Area of triangle:

$\frac{1}{2}(\text{base})(\text{height})$

$\frac{1}{2}[2-(t-3)]\left[1-\frac{1}{2}(t-3)\right]$

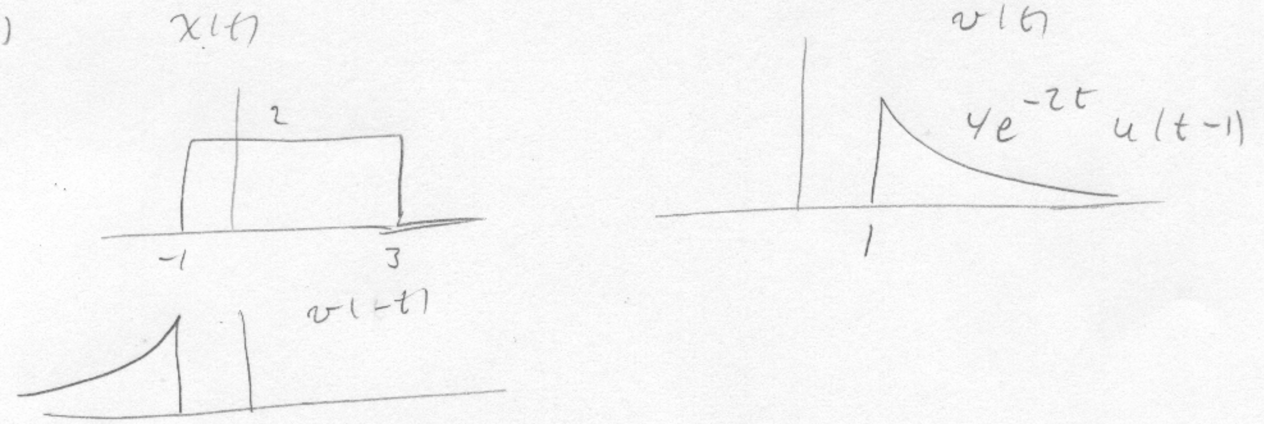
$\frac{1}{4}(t-5)^2$

sum: $-\frac{1}{4}t^2 + \frac{3}{2}t - \frac{5}{4}$

$$y(t) = \begin{cases} 0 & t \le 0 \\ \frac{1}{4}(t-1)^2 & 1 \le t \le 3 \\ -\frac{1}{4}t^2 + \frac{3}{2}t - \frac{5}{4} & 3 \le t \le 5 \\ 0 & t \geq 5 \end{cases}$$

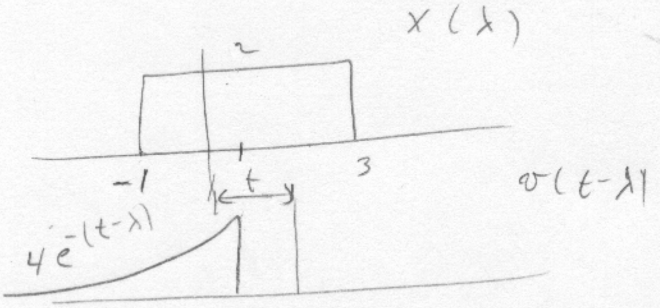
See MATLAB for plot

(e)



For $t < 0$, no overlap; $y(t) = 0$

For $0 \le t \le 4$:



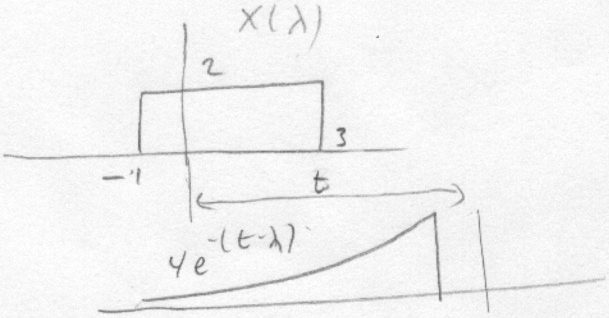
Overlap from $\lambda = -1$ to $\lambda = t-1$

$$y(t) = \int_{-1}^{t-1} x(\lambda) v(t-\lambda)\, d\lambda = \int_{-1}^{t-1} 2 \cdot 4 e^{-(t-\lambda)}\, d\lambda$$

$$= 8e^{-t} \int_{-1}^{t-1} e^{\lambda}\, d\lambda = 8e^{-t} \left(e^{\lambda} \right) \Big|_{-1}^{t-1} = 8e^{-t} \left(e^{t-1} - e^{-1} \right)$$

$$= 8 \left(e^{-1} - e^{-t-1} \right)$$

For $t \geq 4$



overlap from -1 to 3

$$y(t) = \int_{-1}^{3} x(\lambda)\, v(t-\lambda)\, d\lambda = \int_{-1}^{3} 2 \cdot 4 e^{-(t-\lambda)}\, d\lambda$$

$$= 8e^{-t} \int_{-1}^{3} e^{\lambda}\, d\lambda = 8e^{-t}\, e^{\lambda}\Big|_{-1}^{3} = 8e^{-t}\left(e^{3} - e^{-1}\right)$$

$$y(t) = \begin{cases} 0 & t \leq 0 \\ 8(e^{-1} - e^{-t-1}) & 0 \leq t \leq 4 \\ 8(e^{3-t} - e^{-1-t}) & t \geq 4 \end{cases}$$

See MATLAB for plots

6 (a) $h(t) = 2e^{-t}u(t) - \delta(t)$

$x(t) = u(t)$

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} \left[2e^{-\lambda}u(\lambda) - \delta(\lambda)\right] u(t-\lambda)\, d\lambda$$

$$= \underbrace{\int_{-\infty}^{\infty} 2e^{-\lambda} u(\lambda)\, u(t-\lambda)\, d\lambda}_{y_1(t)} - \underbrace{\int_{-\infty}^{\infty} \delta(\lambda)\, u(t-\lambda)\, d\lambda}_{y_2(t)}$$

$y_1(t) = 0$ if $t < 0$

$$y_1(t) = \int_{0}^{t} 2e^{-\lambda}\, d\lambda = -2e^{-\lambda}\Big|_{0}^{t} = 2(1 - e^{-t}) \quad \text{for } t \geq 0$$

$y_2(t) = -u(t)$

$$y(t) = y_1(t) + y_2(t) = 2(1-e^{-t})u(t) - u(t)$$
$$= (1-2e^{-t})u(t)$$

(b) $x(t) = e^{-t}u(t)$

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} [2e^{-\lambda}u(\lambda) - \delta(\lambda)]\, e^{-(t-\lambda)} u(t-\lambda)\, d\lambda$$

$$= \underbrace{\int_{-\infty}^{\infty} 2e^{-\lambda} e^{-(t-\lambda)} u(\lambda)\, u(t-\lambda)\, d\lambda}_{y_1(t)} - \underbrace{\int_{-\infty}^{\infty} \delta(\lambda)\, e^{-(t-\lambda)} u(t-\lambda)\, d\lambda}_{y_2(t)}$$

$y_1(t) = 0$ for $t < 0$

$$y_1(t) = \int_0^t 2e^{-\lambda} e^{-(t-\lambda)}\, d\lambda = 2e^{-t}\int_0^t d\lambda = 2te^{-t} \quad t > 0$$

$$y_2(t) = -e^{-t}u(t)$$

$$y(t) = y_1(t) + y_2(t) = 2te^{-t}u(t) - e^{-t}u(t)$$
$$= (2t-1)e^{-t}u(t)$$

7. Problem 3.26 $\quad y(t) = h(t) * x(t)$

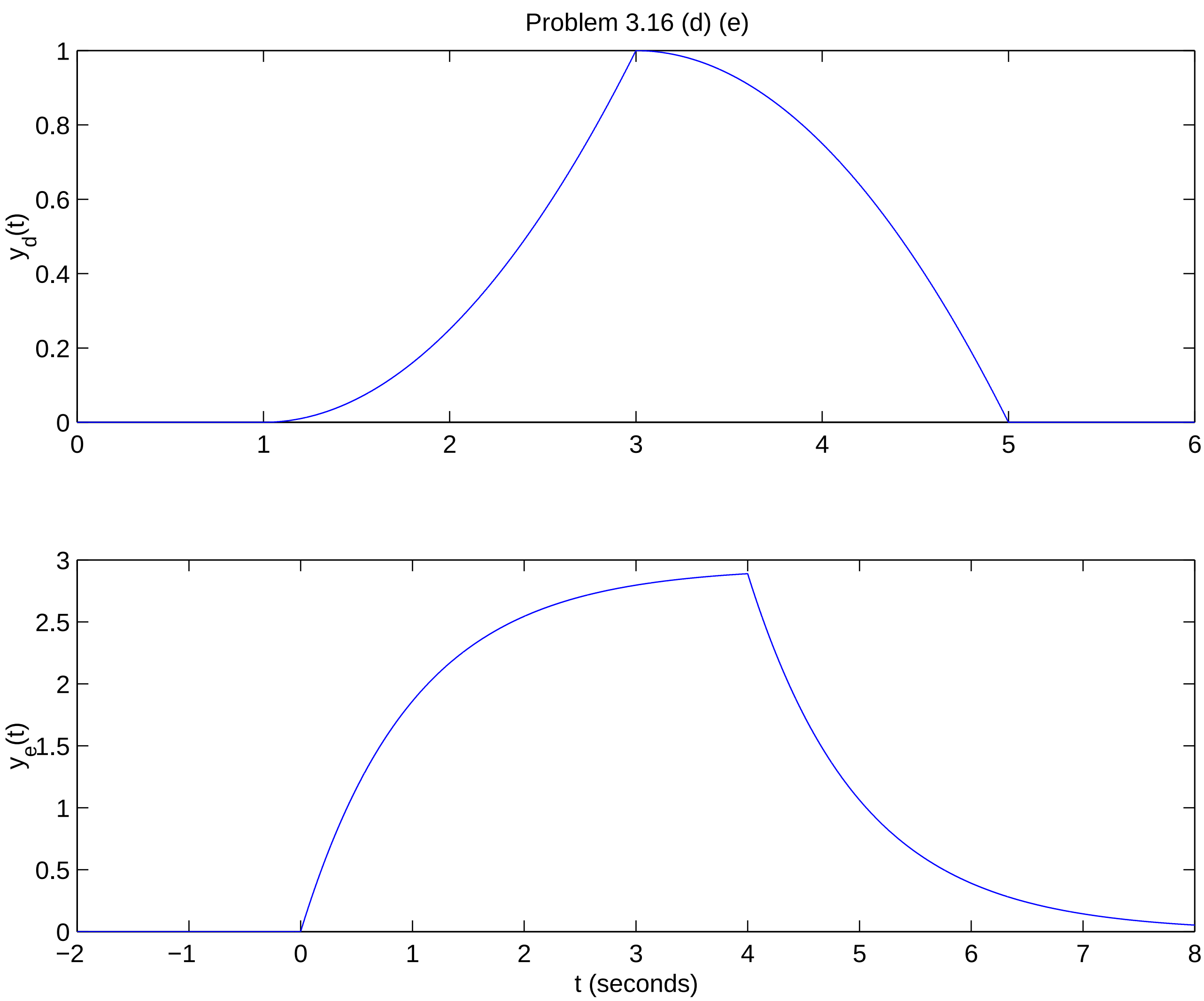
(a) $y_a(t) = h(t) * \dfrac{dx(t)}{dt} = \dfrac{d}{dt}(h(t) * x(t)) = \dfrac{dy(t)}{dt}$

(b) $y_b(t) = h(t) * \dfrac{d^2x(t)}{dt^2} = \dfrac{d}{dt}\left(h(t) * \dfrac{dx(t)}{dt}\right) = \dfrac{d}{dt}(y_a(t)) = \dfrac{d^2y(t)}{dt^2}$

(c) $y_c(t) = h(t) * \int_{-\infty}^{t} x(\lambda)\, d\lambda = \int_{-\infty}^{t} h(\lambda) * x(\lambda) = \int_{-\infty}^{t} y(\lambda)\, d\lambda$

(d) $y_d(t) = h(t) * (x(t) * x(t)) = (h(t) * x(t)) * x(t) = y(t) * x(t)$

```
% HW 4

% Problem 1

% (a)
xa = [1 2 0 1];
ha = [2 3 3];
ya = conv(xa,ha);

% (b)
xb = [3 2 1 1 2];
hb = [4 2 3 2];
yb = conv(xb,hb);

% (c)
xc = [0 0 0 3 1 2];
hc = [4 2 3 2];
yc = conv(xc,hc);

% Problem 3.16
figure(1)
clf

% (d)
t = 0:0.001:6;
yd = ((1/4)*(t-1).^2).*((t>=1)-(t>=3)) + ...
    ((-1/4)*t.^2 + (3/2)*t-(5/4)).*((t>=3)-(t>=5));
subplot(211)
plot(t,yd)
ylabel('y_d(t)');
title('Problem 3.16 (d) (e)')

% (e)
t = -2:0.001:8;
ye = (8*(exp(-1) - exp(-t-1))).*((t>=0)-(t>=4)) + ...
     (8*(exp(3-t)-exp(-1-t))).*(t>=4);
subplot(212)
plot(t,ye)
ylabel('y_e(t)');
xlabel('t (seconds)');

print -dpsc2 p3_16_de.ps
```


Problem 3.16 (d) (e)
y_d(t)
y_e(t)
t (seconds)