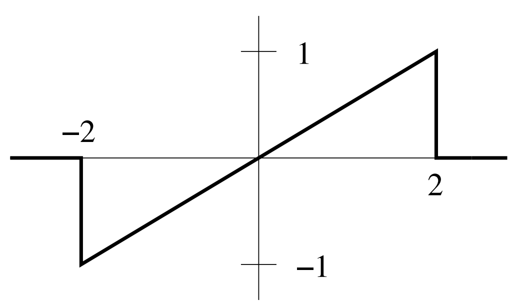
EE 341 - Homework 5

Due September 30, 2005

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots.

1. Problem 4.1
2. Problem 4.6. Do for Figure 4.6 (e) only (on page 196). For parts (ii) and (iii), use MATLAB.
3. Problem 4.7 (a), (b), (c), (d). For (c) and (d), use trigonemetric identities to convert the representations of the signals into ones which do not have multiplications of sinusoids.
4. Problem 4.12 (a) (b).
5. The Fourier transform of $x(t)$ is $p_2(\omega)$. Use the properties of the Fourier transform to find the Fourier transforms of the following functions (without using the inverse Fourier transform to calculating $x(t)$).
   (a) $v(t) = x(5t - 4)$
   (b) $v(t) = tx(t)$
   (c) $v(t) = x(t)e^{j2t}$
   (d) $v(t) = \frac{d^2x(t)}{dt^2}$
   (e) $v(t) = x^2(t)$
6. Problem 4.16 (a) (c).
7. Below are some continuous-time signals. Answer the questions about the Fourier transforms of the signals. Be sure to explain your answers.
   (a)



   i. Is $X(\omega)$ real, imaginary or complex?
   ii. Is $\int_{-\infty}^{\infty} X(\omega)\, dw$ equal to zero?
   iii. Is $X(\omega)$ periodic?

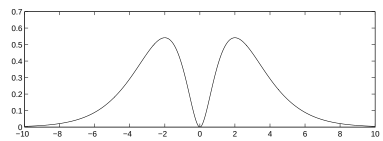
(b) $x(t) = \delta(t-2)$

i. Is $X(\omega)$ real, imaginary or complex?

ii. Is $\int_{-\infty}^{\infty} X(\omega)\, dw$ equal to zero?

iii. Is $X(\omega)$ periodic?

(c)



i. Is $X(\omega)$ real, imaginary or complex?

ii. Is $X(\omega)$ imaginary?

iii. Is $\int_{-\infty}^{\infty} X(\omega)\, dw$ equal to zero?

iv. Is $X(\omega)$ periodic?

EE 341 HW#5

1. Problem 4.1

(a) $x = e^{j\pi/4} + e^{-j\pi/8} = \cos(\pi/4) + j\sin(\pi/4) + \cos(\pi/8) - j\sin(\pi/8)$

$|x| = \sqrt{\left(\cos\frac{\pi}{4} + \cos\frac{\pi}{8}\right)^2 + \left(\sin\frac{\pi}{4} - \sin\frac{\pi}{8}\right)^2} = 1.66$

$\measuredangle x = \tan^{-1}\left(\frac{\sin\pi/4 - \sin\pi/8}{\cos\frac{\pi}{4} + \cos\pi/8}\right) = 0.196$ rad

$x = 1.66\, e^{j0.196}$

(b) $x = (2+5j)\, e^{j10} = \sqrt{2^2+5^2}\, e^{j\tan^{-1}\frac{5}{2}} \cdot e^{j10} = 5.39\, e^{j1.19}\, e^{j10} = 5.39\, e^{j11.19}$

$= 5.39\, e^{j(11.19-4\pi)} = 5.39\, e^{-j1.38}$

(c) $x = 1 + e^{j2} + e^{j4} = 1 + \cos(2) + j\sin(2) + \cos(4) + j\sin(4)$

$= -0.070 + j0.153 = \sqrt{(-0.070)^2 + (0.153)^2}\, e^{j\tan^{-1}\frac{.153}{-0.070}}$

$= 0.168\, e^{j2}$

(d) $x = 1 + e^{j4} = 1 + \cos 4 + j\sin 4 = 0.346 - j0.757$

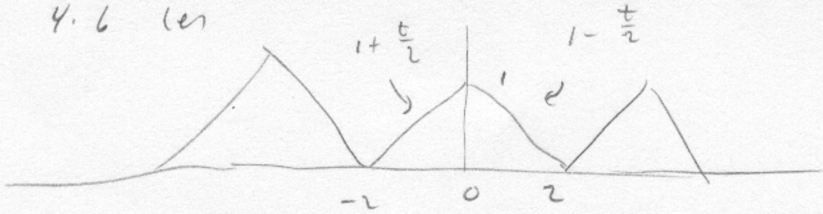
$= \sqrt{(0.346)^2 + (-0.757)^2}\, e^{j\tan^{-1}\frac{-0.757}{0.346}} = 0.832\, e^{-j1.142}$

(e) $x = e^{j(\omega t + \pi/2)} + e^{j(\omega t - \pi/3)}$

$= e^{j\omega t}\left(e^{j\pi/2} + e^{-j\pi/3}\right) = e^{j\omega t}\left[\cos\frac{\pi}{2} + j\sin\frac{\pi}{2} + \cos\frac{\pi}{3} - j\sin\frac{\pi}{3}\right)$

$= e^{j\omega t}(0.5 + j0.134) = 0.5176\, e^{j(\omega t + 0.2618)}$

2. Problem 4.6 (e)



(i) $T = 4 \qquad \omega_0 = \frac{2\pi}{T} = \frac{\pi}{2}$

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jk\omega_0 t}\, dt$$

$k = 0$: $\quad C_k = \frac{1}{4}\int_{-2}^{2} x(t)\, dt = \frac{1}{4}(\text{area under curve}) = \frac{1}{4}\left(\frac{1}{2}\text{base} \times \text{height}\right)$

$$= \frac{1}{8}(4)(1) = \frac{1}{2}$$

$k \neq 0$: $\quad C_k = \frac{1}{4}\int_{-2}^{2} x(t)\, e^{-jk\frac{\pi}{2}t}\, dt = \frac{1}{4}$

$$= \frac{1}{4}\int_{-2}^{0}\left(1+\frac{t}{2}\right)e^{-jk\frac{\pi}{2}t}\, dt + \frac{1}{4}\int_{0}^{2}\left(1-\frac{t}{2}\right)e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1-\cos(k\pi)}{k^2\pi^2}$$

(ii) See MATLAB

(iii) See MATLAB

3. Problem 4.7

(a) $x(t) = \cos(5t+\theta) \qquad \omega_0 = 5 \qquad T = \frac{2\pi}{\omega_0} = \frac{2\pi}{5}$

$$x(t) = \frac{e^{j(5t+\theta)} + e^{-j(5t+\theta)}}{2} = \frac{1}{2}e^{j5t}e^{j\theta} + \frac{1}{2}e^{-j5t}e^{-j\theta}$$

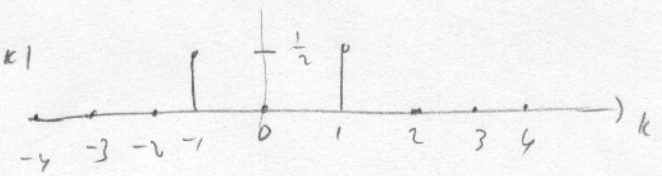
$$= \frac{1}{2}e^{j\theta}e^{j\omega_0 t} + \frac{1}{2}e^{-j\theta}e^{-j\omega_0 t}$$

$$= \sum C_k e^{jk\omega_0 t}$$

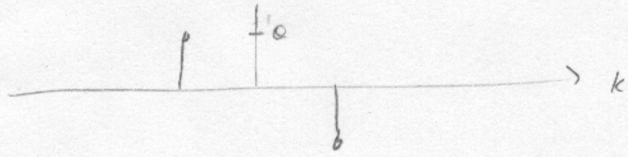
$C_1 = \frac{1}{2}e^{j\theta} \qquad C_{-1} = \frac{1}{2}e^{-j\theta} \qquad C_k = 0$ for all other k

$|C_1| = \frac{1}{2}, \quad \measuredangle C_1 = \theta \qquad |C_2| = \frac{1}{2} \qquad \measuredangle C_2 = -\theta$

$|C_k|$



$\measuredangle C_k$

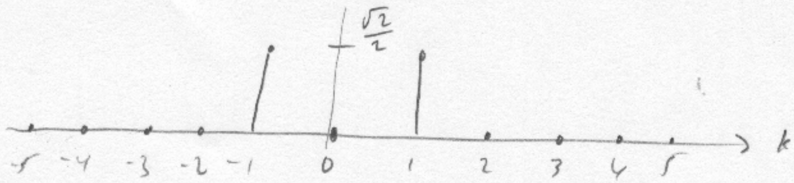


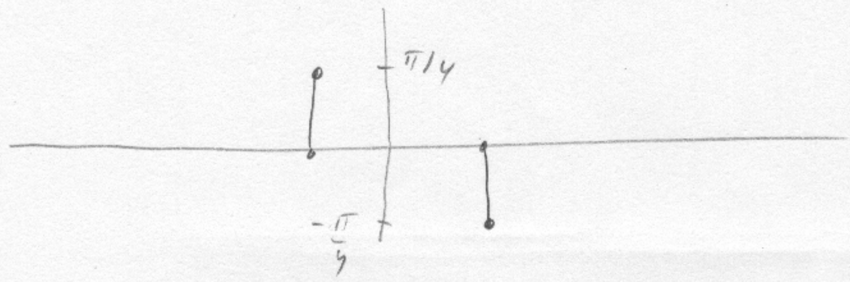
(b) $x(t) = \sin t + \cos t \qquad \omega_0 = 1 \qquad T = \frac{2\pi}{\omega_0} = 2\pi$

$$x(t) = \frac{e^{jt} - e^{-jt}}{2j} + \frac{e^{jt} + e^{-jt}}{2} = \frac{1}{2}(1-j)e^{j\omega_0 t} + \frac{1}{2}(1+j)e^{-j\omega_0 t}$$

$C_1 = \frac{1}{2}(1-j) \qquad C_{-1} = \frac{1}{2}(1+j) \qquad C_k = 0$ for all other K

$|C_1| = \frac{\sqrt{2}}{2} \quad \measuredangle C_1 = -\frac{\pi}{4} \qquad |C_2| = \frac{\sqrt{2}}{2} \quad \measuredangle C_2 = \frac{\pi}{4}$

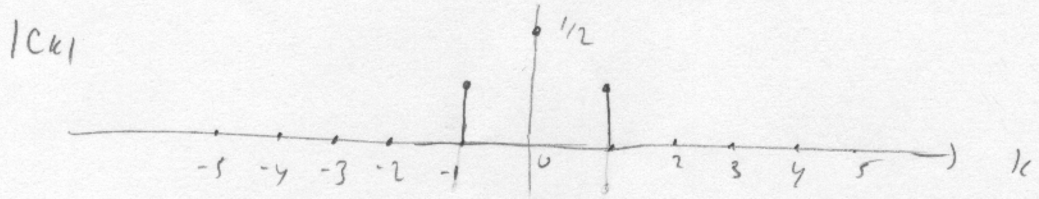


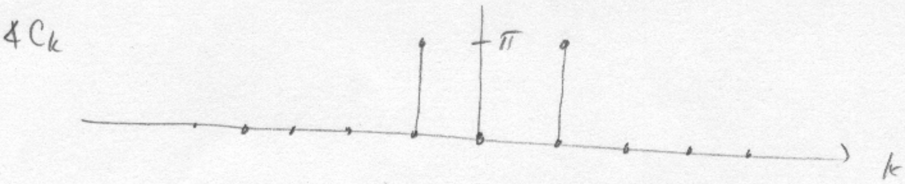


(c) $x(t) = \sin^2 4t = \frac{1}{2} - \frac{1}{2}\cos 8t \qquad \omega_0 = 8 \qquad T = \frac{2\pi}{\omega_0} = \frac{\pi}{4}$

$$x(t) = \frac{1}{2} - \frac{1}{2}\left(\frac{e^{j8t} + e^{-j8t}}{2}\right) = \frac{1}{2} - \frac{1}{4}e^{j\omega_0 t} - \frac{1}{4}e^{-j\omega_0 t}$$

$C_0 = \frac{1}{2} \quad C_1 = -\frac{1}{4} \quad C_{-1} = -\frac{1}{4} \qquad C_k = 0$ for all other k





(d) $x(t) = \cos 2t \sin 3t = \frac{1}{2}\sin t + \frac{1}{2}\sin 5t$

$\omega_1 = 1 \qquad T_1 = \frac{2\pi}{\omega_1} = 2\pi \qquad \omega_2 = 5 \qquad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{5}$

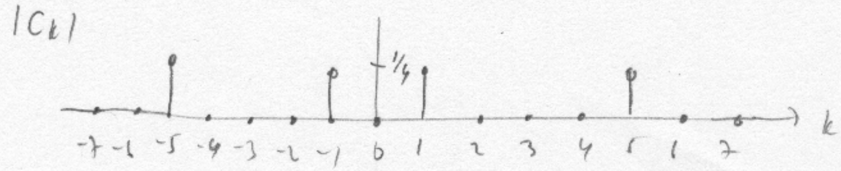
$\frac{T_1}{T_2} = \frac{r}{q} = \frac{2\pi}{2\pi/5} = 5 \qquad T = rT_2 = 5\left(\frac{2\pi}{5}\right) = 2\pi \qquad \omega_0 = \frac{2\pi}{T} = 1$

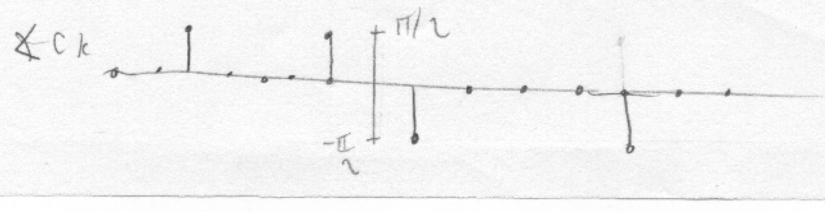
$$x(t) = \frac{1}{2}\,\frac{e^{jt} - e^{-jt}}{2j} + \frac{1}{2}\,\frac{e^{j5t} - e^{-j5t}}{2j}$$

$$= \frac{-j}{4}e^{j\omega_0 t} + \frac{j}{4}e^{-j\omega_0 t} - \frac{j}{4}e^{j5\omega_0 t} + \frac{j}{4}e^{-j5\omega_0 t}$$

$C_1 = -j/4 \qquad C_{-1} = \frac{j}{4} \qquad C_5 = \frac{-j}{4} \qquad C_{-5} = \frac{j}{4}$

$|C_1| = |C_{-1}| = |C_5| = |C_{-5}| = \frac{1}{4} \qquad \measuredangle C_1 = \measuredangle C_5 = -\frac{\pi}{2} \qquad \measuredangle C_{-1} = \measuredangle C_{-5} = \frac{\pi}{2}$





4: Problem 4.12

(a) 4.6(a) is neither

4.6(b) is neither

4.6(c) is neither

4.6(d) is odd

4.6(e) is even

4.6(f) is even

4.6(g) is neither

(b) $x(-t) = x(t)$

$z(-t) = -z(t)$

$$\int_{-\frac{T}{2}}^{T/2} x(t)\, z(t)\, dt = \underbrace{\int_{-T/2}^{0} x(t)\, z(t)\, dt}_{\text{let } \tau = -t} + \underbrace{\int_{0}^{T/2} x(t)\, z(t)\, dt}_{\text{let } \tau = t}$$

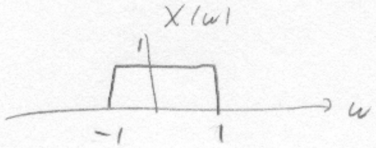
$$= \int_{\frac{T}{2}}^{0} x(-\tau)\, z(-\tau)\, (-d\tau) + \int_0^{T/2} x(\tau)\, z(\tau)\, d\tau$$

$$= \int_0^{T/2} x(-\tau)\, z(-\tau)\, d\tau + \int_0^{T/2} x(\tau)\, z(\tau)\, d\tau$$

$$= -\int_0^{T/2} x(\tau)\, z(\tau)\, d\tau + \int_0^{T/2} x(\tau)\, z(\tau)\, d\tau$$

$$= 0$$

5. $X(\omega) = P_2(\omega)$



(a) $v(t) = x(5t-4) = x(5(t-4/5))$

$$V(\omega) = e^{-j\frac{4}{5}\omega}\,\frac{1}{5}\,P_2\left(\frac{\omega}{5}\right) = \frac{1}{5}e^{-j\frac{4}{5}\omega}\,P_{\frac{2}{5}}(\omega)$$

(b) $v(t) = t\,x(t)$

$$V(\omega) = j\frac{d}{d\omega}X(\omega) = j(\delta(-1) - \delta(1))$$

(c) $v(t) = x(t)\,e^{j2t}$

$$V(\omega) = X(\omega - 2)$$

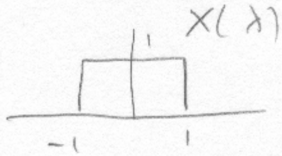
(d) $v(t) = \dfrac{d^2x(t)}{dt^2}$

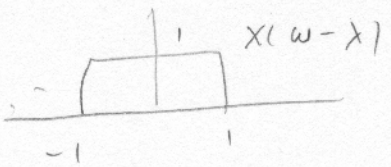
$$V(\omega) = (j\omega)^2 X(\omega) = -\omega^2 P_2(\omega)$$

(e) $v(t) = x^2(t)$

$$V(\omega) = \frac{1}{2\pi} X(\omega) * X(\omega)$$

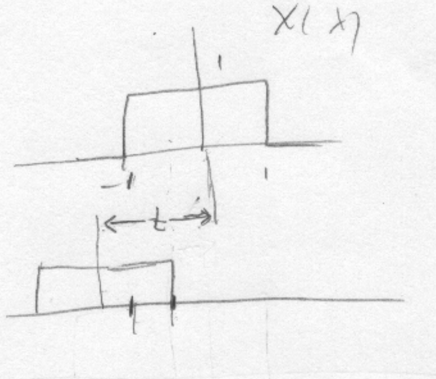
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\lambda)\,X(\omega-\lambda)\,d\lambda$$





$$V(\omega) = \begin{cases} 0 & \omega < -2 \\ \text{next} & -2 < \omega < 0 \\ \text{page} & 0 < \omega < 2 \\ 0 & \omega > -2 \end{cases}$$

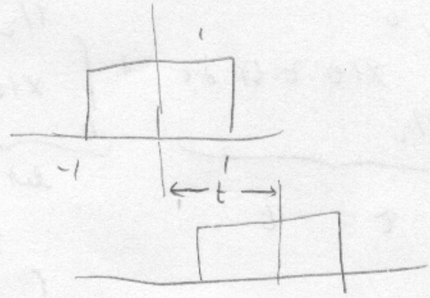
For $-2 < \omega < 0$:



Region of overlap: $\lambda = -1$ to $\lambda = t+1$

area: $[(t+1) - (-1)](1) = t+2$

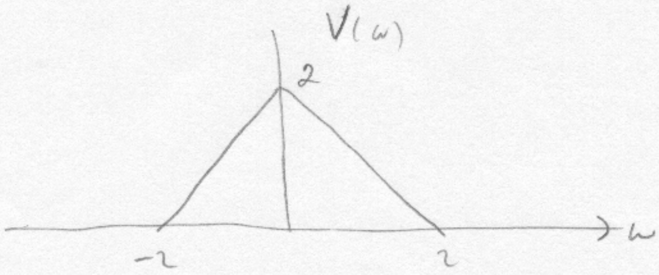
For $0 < \omega < 2$:



Region of overlap: $\lambda = t-1$ to $\lambda = 1$

area: $[(t-1) - (1)](1) = t-2$

$$V(\omega) = \begin{cases} 0 & \omega < -2 \\ t+2 & -2 < \omega < 0 \\ t-2 & 0 < \omega < 2 \\ 0 & \omega > 2 \end{cases}$$



6. Problem 4.16

(a) $x(t) = p_1(t - \frac{3}{2}) - p_1(t - \frac{1}{2})$

$$p_1(t) \leftrightarrow \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

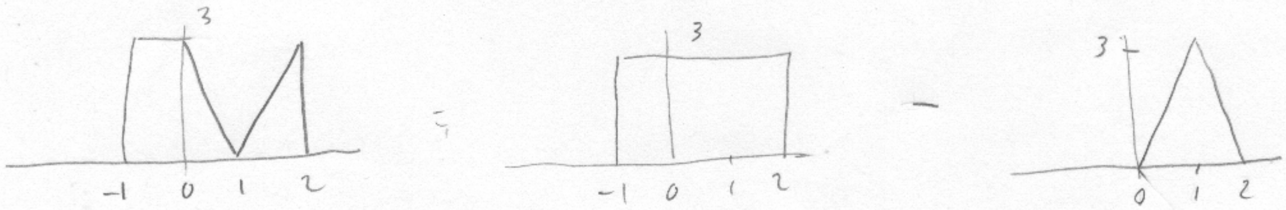
$$p_1(t-c) = e^{-j\omega c} P_1(\omega) = e^{-j\omega c} \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$X(\omega) = e^{-j\omega\frac{3}{2}} \text{sinc}\left(\frac{\omega}{2\pi}\right) - e^{-j\omega\frac{1}{2}} \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

$$= e^{-j\omega} \text{sinc}\left(\frac{\omega}{2\pi}\right)\left[e^{-j\omega\frac{1}{2}} - e^{j\omega\frac{1}{2}}\right]$$

$$= -2j e^{-j\omega} \sin\left(\frac{\omega}{2}\right) \text{sinc}\left(\frac{\omega}{2\pi}\right)$$

(c)



$$x(t) = P_{\frac{3}{2}}(t - \tfrac{1}{2}) - \left(1 - \frac{2|t-1|}{2}\right) P_2(t-1)$$

$$P_\tau(t) \leftrightarrow \tau\, \text{sinc}\left(\frac{\tau\omega}{2\pi}\right) \qquad \left(1 - \frac{2|t|}{\tau}\right) P_\tau(t) \leftrightarrow \frac{\tau}{2} \text{sinc}^2\left(\frac{\tau\omega}{2\pi}\right)$$

$$P_\tau(t-c) \leftrightarrow e^{-j\omega c}\, \tau\, \text{sinc}\left(\frac{\tau\omega}{2\pi}\right) \qquad \left(1 - \frac{2|t-c|}{\tau}\right) P_\tau(t-c) \leftrightarrow e^{-j\omega c} \frac{\tau}{2} \text{sinc}^2\left(\frac{\tau\omega}{2\pi}\right)$$

$$X(\omega) = e^{-j\omega\frac{1}{2}} \frac{3}{2} \text{sinc}\left(\frac{\frac{3}{2}\omega}{2\pi}\right) - e^{-j\omega \cdot 1} \frac{2}{2} \text{sinc}^2\left(\frac{2\omega}{2\pi}\right)$$

$$= \frac{3}{2} e^{-j\omega/2} \text{sinc}\left(\frac{3\omega}{4\pi}\right) - e^{-j\omega} \text{sinc}^2\left(\frac{\omega}{\pi}\right)$$

7. (a) (i) $x(t)$ real, odd $\Rightarrow X(\omega)$ imaginary, odd

(ii) $x(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\,d\omega \Rightarrow \int_{-\infty}^{\infty} X(\omega)\,d\omega = 2\pi\, x(0) = 0$

(iii) If $X(\omega)$ were periodic, it could be written as a sum of sinusoids in ω, $\Rightarrow x(t)$ would have δ-functions.

$x(t)$ doesn't have δ-functions, so $X(\omega)$ not periodic

(b) (i) $x(t)$ neither even nor odd, so $X(\omega)$ complex

(ii) $\int_{-\infty}^{\infty} X(\omega)\,d\omega = 2\pi\, x(0) = 0$

(iii) $x(t)$ has δ-function so $X(\omega)$ periodic

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt = \int_{-\infty}^{\infty} \delta(t-2)e^{-j\omega t}\,d\omega = e^{-j\omega 2} = \cos(2\omega) - j\sin(2\omega)$$

$X(\omega)$ complex, periodic

(c) (i) $x(t)$ real, even $\Rightarrow X(\omega)$ real, even

(ii) See (i)

(iii) $\int_{-\infty}^{\infty} X(\omega)\,d\omega = 2\pi\, x(0) = 0$

(iv) $x(t)$ does not have a δ-function, so $X(\omega)$ not periodic

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% EE 341 Homework #5

% Problem 4.6 (e)

% (ii)

figure(1)
clf
k=-5:5;
c = (1-cos(k*pi))./(k.^2*pi^2);
c(k==0) = 1/2;                        %  Take care of c_0
subplot(211)
stem(k,abs(c));
grid
ylabel('|c_k|');
title('Problem 4.16 (e)')

subplot(212)
stem(k,angle(k)*180/pi);
grid
xlabel('k');
ylabel('\angle c_k');
print -dpsc2 p4_16_3_ii.ps

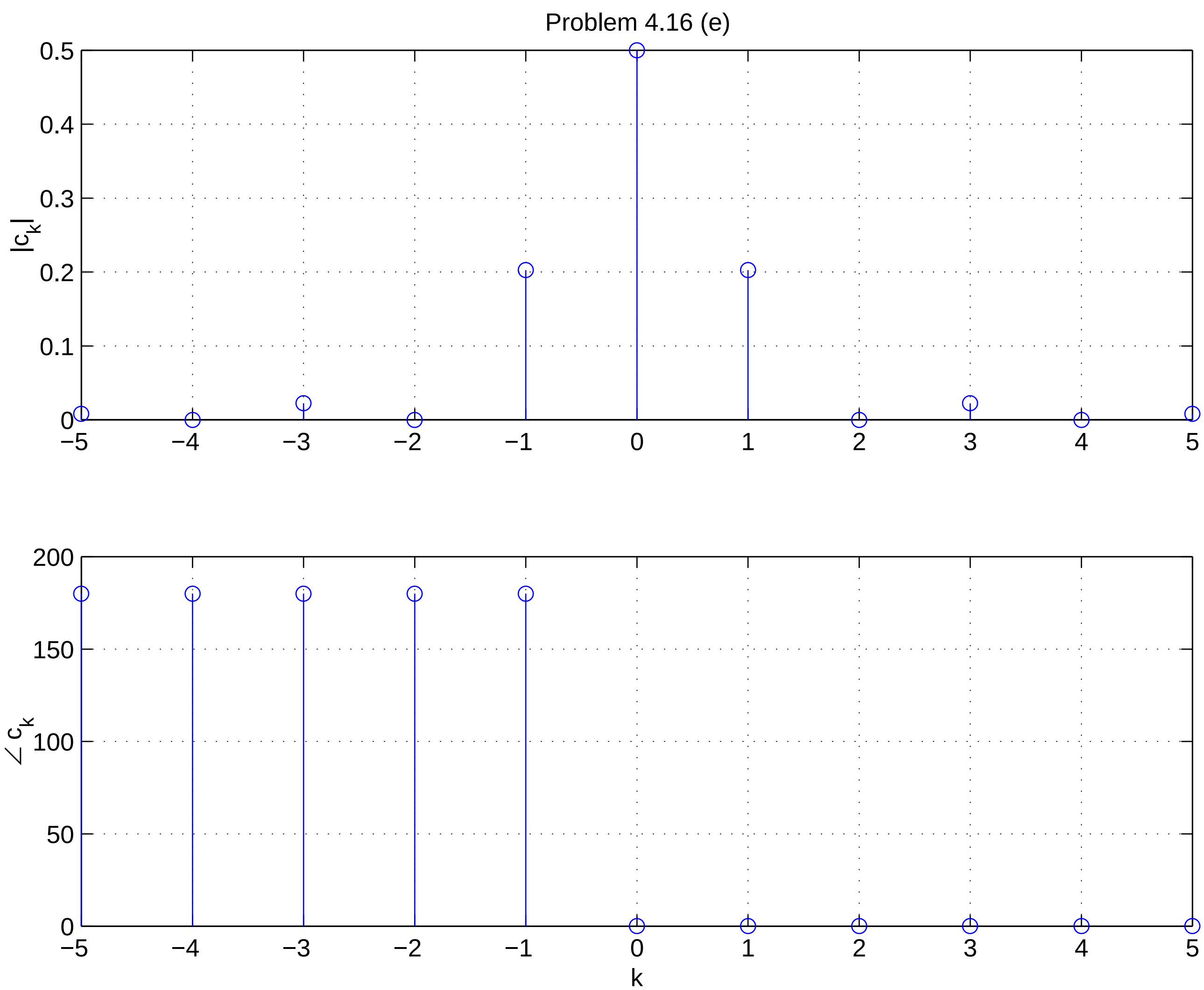
% (iii)

T = 4;
wo = 2*pi/T;
figure(2)
clf
k = -1:1;
c = (1-cos(k*pi))./(k.^2*pi^2);
c(k==0) = 1/2;                        %  Take care of c_0
t = -6:0.01:6;
x1 = zeros(size(t));
for ii = 1:length(k)
    x1 = x1 + c(ii)*exp(j*k(ii)*wo*t);
end
subplot(311)
plot(t,x1);
grid
ylabel('x(t) for N = 1');
title('Problem 4.16 (e)')

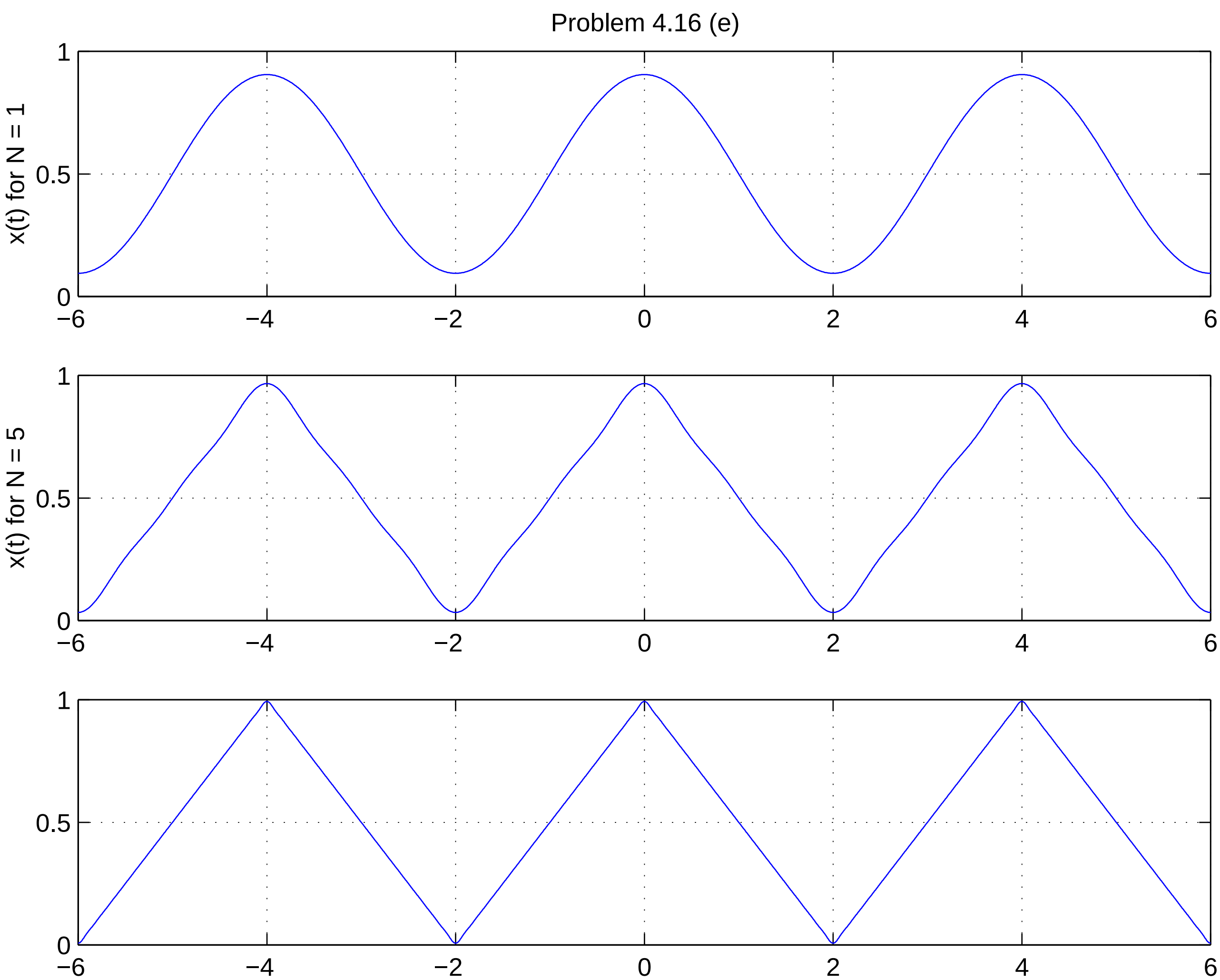
k = -5:5;
c = (1-cos(k*pi))./(k.^2*pi^2);
c(k==0) = 1/2;                        %  Take care of c_0
t = -6:0.01:6;
x2 = zeros(size(t));
for ii = 1:length(k)
    x2 = x2 + c(ii)*exp(j*k(ii)*wo*t);
end
subplot(312)
plot(t,x2);
grid
ylabel('x(t) for N = 5');

k = -30:30;
c = (1-cos(k*pi))./(k.^2*pi^2);
c(k==0) = 1/2;                        %  Take care of c_0
t = -6:0.01:6;
x3 = zeros(size(t));
for ii = 1:length(k)
    x3 = x3 + c(ii)*exp(j*k(ii)*wo*t);
```

```
end
subplot(313)
plot(t,x3);
grid
print -dpsc2 p4_16_3_iii.ps
ylabel('x(t) for N = 30');
xlabel('t (seconds)');
```



Problem 4.16 (e)
0.5
0.4
0.3
0.2
0.1
0
$|c_k|$
−5 −4 −3 −2 −1 0 1 2 3 4 5
200
150
100
50
0
$\angle c_k$
−5 −4 −3 −2 −1 0 1 2 3 4 5
k



Problem 4.16 (e)
x(t) for N = 1
x(t) for N = 5
1
0.5
0
−6
−4
−2
0
2
4
6