EE 341 - Homework 6

Due October 7, 2005

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots.

1. Find the Fourier transforms of the following signals:
   (a) $x(t) = e^{-2t}\cos(2\pi t)u(t)$
   (b) $x(t) = \cos^2(2\pi t)\sin(2\pi t)$
   (c) $x(t) = \text{sinc}(t) * \text{sinc}(2t)$
   (d) $x(t) = (1 - e^{-t})u(t)$
2. Sketch the spectra ($|X(\omega)|$ and $\angle X(\omega)$) for the following signals:
   (a) $x(t) = \text{sinc}(t - 2)$
   (b) $x(t) = \text{sinc}^2(t)\cos(8\pi t)$
   (c) $x(t) = \cos(\pi t - 0.25\pi)$
   (d) $x(t) = \cos(\pi t)p_1(t)$
   (e) $x(t) = \cos(\pi t)p_{100}(t)$
3. Find the Fourier transform of the following signals and sketch their spectra ($|X(\omega)|$ and $\angle X(\omega)$):
   (a) $x(t) = \delta(t + 1) + \delta(t - 1)$
   (b) $x(t) = \delta(t + 1) - \delta(t - 1)$
4. Sketch the spectra ($|Y(\omega)|$ and $\angle Y(\omega)$) where $y(t) = m(t)x(t)$ (the signal $x(t)$ which has been modulated by the signal $m(t)$):
   (a) $X(\omega) = p_1(\omega)$ $m(t) = \cos(\pi t)$
   (b) $X(\omega) = p_4(\omega)$ $m(t) = \cos(\pi t)$
   (c) $X(\omega) = p_1(\omega)$ $m(t) = \cos(10\pi t)$
5. Problem 4.24.

EE 341 HW #6

1. (a) $x(t) = e^{-2t}\cos(2\pi t)\,u(t)$

$$e^{-2t}u(t) \Leftrightarrow \frac{1}{j\omega+2}$$

$$e^{-2t}\cos(2\pi t)\,u(t) \Leftrightarrow \frac{1}{2}\left[\frac{1}{j(\omega+2\pi)+2} + \frac{1}{j(\omega-2\pi)+2}\right]$$

(b) $x(t) = \cos^2(2\pi t)\sin(2\pi t)$

$$\sin(2\pi t) \Leftrightarrow j\pi\left[\delta(\omega+2\pi) - \delta(\omega-2\pi)\right]$$

$$\cos(2\pi t)\sin(2\pi t) \Leftrightarrow j\pi\left[\delta(\omega+2\pi+2\pi) - \delta(\omega-2\pi+2\pi) + \delta(\omega+2\pi-2\pi) - \delta(\omega-2\pi-2\pi)\right]$$

$$= j\pi\left[\delta(\omega+4\pi) - \delta(\omega-4\pi)\right]$$

$$\cos^2(2\pi t)\sin(2\pi t) \Leftrightarrow j\pi\left[\delta(\omega+4\pi+2\pi) - \delta(\omega-4\pi+2\pi) + \delta(\omega+4\pi-2\pi) - \delta(\omega-4\pi-2\pi)\right]$$

$$= j\pi\left[\delta(\omega+6\pi) + \delta(\omega+2\pi) - \delta(\omega-2\pi) - \delta(\omega-6\pi)\right]$$

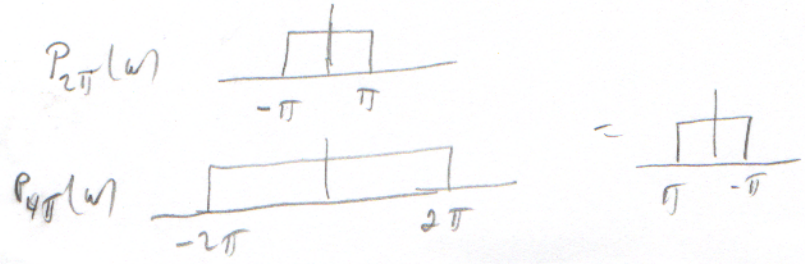
(c) $x(t) = \mathrm{sinc}(t) * \mathrm{sinc}(2t)$

$$\mathrm{sinc}(t) = \frac{1}{2\pi}\left[2\pi\,\mathrm{sinc}\left(\frac{2\pi t}{2\pi}\right)\right] \Leftrightarrow \frac{1}{2\pi}\left(2\pi\,P_{2\pi}(\omega)\right)$$

$$\mathrm{sinc}(2t) = \frac{1}{4\pi}\left[4\pi\,\mathrm{sinc}\left(\frac{4\pi t}{2\pi}\right)\right] \Leftrightarrow \frac{1}{4\pi}\left[2\pi\,P_{4\pi}(\omega)\right]$$

$$\mathrm{sinc}(t) * \mathrm{sinc}(2t) \Leftrightarrow \frac{1}{2\pi}\left(2\pi\,P_{2\pi}(\omega)\right)\frac{1}{4\pi}\left(2\pi\,P_{4\pi}(\omega)\right)$$

$$= \frac{1}{2}P_{2\pi}(\omega)\,P_{4\pi}(\omega)$$



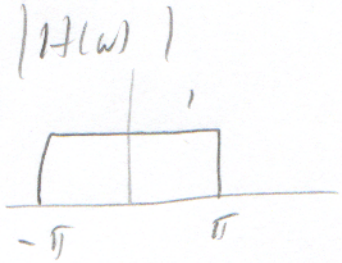
$$\mathrm{sinc}(t) * \mathrm{sinc}(2t) \Leftrightarrow \frac{1}{2}P_{2\pi}(\omega)$$

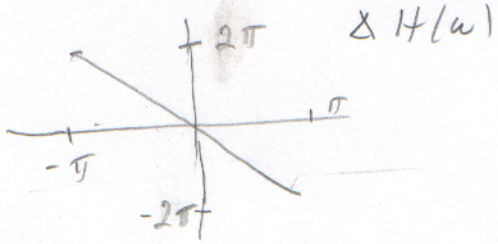
(d) $x(t) = (1-e^{-t})\,u(t)$

$u(t) - e^{-t}u(t) \Leftrightarrow \pi\delta(\omega) + \frac{1}{j\omega} + \frac{1}{j\omega+1}$

2(a) $x(t) = \text{sinc}(t-2)$ $\quad \text{sinc}(t) \Leftrightarrow P_{2\pi}(\omega)$

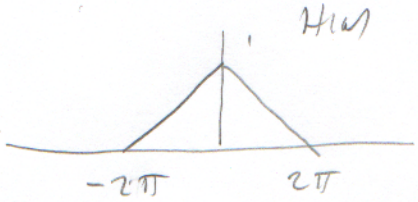
$\text{sinc}(t-2) \Leftrightarrow e^{-j2\omega} P_{2\pi}(\omega)$



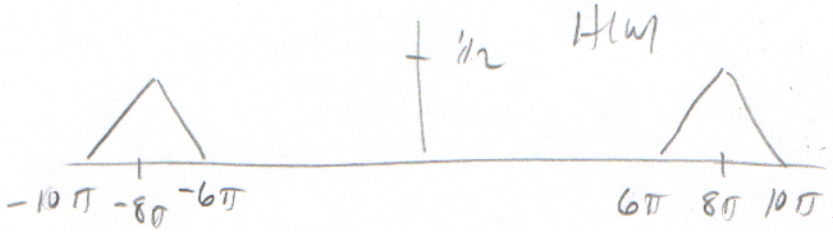


(b) $x(t) = \text{sinc}^2(t)\cos(8\pi t)$

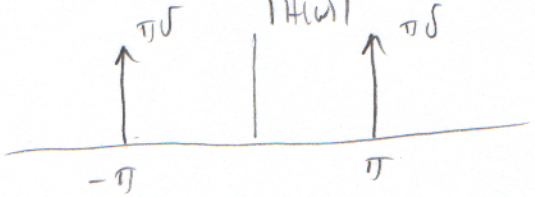
$\text{sinc}^2(t) = \frac{1}{2\pi}\left[\frac{4\pi}{2}\text{sinc}^2\left(\frac{4\pi t}{4\pi}\right)\right] \Leftrightarrow \frac{1}{2\pi}\left[2\pi\left(1-\frac{2|\omega|}{4\pi}\right)P_{4\pi}(\omega)\right]$

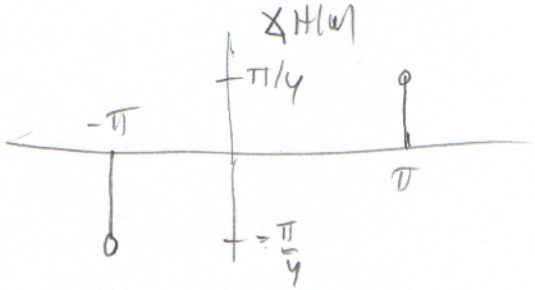


Multiply by $\cos(8\pi t)$ shifts by $\pm 8\pi$) and scales by $\frac{1}{2}$



(c) $x(t) = \cos(\pi t - 0.25\pi) \Leftrightarrow \pi\left[e^{j\frac{\pi}{4}}\delta(\omega+\pi) + e^{-j\frac{\pi}{4}}\delta(\omega-\pi)\right]$





(d) $x(t) = \cos(\pi t)\, P_1(t)$

$P_1(t) \longleftrightarrow \text{sinc}\left(\frac{\omega}{2\pi}\right)$

$\cos(\pi t)\, P_1(t) \Longleftrightarrow \text{sinc}\left(\frac{\omega+\pi}{2\pi}\right) + \text{sinc}\left(\frac{\omega-\pi}{2\pi}\right)$

See MATLAB for plot

(e) $x(t) = \cos(\pi t)\, P_{100}(t)$

$P_{100}(t) \Longleftrightarrow 100\,\text{sinc}\left(\frac{50\omega}{\pi}\right)$

$\cos(\pi t)\, P_{100}(t) \Longleftrightarrow 100\,\text{sinc}\left(\frac{50(\omega+\pi)}{\pi}\right) + \text{sinc}\left(\frac{50\omega-\pi}{\pi}\right)$

See MATLAB for plot

3. (a) $x(t) = \delta(t+1) + \delta(t-1)$

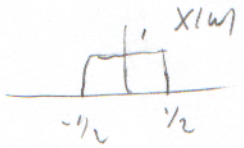
$\delta(t) \Longleftrightarrow 1$

$\delta(t+1) \Longleftrightarrow e^{j\omega} \qquad \delta(t-1) \Longleftrightarrow e^{-j\omega}$

$\delta(t+1) + \delta(t-1) \Longleftrightarrow e^{j\omega} + e^{-j\omega} = 2\cos(\omega)$

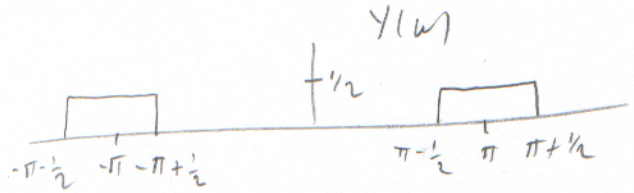
(b) $x(t) = \delta(t+1) - \delta(t-1) \Longleftrightarrow e^{j\omega} - e^{-j\omega} = 2j\sin(\omega)$

4. (a) $X(\omega) = P_1(\omega)$

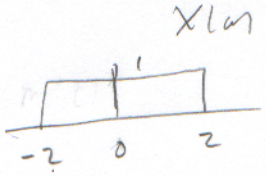


$m(t) = \cos(\pi t)$

$Y(\omega) = \frac{1}{2}[X(\omega+\pi) + X(\omega-\pi)] = \frac{1}{2}[P_1(\omega+\pi) + P_1(\omega-\pi)]$

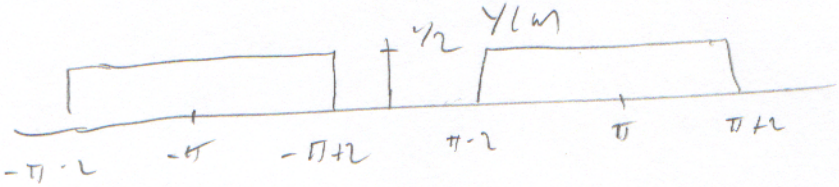


(b) $X(\omega) = P_4(\omega)$

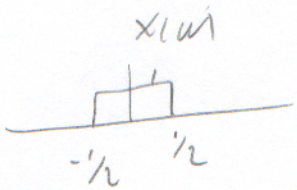


$m(t) = \cos(\pi t)$

$$Y(\omega) = \frac{1}{2}\left[P_4(\omega+\pi) + P_4(\omega-\pi)\right]$$

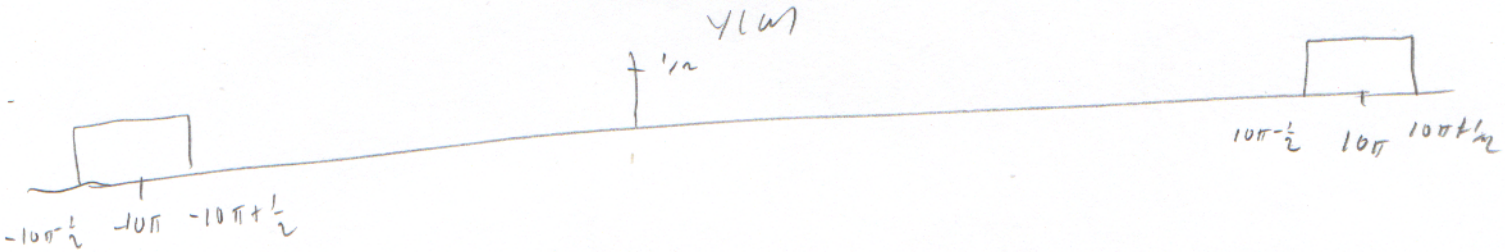


(c) $X(\omega) = P_1(\omega)$



$m(t) = \cos(10\pi t)$

$$Y(\omega) = \frac{1}{2}\left(P_1(\omega+10\pi) + P_1(\omega-10\pi)\right)$$



5. Problem 4.24

$$X(\omega) = \begin{cases} 2 & |\omega| < \pi \\ 0 & \text{otherwise} \end{cases} = 2P_{2\pi}(\omega)$$

$$V(\omega) = X(\omega-\omega_0) + X(\omega+\omega_0) \Rightarrow v(t) = 2x(t)\cos(\omega_0 t)$$

(a) $2P_{2\pi}(\omega) = \frac{1}{\pi}(2\pi P_{2\pi}(\omega)) \Longleftrightarrow \frac{1}{\pi}\left(2\pi \operatorname{sinc}\left(\frac{2\pi t}{2\pi}\right)\right) = 2\operatorname{sinc}(t)$

(b) $v(t) = 2\cos(\omega_0 t)\, x(t) = 4\cos(\omega_0 t)\operatorname{sinc}(t)$

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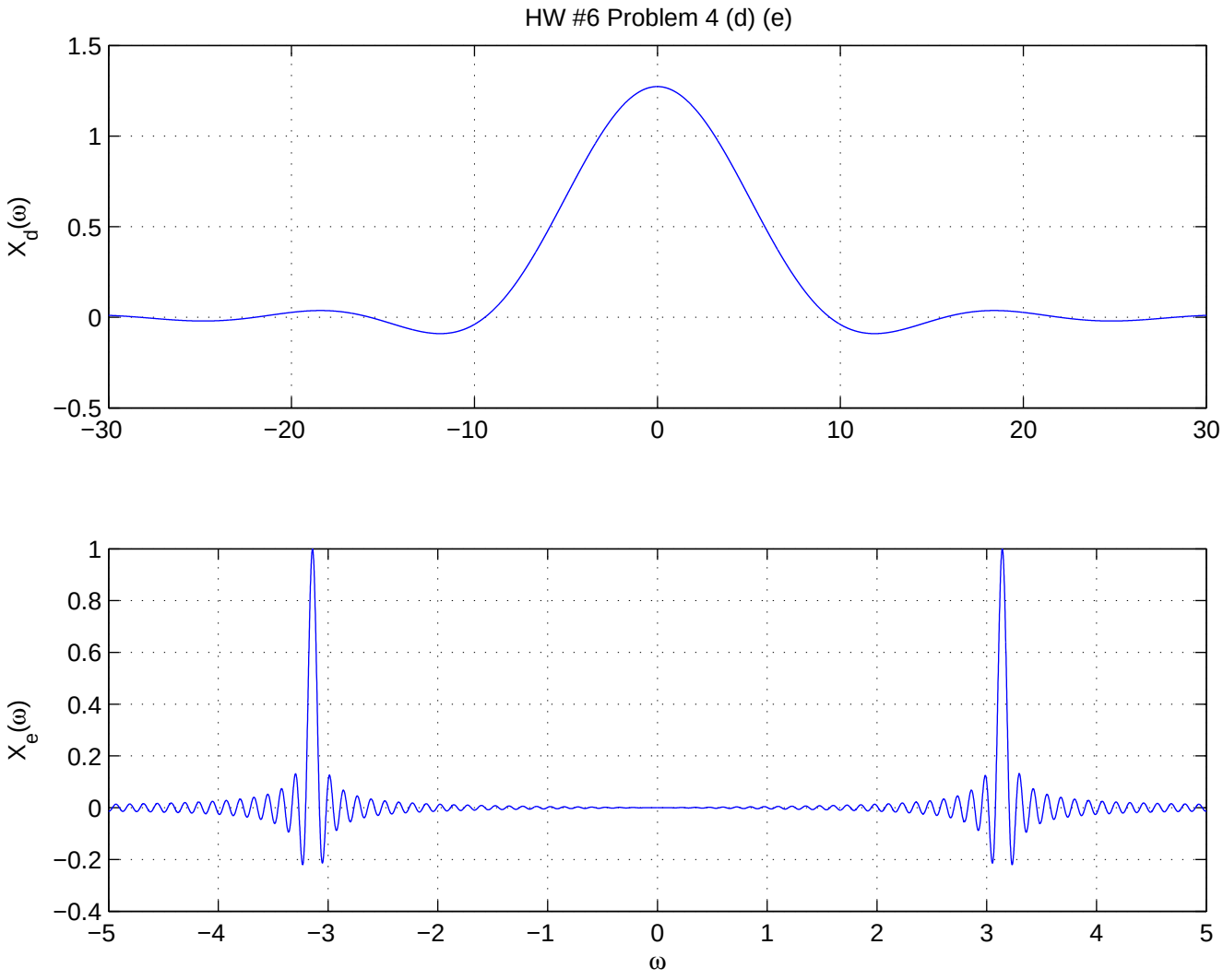
% EE 341 HW #6

% Problem 3
figure(1)
clf

% (d)
w = -30:0.001:30;
X = sinc((w+pi)/(2*pi)) + sinc((w-pi)/(2*pi));
subplot(211)
plot(w,X);
grid
ylabel('X_d(\omega)')
title('HW #6 Problem 4 (d) (e)')

% (e)
w = -5:0.001:5;
X = sinc(50*(w+pi)/pi) + sinc(50*(w-pi)/pi);
subplot(212)
plot(w,X);
grid
ylabel('X_e(\omega)')
xlabel('\omega')

print -dpsc2 hw6_p4.ps
```


HW #6 Problem 4 (d) (e)
$X_d(\omega)$
$X_e(\omega)$
ω