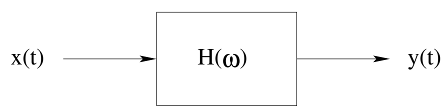
EE 341 - Homework 7

Due October 12, 2005

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots.

1. Problem 5.13
2. Problem 5.19
3. Problem 5.20
4. Problem 5.22
5. Problem 5.23
6. Problem 5.25
7. Problem 5.27
8. Consider the following system:



The frequency response of the filter is

$$H(\omega) = 1 - e^{-j\omega/2}$$

The input $x(t)$ is:

$$x(t) = 5 + 2\cos(\pi\, t) + 3\sin(2\,\pi\, t)$$

Find the output $y(t)$.

EE 341
HW # 7

1. Problem 5.13

$$x(t) = 1 + 4\cos(2\pi t) + 8\sin(3\pi t - 90^0)$$
$$y(t) = 2 - 2\sin(2\pi t)$$

(a) Can find $H(\omega)$ for $\omega = 0,\ 2\pi,\ 3\pi$

(b) $\omega = 0$: output is twice input, no phase change $H(0) = 2$

$\omega = 2\pi$: output is $-\frac{1}{2}$ of input, no phase change $H(2\pi) = -\frac{1}{2}$

$\omega = 3\pi$: output is zero; $H(\omega) = 0$

2. Problem 5.19

$$H(\omega) = \begin{cases} 6e^{-j2\omega} & \omega > 3,\ \omega < -3 \\ 0 & \text{otherwise} \end{cases}$$

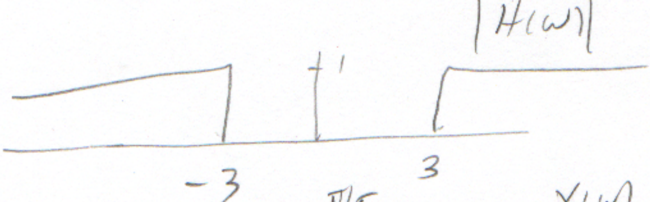
(a) $H(\omega) = 1 - P_6(\omega)e^{-j2\omega}$

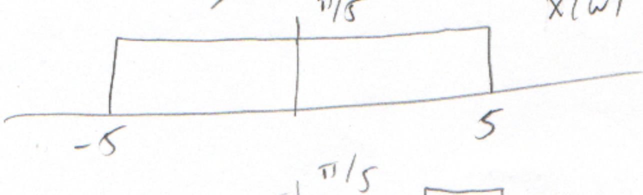
$1 \Leftrightarrow \delta(t)$ $\qquad P_6(\omega) \Leftrightarrow \frac{3}{\pi}\,\text{sinc}\left(\frac{3t}{\pi}\right)$

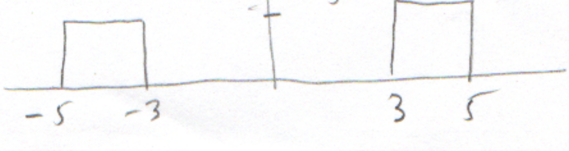
$P_6(\omega)e^{-j2\omega} \Leftrightarrow \frac{3}{\pi}\,\text{sinc}\left(\frac{3(t-t_0)}{\pi}\right)$

$h(t) = 1 - \frac{3}{\pi}\,\text{sinc}\left(\frac{3(t-t_0)}{\pi}\right)$

(b) $x(t) = \text{sinc}\,\frac{5t}{\pi} = \text{sinc}\,\frac{10t}{2\pi}$ $\qquad X(\omega) = \frac{\pi}{5}P_{10}(\omega)$







$Y(\omega) = H(\omega)X(\omega)$
$= \frac{\pi}{5}\left(P_{10}(\omega) - P_6(\omega)\right)e^{-j2\omega}$

$$Y(\omega) = \frac{2\pi}{10} P_{10}(\omega) e^{-j2\omega} = \frac{3}{5}\frac{2\pi}{6} P_6(\omega) e^{-j2\omega}$$

$$y(t) = \text{sinc}\left(\frac{10(t-t_0)}{2\pi}\right) - \frac{3}{5}\,\text{sinc}\left(\frac{6(t-t_0)}{2\pi}\right)$$

$$y(t) = \text{sinc}\left(\frac{5(t-t_0)}{\pi}\right) - \frac{3}{5}\,\text{sinc}\left(\frac{3(t-t_0)}{\pi}\right)$$

(c) $T = 2 \qquad \omega_0 = \frac{2\pi}{T} = \pi$

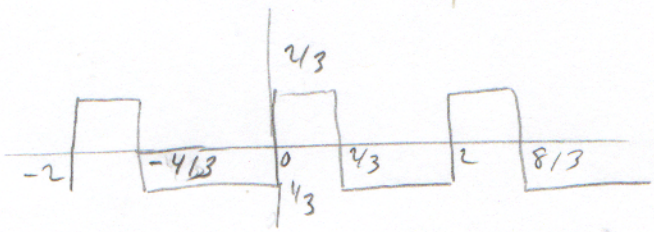
Input signal has components with freq $0, \pm\omega_0, \pm 2\omega_0, \ldots$

Filter will block 0 freq (DC), pass all other freqs.

The DC component of $x(t)$ is

$$C_0^x = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{2}\int_0^{2/3} dt = \frac{1}{3}$$

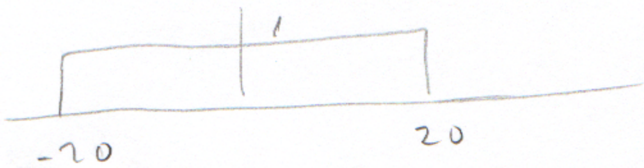
System shifts signal down by $1/3$



3. Problem 5.20

$$x(t) = 4 + 2\cos(10t + \pi/4) + 3\cos(30t - \pi/2)$$

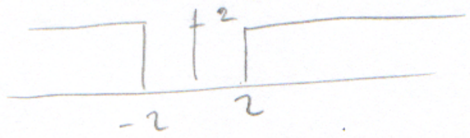
a)



Filter passes DC, 10 rad/sec; blocks 30 rad/sec

$$y(t) = 4 + 2\cos(10t + \pi/4)$$

(b)



Filter blocks DC, passes 10 rad/sec & 30 rad/sec with a gain of 2

$y(t) = 4\cos(10t + \pi/4) + 6\cos(30t - \pi/2)$

(c) $H(\omega) = \text{sinc}(\omega/20)$

$H(0) = \text{sinc}(0) = 1$

$H(10) = \text{sinc}\left(\frac{10}{20}\right) = \text{sinc}\left(\frac{1}{2}\right)$

$H(30) = \text{sinc}\left(\frac{30}{20}\right) = \text{sinc}\left(\frac{3}{2}\right)$

$y(t) = H(0)\cdot 4 + H(10)\cdot 2\cos(10t + \pi/4) + H(30)\cdot 3\cos(30t - \pi/2)$

$= 4 + 1.274\cos(10t + \pi/4) + 0.636\cos(30t - \pi/2)$

(d) From the graph,

$H(0) = 0$

$H(10) = 0.36e^{j2.2}$

$H(30) = 0.92e^{j0.8}$

$y(t) = 0\times 4 + 0.36\times 2\cos(10t + \pi/4 + 2.2) + 0.92\times 3\cos(30t - \pi/2 + 0.8)$

$= 0.72\cos(10t + 3) + 2.76\cos(30t - 0.77)$

4. Problem 5.22

$x(t) = \text{sinc}\left(\frac{t}{2\pi}\right)(\cos 3t)^2 + \text{sinc}\left(\frac{t}{2\pi}\right)\cos t$

$= \text{sinc}\left(\frac{t}{2\pi}\right)\left(\frac{1}{2} + \frac{1}{2}\cos 6t\right) + \text{sinc}\left(\frac{t}{2\pi}\right)\cos t$

$= \frac{1}{2}\text{sinc}\left(\frac{t}{2\pi}\right) + \frac{1}{2}\text{sinc}\left(\frac{t}{2\pi}\right)\cos 6t + \text{sinc}\left(\frac{t}{2\pi}\right)\cos t$

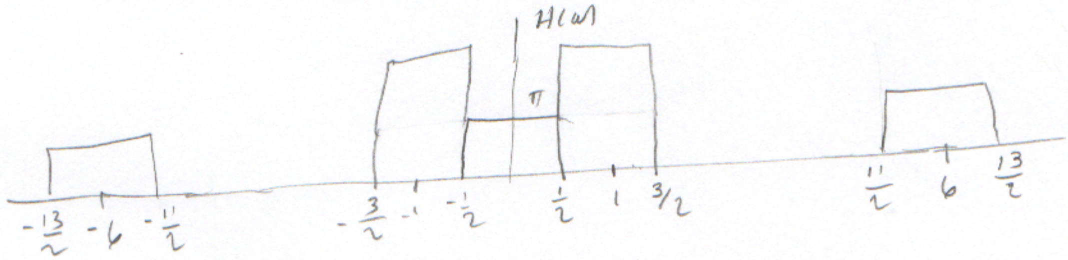
$y(t) = \text{sinc}\left(\frac{t}{2\pi}\right)$

(a) $\text{sinc}\left(\frac{t}{2\pi}\right) \Leftrightarrow 2\pi p_1(\omega)$

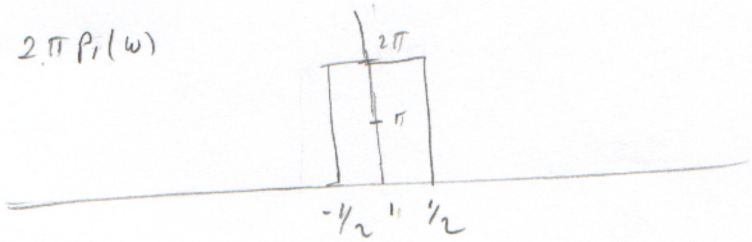
$$\text{sinc}\left(\frac{t}{2\pi}\right)\cos 6t \Leftrightarrow 2\pi\left[p_1(\omega+6) + p_1(\omega-6)\right]$$

$$\text{sinc}\left(\frac{t}{2\pi}\right)\cos t \Leftrightarrow 2\pi\, p_1(\omega+1) + 2\pi\, p_1(\omega-1)$$

$$H(\omega) = \pi p_1(\omega) + \pi p_1(\omega+6) + \pi p_1(\omega-6) + 2\pi p_1(\omega+1) + 2\pi p_1(\omega-1)$$



$Y(\omega) = 2\pi p_1(\omega)$

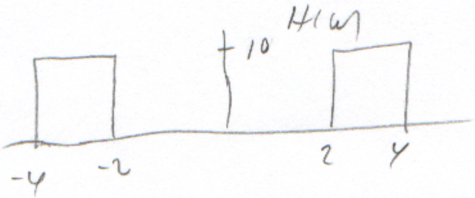


$H(\omega)$ passes signals from $-1 < \omega < 1$ with a gain of 2, and blocks all other signals. There is no change in phase

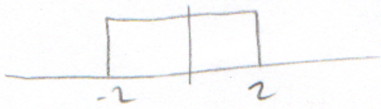
$$H(\omega) = \begin{cases} 2 & -1 < \omega < 1 \\ 0 & |\omega| > 1 \end{cases}$$

(b) See MATLAB

5. Problem 5.23

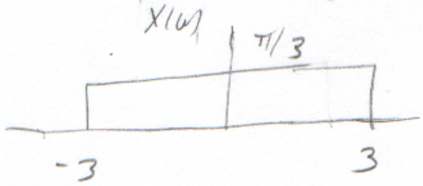


(a) $x(t) = \text{sinc}\left(\frac{2t}{\pi}\right) = \frac{1}{4}\left[4\left(\frac{4t}{2\pi}\right)\right]$ $\qquad X(\omega) = \frac{\pi}{2} p_4(\omega)$

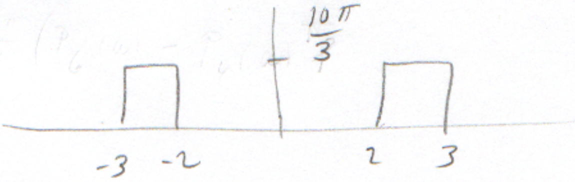


No signal gets through, $Y(\omega) = 0$, $y(t) = 0$

(b) $x(t) = \text{sinc}(3t/\pi) = \frac{1}{6}\left(6\,\text{sinc}\left(\frac{6t}{2\pi}\right)\right)$; $X(\omega) = \frac{\pi}{3} P_6(\omega)$



Filter passes frequencies from 2 to 3 rad/s, with a gain of 10 and a phase shift of $e^{-j4\omega}$



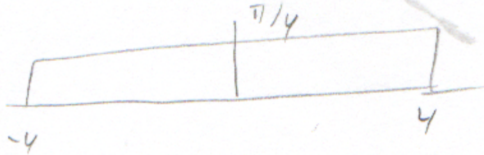
$$Y(\omega) = \frac{10\pi}{3}\left(P_6(\omega) - P_4(\omega)\right)e^{-j4\omega}$$

$$P_\tau(\omega) \Leftrightarrow \frac{\tau}{2\pi}\,\text{sinc}\left(\frac{\tau t}{2\pi}\right) \qquad P_\tau(\omega)e^{-j4\omega} \Leftrightarrow \frac{\tau}{2\pi}\,\text{sinc}\left(\frac{\tau(t-4)}{2\pi}\right)$$

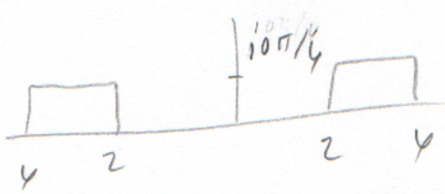
$$y(t) = \frac{10\pi}{3}\left(\frac{6}{2\pi}\,\text{sinc}\left(\frac{6(t-4)}{2\pi}\right) - \frac{4}{2\pi}\,\text{sinc}\left(\frac{4(t-4)}{2\pi}\right)\right)$$

$$= 10\,\text{sinc}\left(\frac{3(t-4)}{\pi}\right) - \frac{20}{3}\,\text{sinc}\left(\frac{2(t-4)}{\pi}\right)$$

(c) $x(t) = \text{sinc}(4\pi t) = \frac{1}{8}\left(8\,\text{sinc}\left(\frac{8t}{2\pi}\right)\right) \Leftrightarrow \frac{\pi}{4} P_8(\omega)$



Passes 2 to 4 rad/sec, gain of 10 phase of $e^{-j4\omega}$



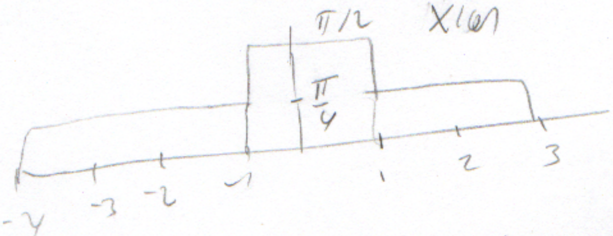
$$Y(\omega) = \frac{10\pi}{4}\left(P_8(\omega) - P_4(\omega)\right)e^{-j4\omega}$$

$$y(t) = \frac{10\pi}{4}\left(\frac{8}{2\pi}\,\text{sinc}\left(\frac{8(t-4)}{2\pi}\right) - \frac{4}{2\pi}\,\text{sinc}\left(\frac{4(t-4)}{2\pi}\right)\right)$$

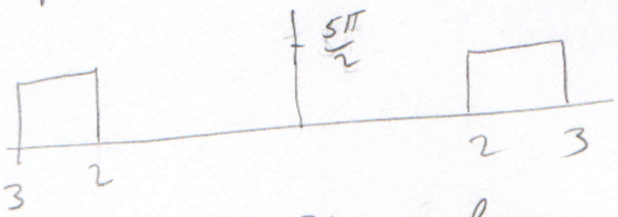
$$= 10\,\text{sinc}\left(\frac{4(t-4)}{2\pi}\right) - 5\,\text{sinc}\left(\frac{2(t-4)}{\pi}\right)$$

(d) $x(t) = \text{sinc}\left(\frac{2t}{\pi}\right)\cos t$

$\text{sinc}\,\frac{2t}{\pi} \Leftrightarrow \frac{\pi}{2} P_4(\omega)$ $\quad$ $\text{sinc}\left(\frac{2t}{\pi}\right)\cos t \Leftrightarrow \frac{\pi}{4}\left[P_4(\omega+1) + P_4(\omega-1)\right]$



Filter passes 2 to 3 rad/sec, gain 10, phase shift $e^{-j4\omega}$

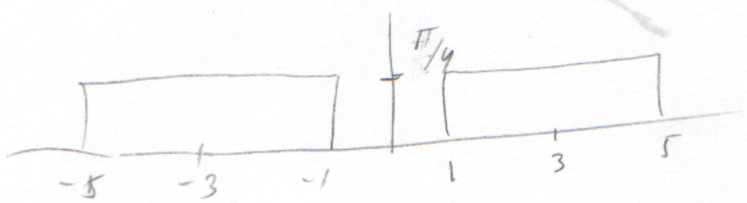


Same as (b) except 3/4 as large

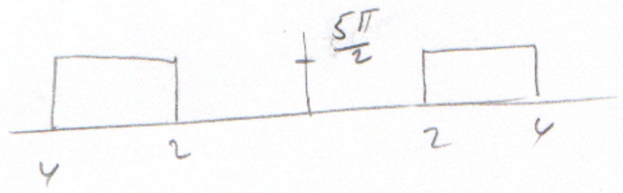
$$y(t) = \frac{3}{4}\left[10\,\text{sinc}\left(\frac{3(t-4)}{\pi}\right) - \frac{20}{3}\,\text{sinc}\left(\frac{2(t-4)}{\pi}\right)\right]$$

$$= \frac{15}{2}\,\text{sinc}\left(\frac{3(t-4)}{\pi}\right) - 5\,\text{sinc}\left(\frac{2(t-4)}{\pi}\right)$$

(e) $x(t) = \text{sinc}\left(\frac{2t}{\pi}\right)\cos(3t) \Leftrightarrow \frac{\pi}{4}\left(P_4(\omega+3) + P_4(\omega-3)\right)$



Filter passes from 2 to 4 rad/sec, gain 10, phase $e^{-j4\omega}$



Same as (c)

$$y(t) = 10\,\text{sinc}\left(\frac{4(t-4)}{\pi}\right) - 5\,\text{sinc}\left(\frac{2(t-4)}{\pi}\right)$$

(f) $x(t) = \text{sinc}\left(\frac{2t}{\pi}\right)\cos 6t \Leftrightarrow \frac{\pi}{4}\left(P_4(\omega+6) - P_4(\omega-6)\right)$

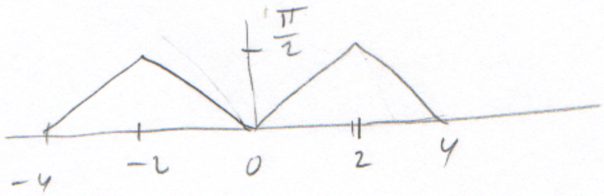
Filter blocks all, $Y(\omega) = 0$, $y(t) = 0$

(g) $x(t) = \text{sinc}^2\left(\frac{t}{\pi}\right)\cos(2t)$

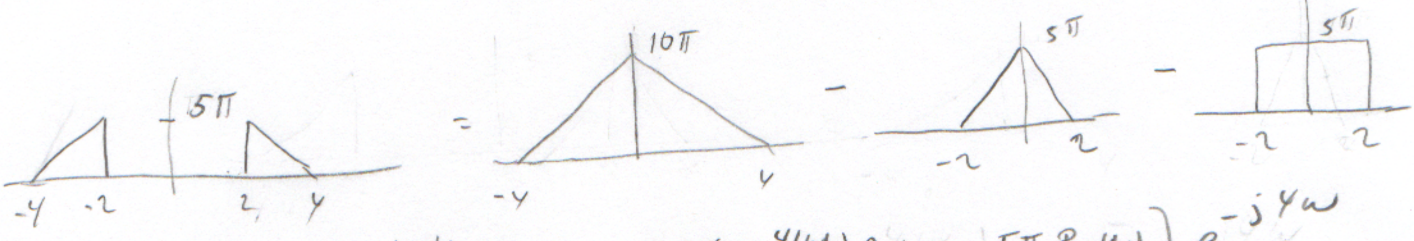
$$\text{sinc}^2\frac{t}{\pi} = \frac{1}{2}\left(\frac{4}{2}\text{sinc}^2\frac{4t}{4\pi}\right) \Leftrightarrow \frac{1}{2}\left(2\pi\left(1 - \frac{2|\omega|}{4}\right)P_4(\omega)\right)$$

$$= \pi\left(1 - \frac{|\omega|}{2}\right)P_4(\omega)$$

$$\text{sinc}^2\left(\frac{t}{\pi}\right)\cos 2t \Leftrightarrow \frac{1}{2}\left[\pi\left(1 - \frac{|\omega+2|}{2}\right)P_4(\omega+2) + \pi\left(1 - \frac{|\omega-2|}{2}\right)P_4(\omega-2)\right)$$



Passes 2 to 4 rad/sec, gain 10 phase shift $e^{-j4\omega}$



$$Y(\omega) = \left[10\pi\left(1 - \frac{2|\omega|}{8}\right)P_8(\omega) - 5\pi\left(1 - \frac{4|\omega|}{2}\right)P_4(\omega) - 5\pi P_4(\omega)\right]e^{-j4\omega}$$

$$= \left[5\left(2\pi\left(1 - \frac{2|\omega|}{8}\right)\right)P_8(\omega) - \frac{5}{2}\left[2\pi\left(1 - \frac{4|\omega|}{2}\right)\right]P_4(\omega) - \frac{5}{2}\left[2\pi P_4(\omega)\right]\right]e^{-j4\omega}$$

$$y(t) = 5\left(\frac{8}{2}\text{sinc}^2\left(\frac{8(t-4)}{4\pi}\right)\right) - \frac{5}{2}\left(\frac{4}{2}\text{sinc}^2\left(\frac{4(t-4)}{4\pi}\right)\right) - \frac{5}{2}\left(4\,\text{sinc}\left(\frac{4(t-4)}{2\pi}\right)\right)$$

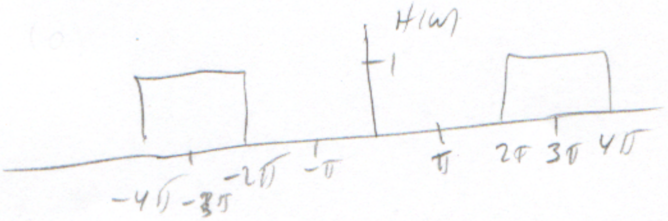
$$= 20\,\text{sinc}^2\left(\frac{2(t-4)}{\pi}\right) - 5\,\text{sinc}^2\left(\frac{t-4}{\pi}\right) - 10\,\text{sinc}\left(\frac{2(t-4)}{\pi}\right)$$

See MATLAB for plots

6. Problem 5.25

$T = 2 \Rightarrow \omega_0 = \frac{2\pi}{T} = \pi$

$$c_k = \begin{cases} 0 & k = 0 \\ 0 & k \text{ even} \\ 1 & k \text{ odd} \end{cases}$$



Only frequency which gets through are 3π and -3π

Signal at 3π shifted by $\pi/2$ $\quad H(3\pi) = e^{j\pi/2} = j$

Signal at -3π shifted by $-\pi/2$ $\quad H(-3\pi) = e^{-j\pi/2} = -j$

$y(t) = H(3\pi)\, c_3^x\, e^{j3\omega_0 t} + H(-3\pi)\, c_{-3}^x\, e^{-j3\omega_0 t}$

$= j e^{j3\pi t} - j e^{-j3\pi t}$

$= -2 \dfrac{e^{j3\pi t} - e^{-j3\pi t}}{2j} = -2\sin(3\pi t)$

7. Problem 5.27

$h(t) = \frac{1}{t}$

This is not in table But $\frac{1}{\omega}$ is in tables so use duality

$\frac{1}{j\omega} \Leftrightarrow -0.5 + u(t)$

$X(t) \Leftrightarrow 2\pi\, x(-\omega)$

$\frac{1}{jt} \Leftrightarrow 2\pi\,[-0.5 + u(-\omega)]$

$$\frac{1}{t} \Leftrightarrow j2\pi\,[-0.5 + u(-\omega)] = \begin{cases} j\pi & \omega < 0 \\ -j\pi & \omega > 0 \end{cases}$$

$x(t) = A\cos w_0 t$

$y(t) = |H(w_0)|\, A\cos(w_0 t + \measuredangle H(w_0))$

$|H(w_0)| = \pi \qquad \measuredangle H(w_0) = -90^\circ$

$y(t) = A\pi\cos(w_0 t - 90^\circ) = A\pi\sin(w_0 t)$

A Hilbert transformer changes $\cos(w_0 t)$ to $\sin(w_0 t)$

8. $x(t) = 5 + 2\cos(\pi t) + 3\sin(2\pi t)$

$y(t) = H(0)\cdot 5 + H(\pi)\cdot 2\cos(\pi t + \measuredangle H(\pi)) + 3\sin(2\pi t + \measuredangle H(2\pi))$

$H(w) = 1 - e^{-jw/2}$

$H(0) = 1 - e^0 = 1 - 1 = 0$

$H(\pi) = 1 - e^{-j\pi/2} = 1 - j \qquad |H(\pi)| = \sqrt{1+1} = \sqrt{2} \qquad \measuredangle H(\pi) = -45^\circ$

$H(2\pi) = 1 - e^{-j\pi} = 1 - (-1) = 2$

$y(t) = 0.5 + \sqrt{2}\cdot 2\cos(\pi t - 45^\circ) + (-2)\, 3\sin(2\pi t)$

$= 2\sqrt{2}\cos(\pi t - 45^\circ) - 6\sin(2\pi t)$

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% EE 341 Homework #7

% Problem 5.22 (b)

figure(1)
clf
subplot(211)
t = -30:0.1:30;
x = sinc(t/(2*pi)).*cos(3*t).^2 + sinc(t/(2*pi)).*cos(t);
plot(t,x)
grid
ylabel('x(t)')
title('Problem 5.22 (b)')
subplot(212)
t = -30:0.1:30;
y = sinc(t/(2*pi));
plot(t,y)
axis([-30 30 -1 2])
grid
ylabel('y(t)')
xlabel('t (seconds)')
print -dpsc2 p5_22_b.ps

% Problem 5.23

figure(2)
clf

% (a) y(t) = 0, so I will not plot

% (b)

t = -20:0.01:20;
subplot(311)
y_b = 10*sinc(3*(t-4)/(2*pi)) - (20/3)*sinc(2*(t-4)/pi);
plot(t,y_b)
grid
ylabel('y_b(t)')
title('Problem 5.23 (b) (c) (d)')

subplot(312)
y_c = 10*sinc(4*(t-4)/(2*pi)) - 5*sinc(2*(t-4)/pi);
plot(t,y_c)
ylabel('y_c(t)')
grid

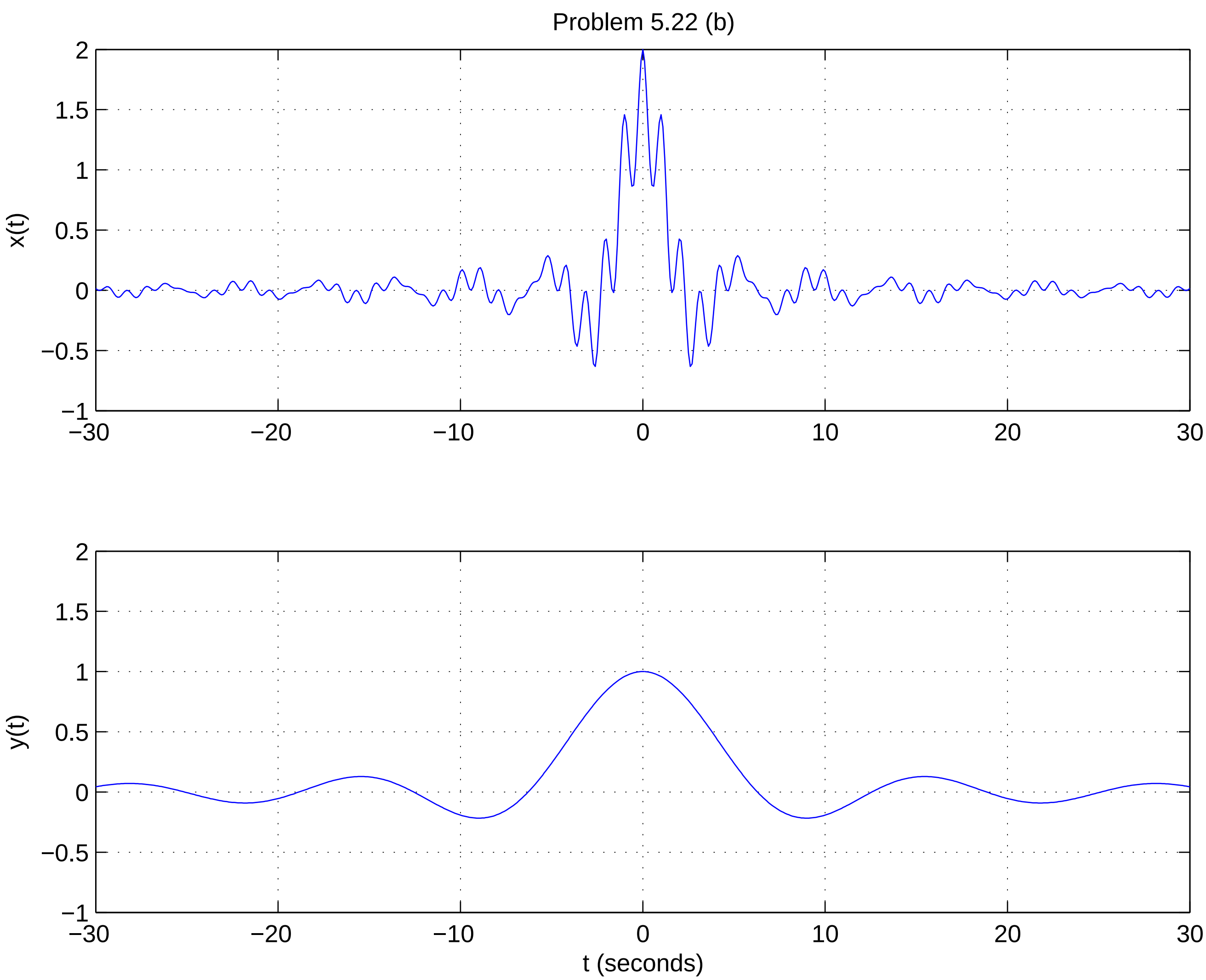
subplot(313)
y_d = (15/2)*sinc(3*(t-4)/(2*pi)) - 5*sinc(2*(t-4)/pi);
plot(t,y_d)
ylabel('y_d(t)')
grid
xlabel('t (seconds)')
print -dpsc2 p5_23_bcd.ps

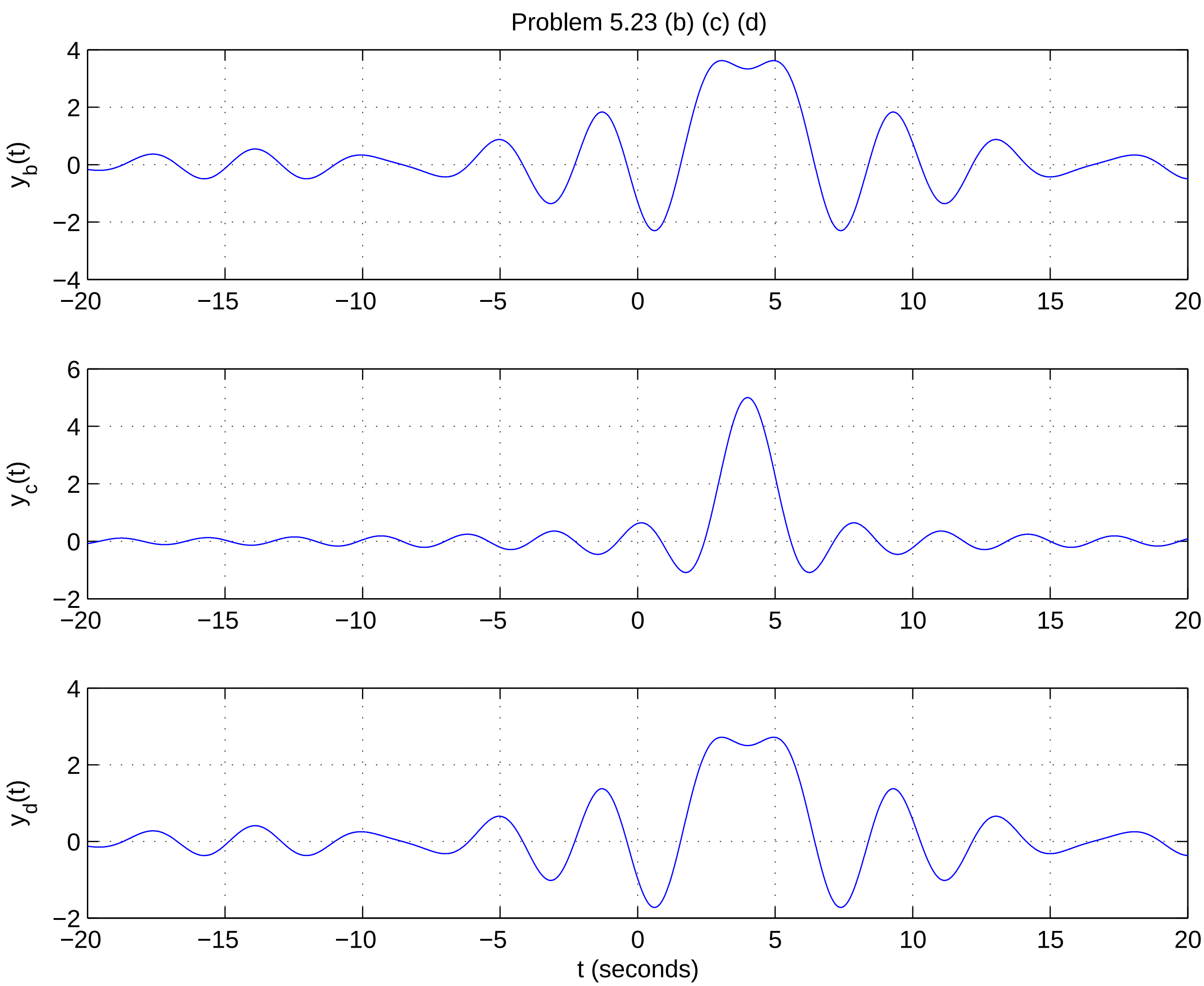
figure(3)
clf

subplot(311)
y_e = 10*sinc(4*(t-4)/(2*pi)) - 5*sinc(2*(t-4)/pi);
plot(t,y_e)
grid
ylabel('y_e(t)')
title('Problem 5.23 (e) (g)')

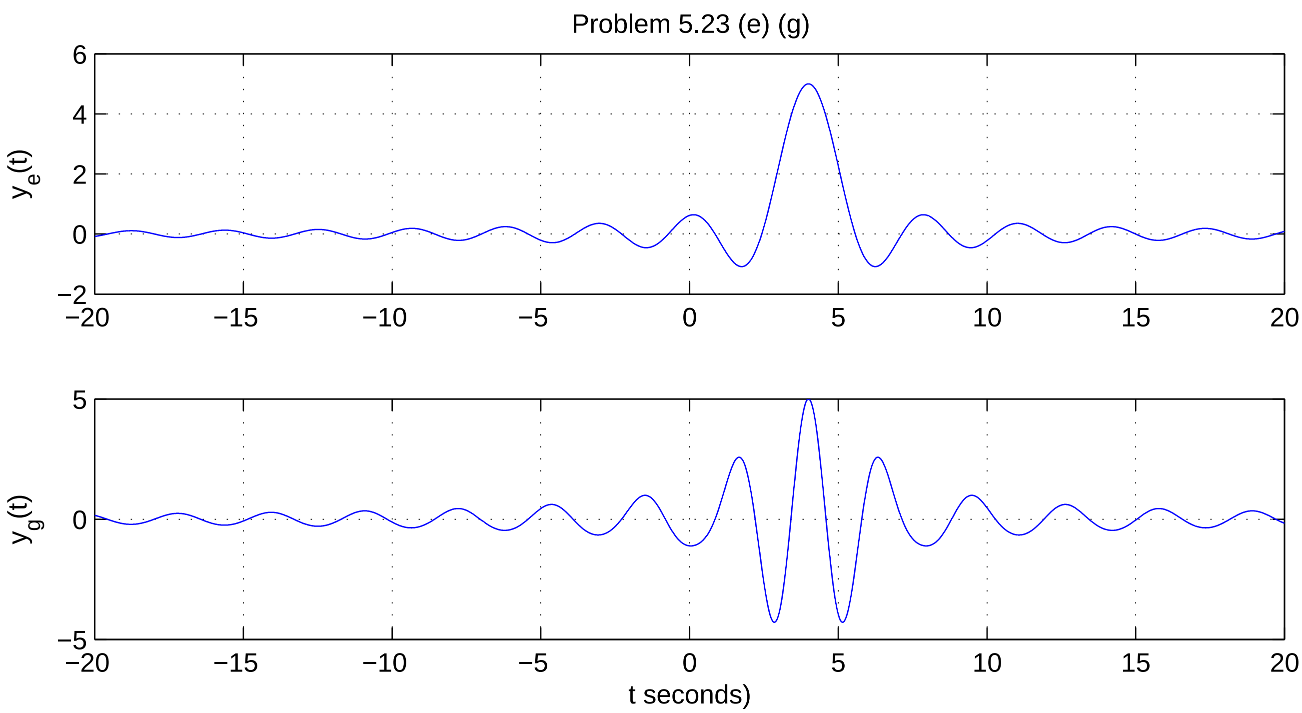
% (f) is zero, so I won't plot

subplot(312)
y_g = 20*sinc(2*(t-4)/pi).^2 - 5*sinc((t-4)/pi).^2 - 10*sinc(2*(t-4)/pi);
plot(t,y_g);
grid
ylabel('y_g(t)')
xlabel('t seconds)')
print -dpsc2 p5_23_eg.ps
```


Problem 5.22 (b)
x(t)
2
1.5
1
0.5
0
−0.5
−1
−30
−20
−10
0
10
20
30
y(t)
t (seconds)



Problem 5.23 (b) (c) (d)
$y_b(t)$
$y_c(t)$
$y_d(t)$
t (seconds)
4
2
0
−2
−4
6
−20
−15
−10
−5
5
10
15
20


Problem 5.23 (e) (g)
$y_e(t)$
$y_g(t)$
t seconds)
6
4
2
0
−2
5
−5
−20
−15
−10
−5
0
5
10
15
20