EE 341 - Homework 8

Due October 19, 2005

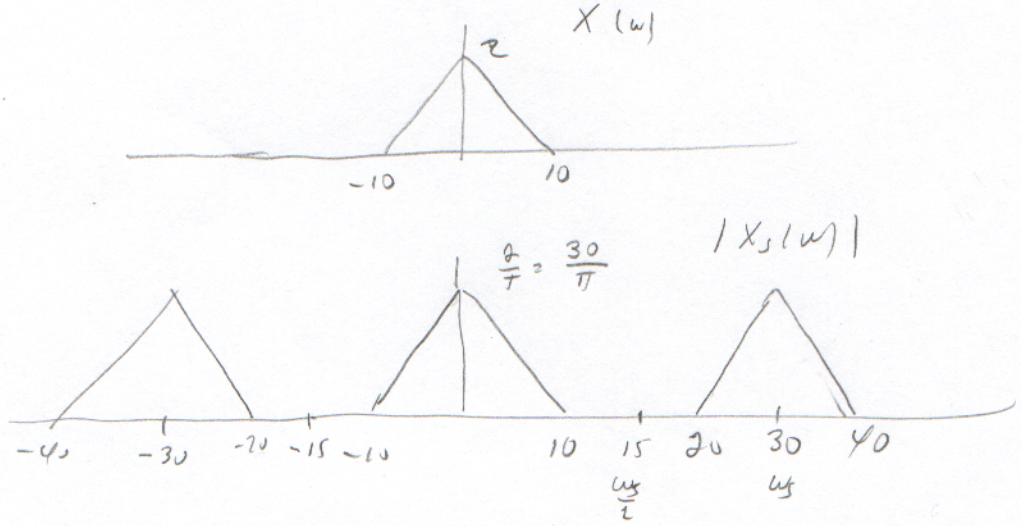
For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots.

1. Problem 5.31
2. Problem 5.34
3. Problem 5.36
4. Let $x(t) = 8\cos(2\pi t) + 6\cos(22\pi t) + 6\sin(32\pi t) + \cos(58\pi t) + \sin(66\pi t)$
   (a) What minimum sampling frequency F_s should be used to avoid aliasing?
   (b) $x(t)$ is sampled at $F_s = 80$ Hz. Sketch the sampled spectrum for $X_s(\omega)$.
   (c) The sampled signal from Part (b) is passed through a low pass filter with a bandwidth of 40 Hz. Sketch the spectrum of the signal $Y_s(\omega)$ which comes out of the low pass filter.
   (d) What frequencies are present in $y_s(t)$ from Part (c)?
5. Repeat Problem 3 for the sampling frequency $F_s = 10$ Hz.
6. The signal $x(t) = \cos(400\pi t) + \sin(800\pi t)$ is sampled with a sampling frequency $F_s = 500$ Hz.
   (a) Use MATLAB to plot the signals $\cos(400\pi t)$, $\sin(800\pi t)$ and $x(t)$. Use a time interval so you have several cycles of each signal, and a time spacing small enough so that the plots look smooth.
   (b) Let $x[n] = \cos(400\pi n T_s) + \sin(800\pi n T_s)$, where $T_s = 1/F_s$. Use the MATLAB `stem` function to plot $\cos(400\pi n T_s)$, $\sin(800\pi n T_s)$ and $x[n]$ (where n is an integer). Compare the sampled signals to the continuous-time signals from Part (a).

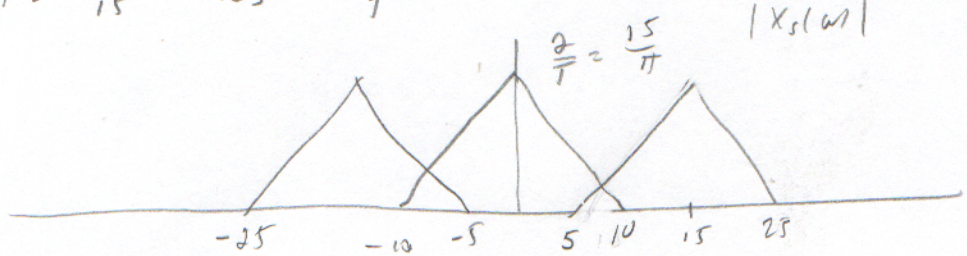
EE 341 HW # 8

1. Problem 5.31

(a) $T = \pi/15$ $\quad \omega_s = \frac{2\pi}{T} = 30$ rad/s



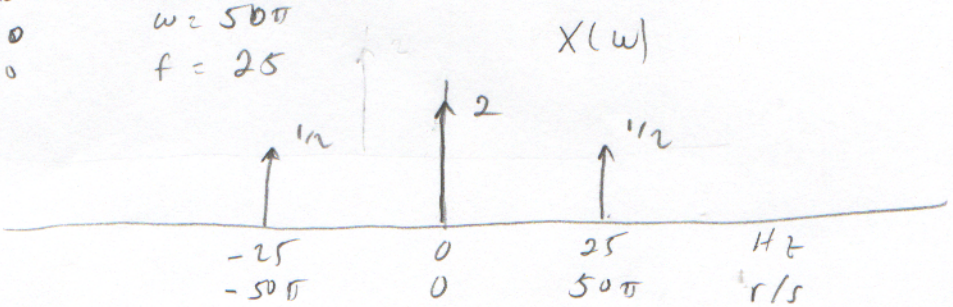
(b) $T = \frac{2\pi}{15}$ $\quad \omega_s = \frac{2\pi}{T} = 15$



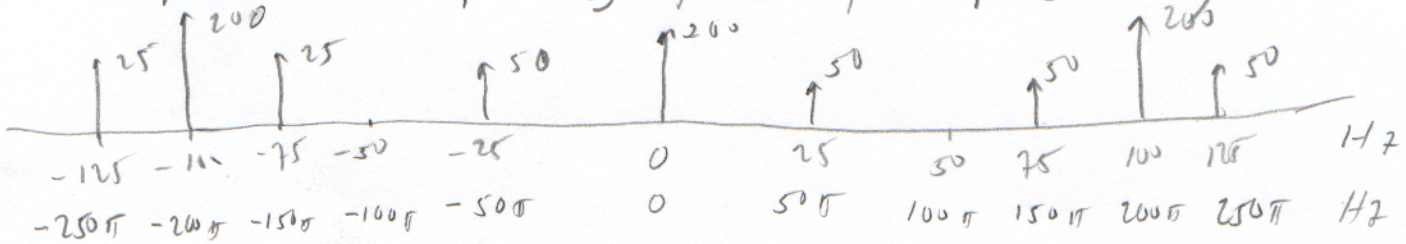
2. Problem 5.34

$x(t) = 2 + \cos(50\pi t)$ $\quad T = 0.01$ s $\quad \omega_s = \frac{2\pi}{T} = 200$ rad/s $\quad F_s = \frac{1}{T} = 100$ Hz

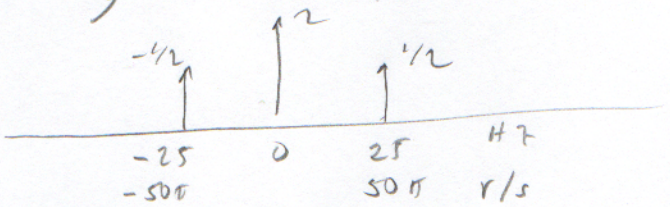
$2 \rightarrow \omega = 0,\ f = 0$ $\quad \cos(50\pi t) \rightarrow \omega = 50\pi,\ f = 25$



(a) Amplitude multiplied by $\frac{1}{T} = 100$; repeats every 100 Hz



(b) Perfect reconstructor keeps frequencies from -50 Hz to 50 Hz, and multiplies by $T = 0.01$



$$Y(\omega) = 2\delta(\omega) + \tfrac{1}{2}\delta(\omega + 50\pi) + \tfrac{1}{2}\delta(\omega - 50\pi)$$

$$y(t) = 2 + \cos(50\pi t)$$

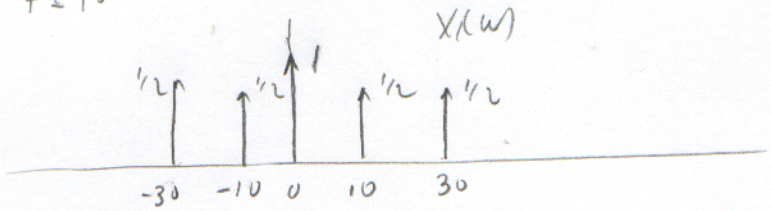
(c) $x(n) = x(nT) = 2 + \cos(50\pi\, n\,(0.01)) = 2 + \cos\left(\frac{\pi n}{2}\right)$

3. Problem 5.36

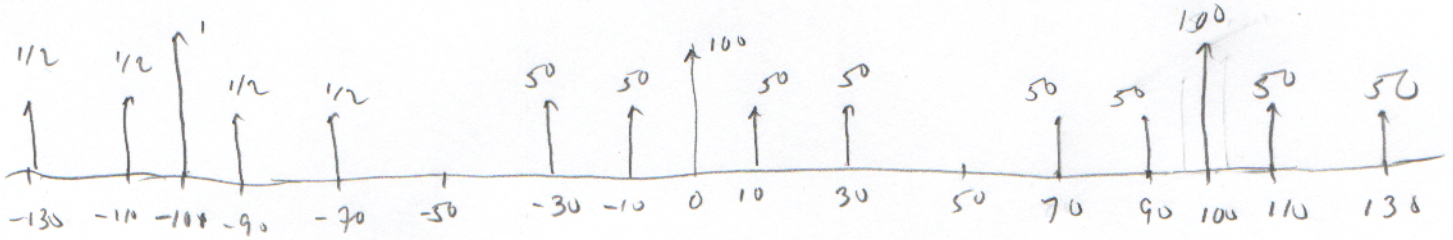
$x(t) = 1 + \cos(20\pi t) + \cos(60\pi t)$, $T = 0.01$ $F_s = \frac{1}{T} = 100$ Hz

$\omega = 0$, $f = 0$; $\omega = 20\pi$, $f = 10$; $\omega = 60\pi$, $f = 30$

$\omega_s = \frac{2\pi}{T} = 200$ rad/s



(a) Amplitude multiplied by $\frac{1}{T} = 100$, repeats every 100 Hz, 200π r/s



(b) $B = 30$ Hz (bandwidth) $F_s > 2B$, so no aliasing

$$y(t) = 1 + \cos(20\pi t) + \cos(60\pi t)$$

(c) $x(n) = x(nT) = 1 + \cos(20\pi\, n\,(0.01)) + \cos(60\pi\, n\,(0.01))$

$$= 1 + \cos(0.2\pi n) + \cos(0.6\pi n)$$

4. $x(t) = 8\cos(2\pi t) + 6\cos(22\pi t) + 6\sin(32\pi t) + \cos(58\pi t) + \sin(66\pi t)$

1 Hz, 11 Hz, 16 Hz, 29 Hz, 33 Hz

(a) Max freq is 33 Hz. To avoid aliasing need $F_s > 2B = 66$ Hz

(b) See MATLAB for sketch

(c) The signal is sampled fast enough to avoid aliasing, so the sampled signal contains the frequencies of the original signal, scaled by $\frac{1}{T} = 80$, plus repeats every 80 Hz.

(c) The filter cuts out all repeats, so the filtered signal is the same as the original signal, except for amplitude scaling. The problem does not state the gain of the filter. I will assume a gain of $\frac{1}{80}$, so that the filtered signal is identical to the original signal.
See MATLAB for plots.

(d) 1 Hz, 11 Hz, 16 Hz, 29 Hz and 33 Hz

5. $x(t) = 1 + \cos(20\pi t) + \cos(60\pi t)$, $F_s = 10$ Hz, $T = 0.1$ sec

$f=0$, $f=10$, $f=30$

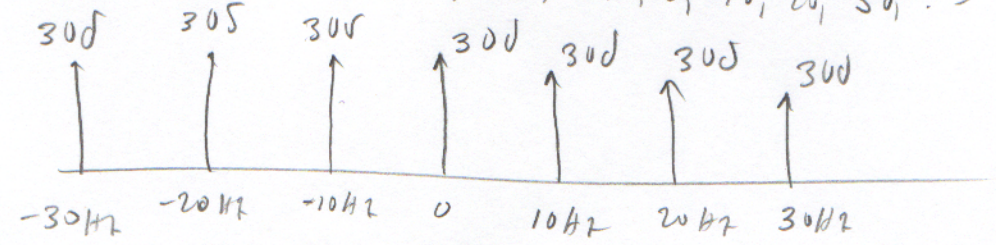
(a) Amplitude multiplied by $\frac{1}{T} = 10$; repeats every 10 Hz

$\cos(20\pi t)$ has a δ fnc of amplitude 5 at 10 Hz and every multiple of ± 10 Hz: ... $-30, -20, -10, 0, 10, 20, 30$, ...
It also has a δ fnc of amplitude 5 at -10 Hz and every multiple of -10 Hz.

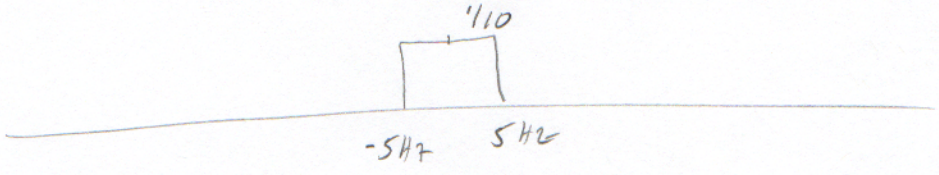
The same will be true for $\cos(60\pi t)$

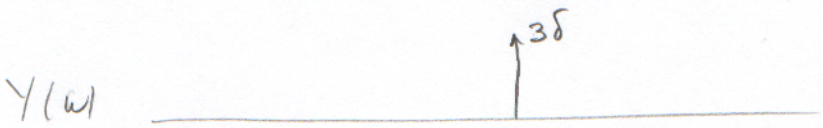
The signal 1 has a δ fnc of amplitude 10 at 0 Hz and every multiple of 10 Hz: ... $-30, -20, -10, 0, 10, 20, 30$ ...

When you add all these together you have δ fncs of amplitude 30 at $-30, -20, -10, 0, 10, 20, 30, \ldots$



(b)





$Y(\omega) = 3\delta(\omega)$

$y(t) = 3$

(c) $x(n) = x(nT) = 1 + \cos(20\pi n(0.1)) + \cos(60\pi n(0.1))$

$= 1 + \cos(2\pi n) + \cos(6\pi n)$

$= 1 + 1 + 1$

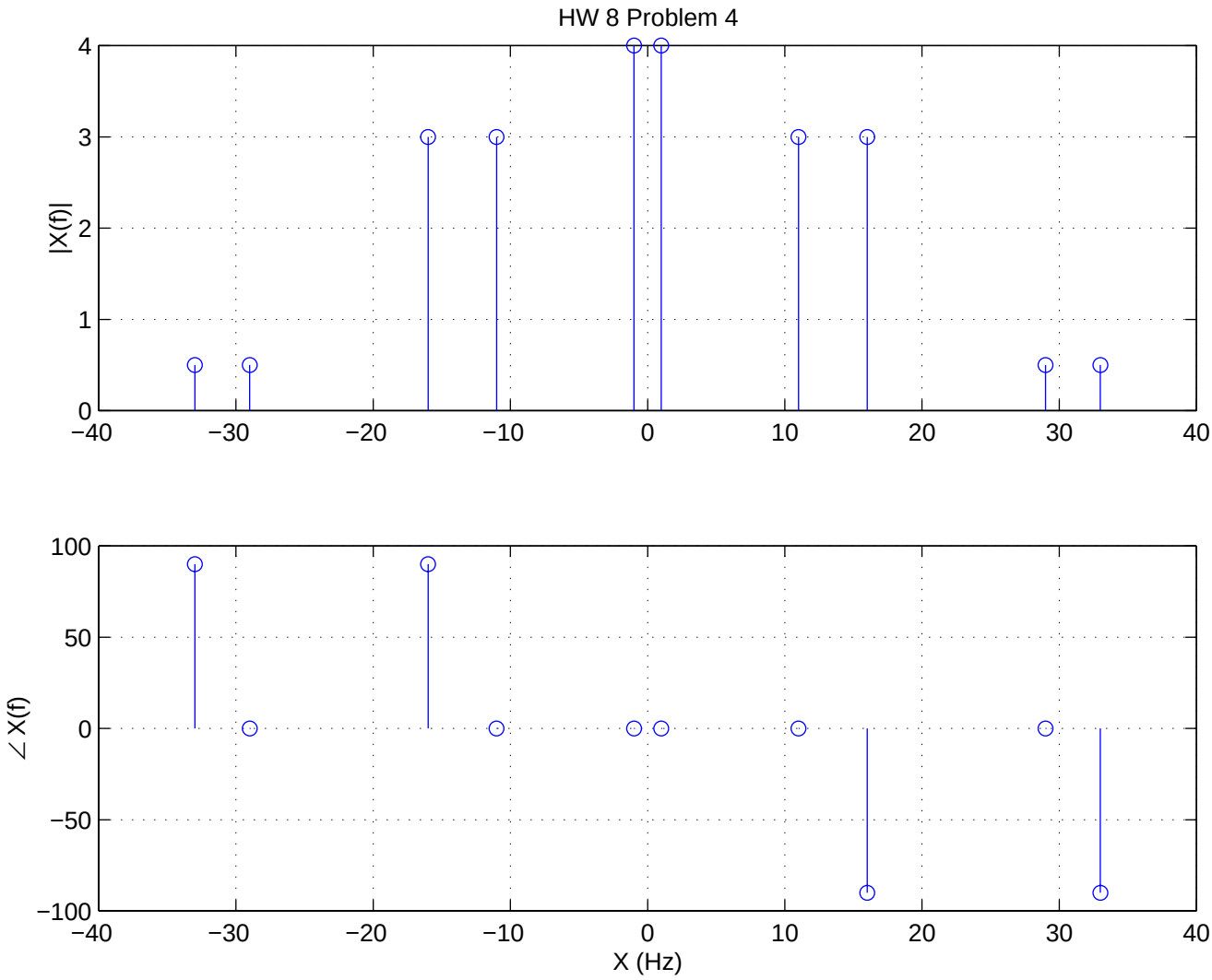
$= 3$

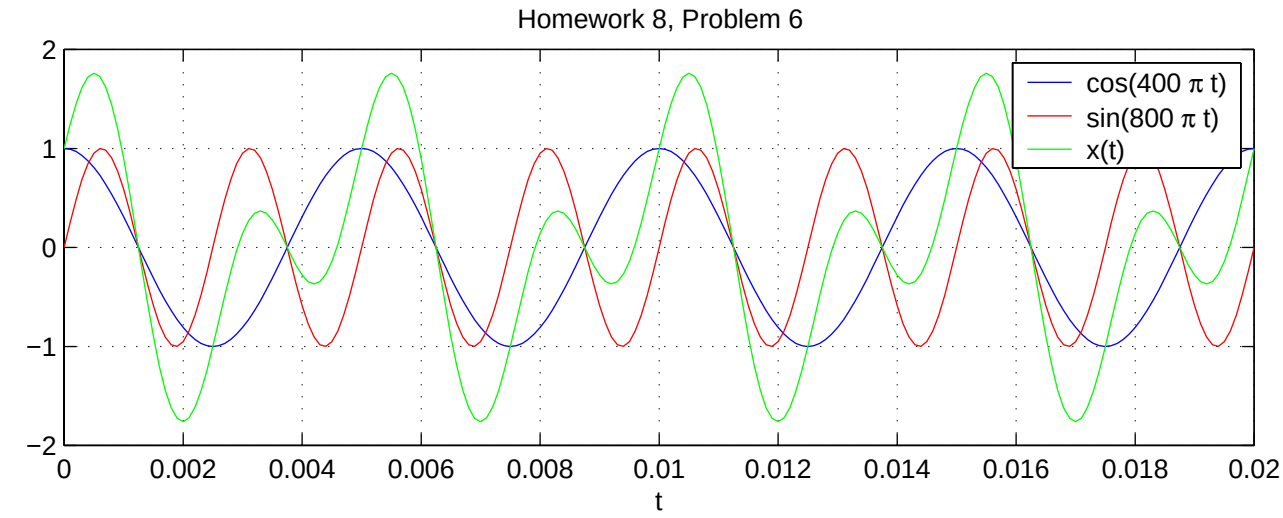
6(a) See MATLAB

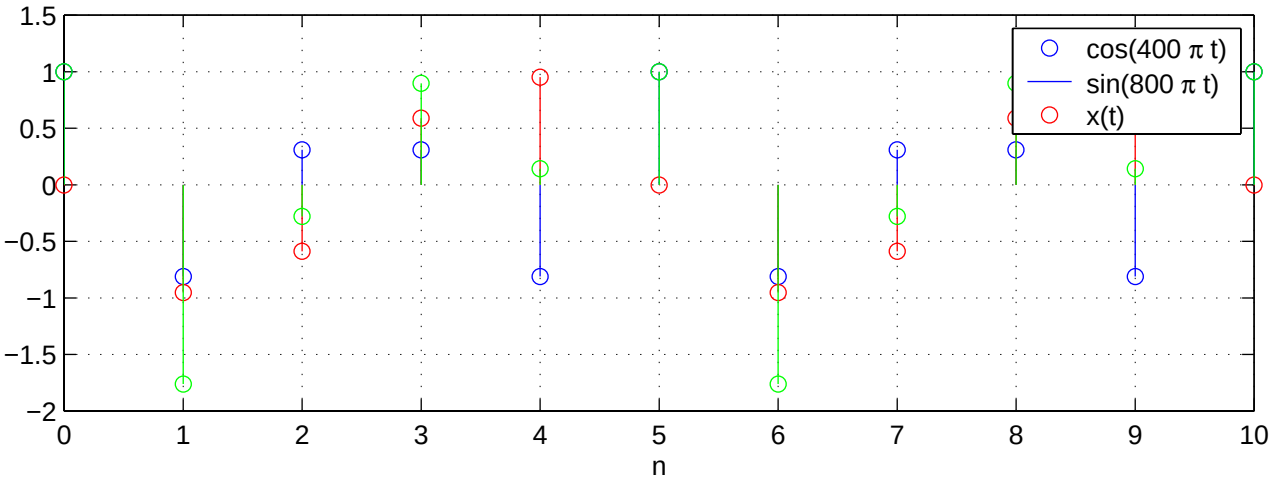
(b) See MATLAB. The signal has frequencies 200 Hz and 400 Hz. It is sampled at 500 Hz. The 400 Hz signal is aliased to −100 Hz, so the sampled signal does not show the 400 Hz freq

```
% EE 341 Homework #8

% Problem 4 (b)

% x(t) = 8 cos(2 pi t) + 6 cos(22 pi t) + 6 sin(32 pi t) + cos(58 pi t)
%        + sin(66 pi t)

% 8 cos(2 pi t) has delta functions of height 4 at -1 Hz and 1 Hz
% 6 cos(22 pi t) has delta functions of height 3 at -11 Hz and 11 Hz
% 6 sin(32 pi t) has delta functions of height 3j at -16 Hz and -3j at 16 Hz
% cos(58 pi t) has delta functions of height 1/2 at -29 Hz and 29 Hz
% sin(66 pi t) has delta functions of height j/2 at -33 Hz and -j/2 at 33 Hz

figure(1)
clf
f = [-33 -29 -16 -11 -1 1 11 16 29 33];
Xmag = [1/2 1/2 3 3 4 4 3 3 1/2 1/2];
Xphase = [90 0 90 0 0 0 0 -90 0 -90];
subplot(211)
stem(f,Xmag);
grid
ylabel('|X(f)|')
title('HW 8 Problem 4')
subplot(212)
stem(f,Xphase)
ylabel('\angle X(f)')
xlabel('X (Hz)')
grid
print -dpsc2 hw08_p4.ps

% Problem 6

% x(t) = cos(400 pi t) + sin(800 pi t);  Fs = 500 Hz

figure(2)
clf
% (a)
% Choose dt = 0.0001 to make plot smooth
dt = 0.0001;
t = 0:dt:0.02;
subplot(211)
x = cos(400*pi*t) + sin(800*pi*t);
plot(t,cos(400*pi*t));
hold on
plot(t,sin(800*pi*t),'r');
plot(t,x,'g');
grid
legend('cos(400 \pi t)','sin(800 \pi t)','x(t)')
xlabel('t')
title('Homework 8, Problem 6')

% (b)
Fs = 500;
Ts = 1/Fs;
t = 0:Ts:0.02;
n = t/Ts;
subplot(212)
x = cos(400*pi*n*Ts)+sin(800*pi*n*Ts);
stem(n,cos(400*pi*n*Ts));
hold on
stem(n,sin(800*pi*n*Ts),'r');
stem(n,x,'g');
grid
legend('cos(400 \pi t)','sin(800 \pi t)','x(t)')
```

```
xlabel('n')
print -dpsc2 hw08_p6.ps
```


HW 8 Problem 4
|X(f)|
∠ X(f)
X (Hz)
4
3
2
1
0
100
50
0
−50
−100
−40
−30
−20
−10
0
10
20
30
40


Homework 8, Problem 6
cos(400 π t)
sin(800 π t)
x(t)
t


cos(400 π t)
sin(800 π t)
x(t)
n