EE 341 - Homework 10

Due November 2, 2005

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots. Problems from the text tell you to use the M-file `dft`. You can use the built-in MATLAB function `fft` instead.

1. Problem 7.9 (b), (d), (f).
2. Problem 7.10.
3. Problem 7.12.
4. Problem 7.22. Do signals (i), (ii), (iii), (vi).
5. Problem 7.30 (a) (b) (e).

EE 341
Hw # 10

1. Problem 7.9

(b) $x = \{1, 0, -1, 0\}$
$\uparrow$

$$X_k = \sum_{n=0}^{3} x(n)\, e^{-j\frac{2\pi kn}{4}}$$

$$X_k = x(0)e^{0} + x(1)e^{-j2\pi k/4} + x(2)e^{-j4\pi k/4} + x(3)e^{-j6\pi k/4}$$

$$= 1 + 0 - 1e^{-j\pi k} + 0 = 1 - e^{-j\pi k}$$

$$X_0 = 1 - e^{0} = 1 - 1 = 0$$

$$X_1 = 1 - e^{-j\pi} = 1 - (-1) = 2$$

$$X_2 = 1 - e^{-j2\pi} = 1 - 1 = 0$$

$$X_3 = 1 - e^{-j3\pi} = 1 - (-1) = 2$$

$$X_k = \{0, 2, 0, 2\}$$

(d) $x = \{-1 \quad 1 \quad 1 \quad 1\}$

$$X_k = (-1)e^{0} + (1)e^{-j2\pi k/4} + (1)e^{-j4\pi k/4} + (1)e^{-j6\pi k/4}$$

$$= -1 + e^{-j\pi k/2} + e^{-j\pi k} + e^{-j3\pi k/2}$$

$$X_0 = -1 + 1 + 1 + 1 = 2$$

$$X_1 = -1 + e^{-j\pi/2} + e^{-j\pi} + e^{-j3\pi/2} = -1 - j - 1 + j = -2$$

$$X_2 = -1 + e^{-j\pi} + e^{-j2\pi} + e^{-j3\pi} = -1 - 1 + 1 - 1 = 2$$

$$X_3 = -1 + e^{-j3\pi/2} + e^{-j3\pi} + e^{-j9\pi/2} = -1 + j - 1 - j = -2$$

$$X_k = \{-2, 2, -2, 2\}$$

(f). $x = \{1, -1, 1, -1\}$

$$X_k = (1)e^{0} + (-1)e^{-j2\pi k/4} + (1)e^{-j4\pi k/4} + (-1)e^{-j6\pi k/4}$$

$$= 1 - e^{-j\pi k/2} + e^{-j\pi k} - e^{-j3\pi k/2}$$

$$X_0 = 1 - 1 + 1 - 1 = 0$$

$$X_1 = 1 - e^{-j\pi/2} + e^{-j\pi} - e^{-j3\pi/2} = 1 - (-j) + (-1) - j = 0$$

$$X_2 = 1 - e^{-j\pi} + e^{-j2\pi} - e^{-j3\pi} = 1 - (-1) + 1 - (-1) = 4$$

$$X_3 = 1 - e^{-j3\pi/2} + e^{-j3\pi} - e^{-j9\pi/2} = 1 - j + (-1) + j = 0$$

$$X_k = \{0, 0, 4, 0\}$$
$\uparrow$

2. Problem 7.10 See MATLAB

3. Problem 7.12 See MATLAB
Expect to have random amounts of all frequencies. That is what we get.

4. Problem 7.22

$$H(\Omega) = \begin{cases} e^{-j3\Omega} & \frac{\pi}{2} \le |\Omega| \le \pi \\ 0 & 0 \le |\Omega| < \pi/2 \end{cases}$$

Note: $H(\Omega) = e^{-j3\Omega}(1 - H_{LP}(\Omega))$ where

$$H_{LP}(\Omega) = \begin{cases} 1 & 0 \le |\Omega| < \frac{\pi}{2} \\ 0 & \frac{\pi}{2} \le |\Omega| < \pi \end{cases}$$

$1 - H_{LP}(\Omega) \leftrightarrow \delta(n) - \frac{B}{\pi}\text{sinc}\left(\frac{B}{\pi}n\right)$ (Table 7.1)

$$= \delta(n) - \frac{1}{2}\text{sinc}\left(\frac{n}{2}\right)$$

$$h(n) = e^{-j3\Omega}(1 - H_{LP}(\Omega)) \longleftrightarrow \delta(n-3) - \frac{1}{2}\text{sinc}\left(\frac{n}{2}\right)$$

(time shift)

(b)

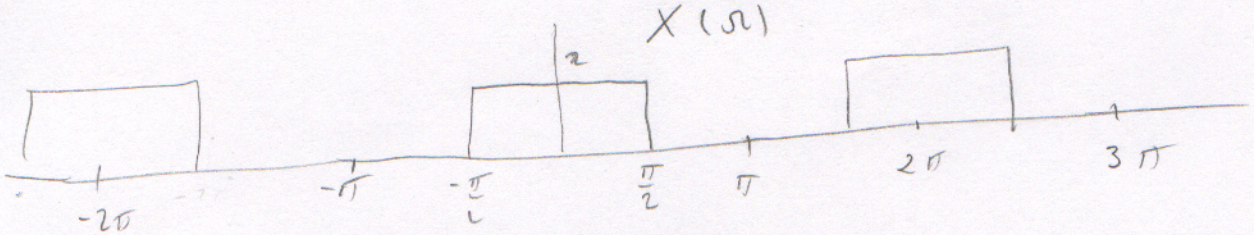
(i) $x(n) = \cos\left(\frac{\pi n}{4}\right)$ $\Omega_0 = \frac{\pi}{4}$ is blocked completely

$y(n) = 0$

(ii) $x(n) = \cos(3\pi n/4)$ $\Omega_0 = \frac{3\pi}{4}$ is passed unchanged except for a delay of 3 samples

$y(n) = \cos(3\pi(n-3)/4)$

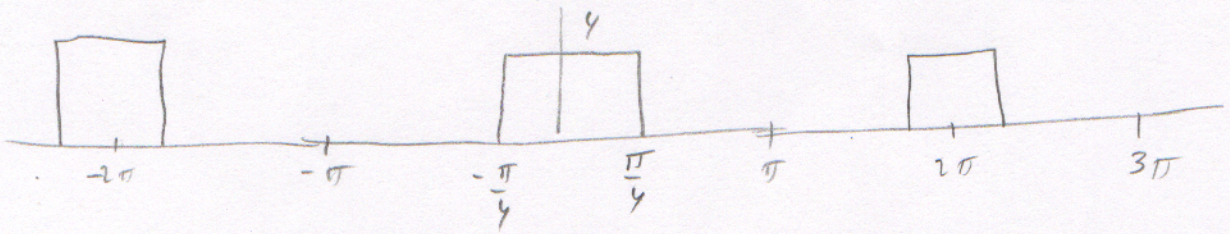
(iii) $x(n) = \text{sinc}(n/2) = 2\,\frac{\pi/2}{\pi}\text{sinc}\left(\frac{\pi/2}{\pi}n\right) \longleftrightarrow 2\sum_{k=-\infty}^{\infty} P_{\pi}(\Omega + 2\pi k)$



Blocked completely;

$y(n) = 0$

(iv) $x(n) = \text{sinc}(n/4) = 4\,\frac{\pi/4}{\pi}\text{sinc}\left(\frac{\pi/4}{\pi}n\right) \longleftrightarrow 4\sum_{k=-\infty}^{\infty} P_{\pi/2}(\Omega + 2\pi k)$

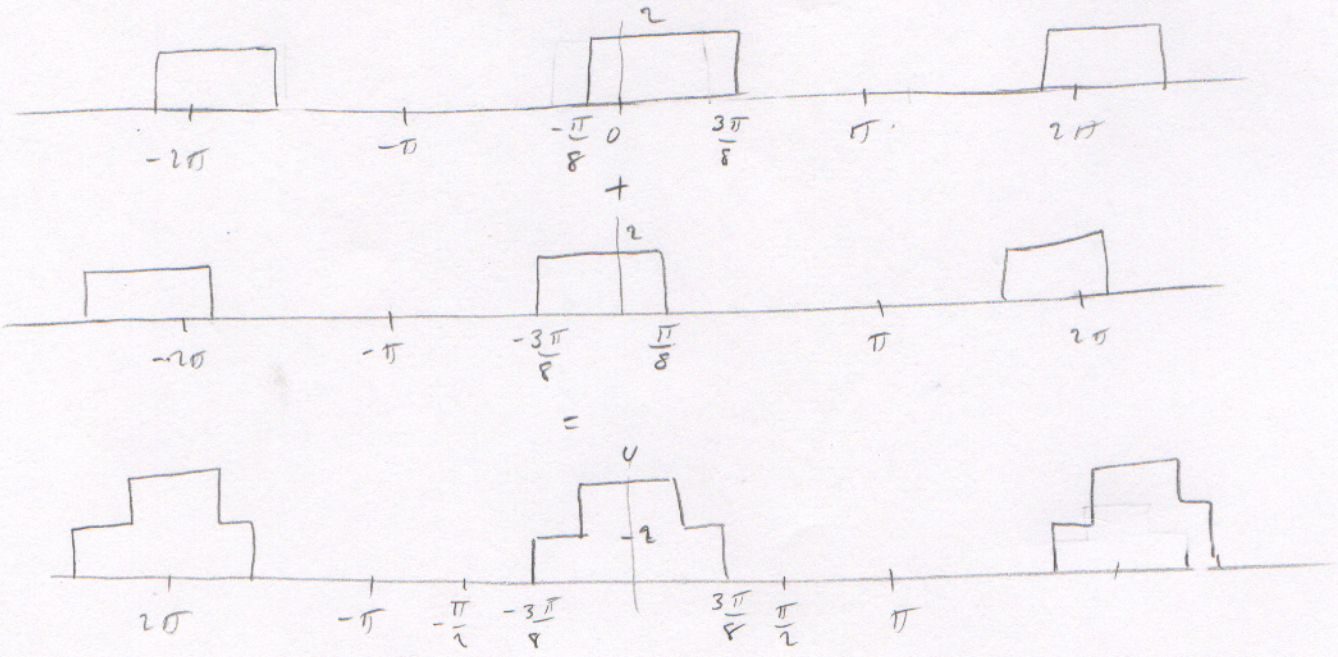


Blocked completely;

$y(n) = 0$

(iv) $x(n) = \text{sinc}(\pi/4)\cos(\pi n/8)$

This is signal (iii), shifted by $\pm\frac{\pi}{8}$

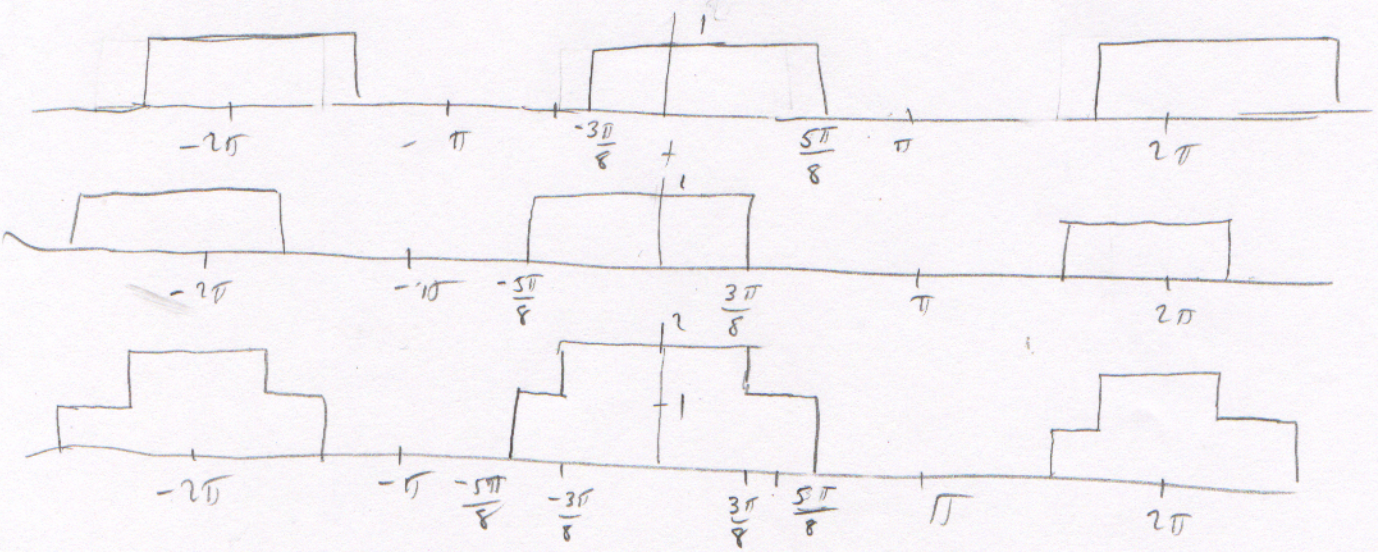


Signal blocked completely

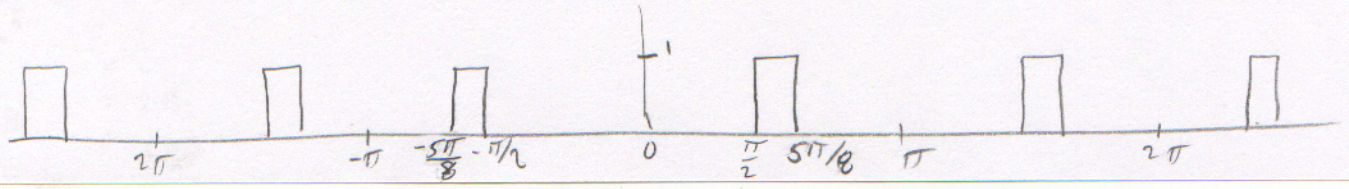
$y(n) = 0$

(v) $x(n) = \text{sinc}(n/2)\cos(\pi n/8)$

This is signal (iii) shifted by $\pm\frac{\pi}{8}$



After filter



This can be written as

$$Y(\Omega) = \left[P_{5\pi/4}(\Omega) - P_{\frac{\pi}{2}}(\Omega) \right] e^{-3j\Omega}$$

$$y(n) = \frac{5\pi/4}{\pi} \text{sinc}\left(\frac{5\pi/4}{\pi}(n-3) \right) - \frac{\pi/2}{\pi} \text{sinc}\left(\frac{\pi/2}{\pi}(n-3) \right)$$

$$= \frac{5}{4} \text{sinc}\left(\frac{5}{4}(n-3) \right) - \frac{1}{2} \text{sinc}\left(\frac{n-3}{2} \right)$$

(c) See MATLAB

5. Problem 7.30

$$y(n+1) - 0.9\, y(n) = 0.1\, x(n+1)$$

$$y(n) - 0.9\, y(n-1) = 0.1\, x(n)$$

$$h(n) - 0.9\, h(n-1) = 0.1\, \delta(n)$$

(a) Homogenous solution

$$h_h(n) - 0.9 h_h(n-1) = 0$$

$$\lambda - 0.9 = 0$$

$$\lambda = 0.9$$

$$h_h(n) = A\,(0.9)^n$$

Particular solution: $h_p(n) = K\,\delta(n)$ (for impulse input)

$$h(n) = A\,(0.9)^n + K\delta(n)$$

$$h(0) = 0.9\, h(-1) + 0.1\, \delta(0) = 0 + 0.1 = 0.1$$

$$h(1) = 0.9\, h(0) + 0.1\, \delta(1) = (0.9)(0.1) = 0.09$$

$$h(0) = A\,(0.9)^0 + K\delta(0) = A + K = 0.1$$

$$h(1) = A(0.9)^1 + K\delta(1) = 0.9A + 0 = 0.09 \Rightarrow A = 0.1$$

$$K = 0$$

$h(n) = 0.1\,(0.9)^n u(n)$

(b) $H(\Omega) = 0.1 \dfrac{1}{1 - 0.9\,e^{-j\Omega}}$

See MATLAB for sketch

(c) Wasn't assigned

$x(n) = 1 + \sin\left(\frac{\pi}{4} n\right) + \sin\left(\frac{\pi}{2} n\right)$

$1: \Omega_0 = 0, \quad H(\Omega_0) = 0.1 \dfrac{1}{1 - 0.9\,e^{-0}} = 0.1\left(\frac{1}{0.1}\right) = 1$

$\sin\left(\frac{\pi}{4} n\right): \Omega_0 = \frac{\pi}{4} \quad H(\Omega_0) = 0.1 \dfrac{1}{1 - 0.9\,e^{-j\pi/4}} = 0.136\,e^{-j60^\circ}$

$\sin\left(\frac{\pi}{2} n\right): \Omega_0 = \frac{\pi}{2} \quad H(\Omega_0) = 0.1 \dfrac{1}{1 - 0.5\,e^{-j\pi/2}} = 0.074\,e^{-j42^\circ}$

$y(n) = 1 + 0.136 \sin\left(\frac{\pi}{4} n - 60^\circ\right) + 0.074 \sin\left(\frac{\pi}{2} n - 42^\circ\right)$

(d) Wasn't assigned

$x(n) = u(n) - u(n-3) = \delta(n) + \delta(n-1) + \delta(n-2)$

$y(n) = h(n) + h(n-1) + h(n-2)$

$= 0.1(0.9)^n u(n) + 0.1(0.9)^{n-1} u(n-1) + 0.1(0.9)^{n-2} u(n-2)$

(e) See MATLAB

Take fft of $h(n)$ and $x(n)$

Multiply

Get $y(n)$ by taking inverse fft

I plotted $y(n)$ calculated from (d) and by using the fft. The two answers agree quite well.

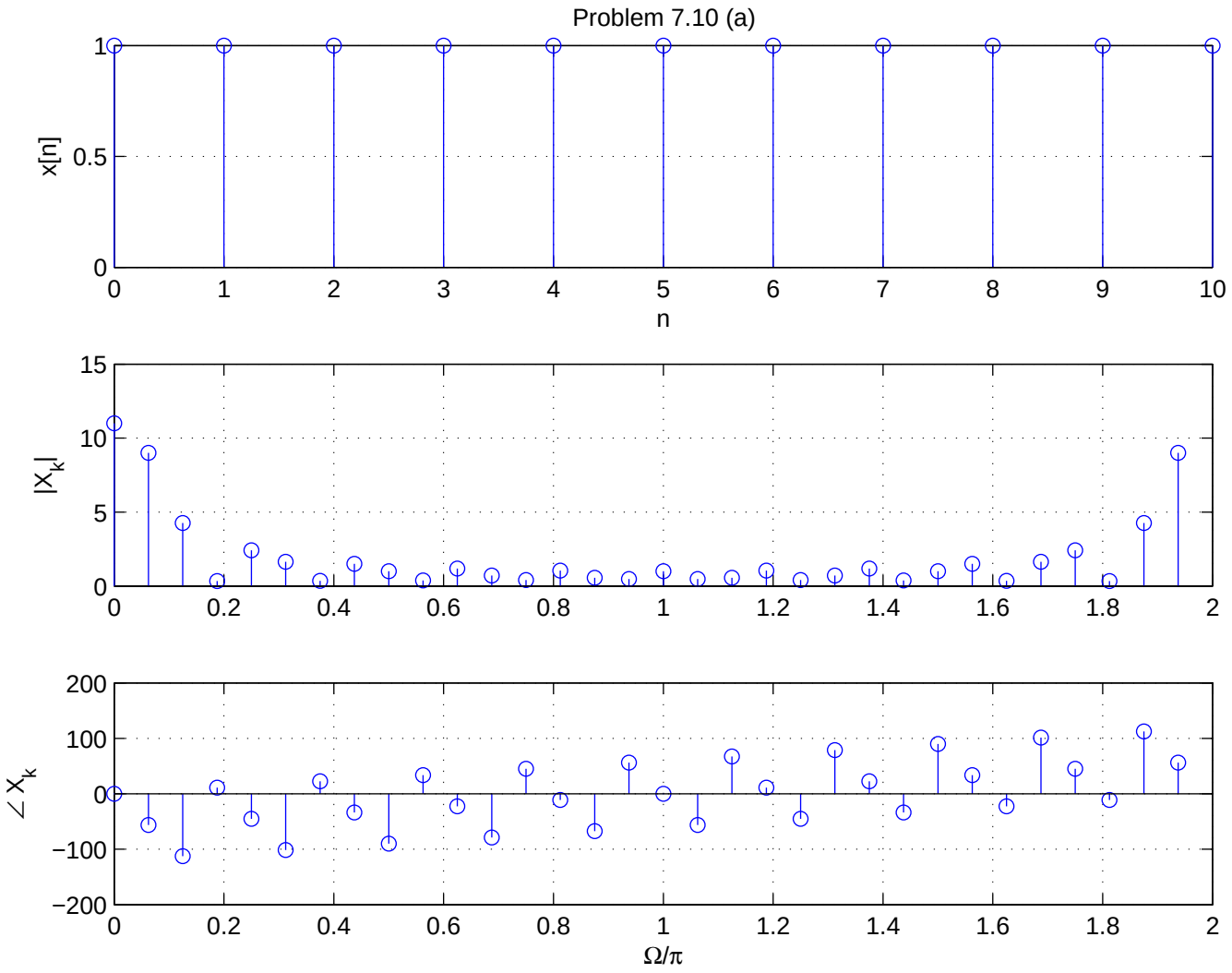
```
% EE 341 Homework 10

% Problem 7.10

% (a)
figure(1)
clf
n=0:10;
x=ones(size(n));
subplot(311)
stem(n,x)
grid
xlabel('n')
ylabel('x[n]')
title('Problem 7.10 (a)')
Xk=fft(x,32);
k=0:31;
Wk = 2*pi*k/32;
subplot(312)
stem(Wk/pi,abs(Xk))
grid
ylabel('|X_k|')
subplot(313)
stem(Wk/pi,angle(Xk)*180/pi)
grid
ylabel('\angle X_k')
xlabel('\Omega/\pi')
print -dpsc2 p7_10_a.ps

% (b)
figure(2)
clf
n=0:20;
x=zeros(size(n));
x((n>=0) & (x <= 10)) = 1;
x((n>=11) & (x <= 20)) = -1;
subplot(311)
stem(n,x)
grid
xlabel('n')
ylabel('x[n]')
title('Problem 7.10 (b)')
Xk=fft(x,32);
k=0:31;
Wk = 2*pi*k/32;
subplot(312)
stem(Wk/pi,abs(Xk))
grid
ylabel('|X_k|')
subplot(313)
stem(Wk/pi,angle(Xk)*180/pi)
grid
ylabel('\angle X_k')
xlabel('\Omega/\pi')
print -dpsc2 p7_10_b.ps

% (c)
figure(3)
clf
n=0:20;
x=n;
subplot(311)
stem(n,x)
grid
```

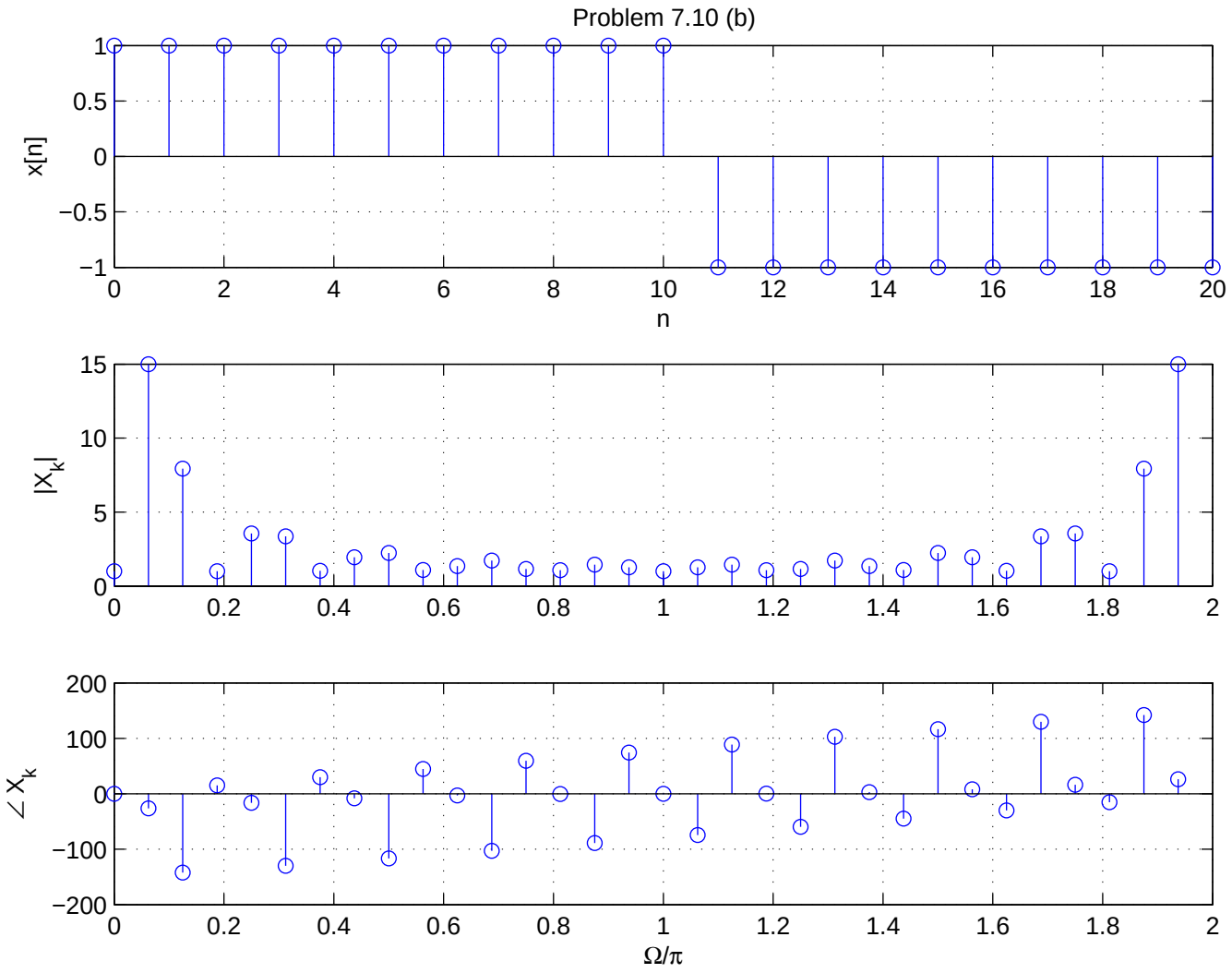
```
xlabel('n')
ylabel('x[n]')
title('Problem 7.10 (c)')
Xk=fft(x,32);
k=0:31;
Wk = 2*pi*k/32;
subplot(312)
stem(Wk/pi,abs(Xk))
grid
ylabel('|X_k|')
subplot(313)
stem(Wk/pi,angle(Xk)*180/pi)
grid
ylabel('\angle X_k')
xlabel('\Omega/\pi')
print -dpsc2 p7_10_c.ps

% (d)
figure(4)
clf
n=0:20;
x = [0 1 2 3 4 5 6 7 8 9 10 10 9 8 7 6 5 4 3 2 1];
subplot(311)
stem(n,x)
grid
xlabel('n')
ylabel('x[n]')
title('Problem 7.10 (d)')
Xk=fft(x,32);
k=0:31;
Wk = 2*pi*k/32;
subplot(312)
stem(Wk/pi,abs(Xk))
grid
ylabel('|X_k|')
subplot(313)
stem(Wk/pi,angle(Xk)*180/pi)
grid
ylabel('\angle X_k')
xlabel('\Omega/\pi')
print -dpsc2 p7_10_d.ps

% (e)
figure(5)
clf
n=0:10;
x = cos(10*pi*n/11);
subplot(311)
stem(n,x)
grid
xlabel('n')
ylabel('x[n]')
title('Problem 7.10 (e)')
Xk=fft(x,32);
k=0:31;
Wk = 2*pi*k/32;
subplot(312)
stem(Wk/pi,abs(Xk))
grid
ylabel('|X_k|')
subplot(313)
stem(Wk/pi,angle(Xk)*180/pi)
grid
ylabel('\angle X_k')
```

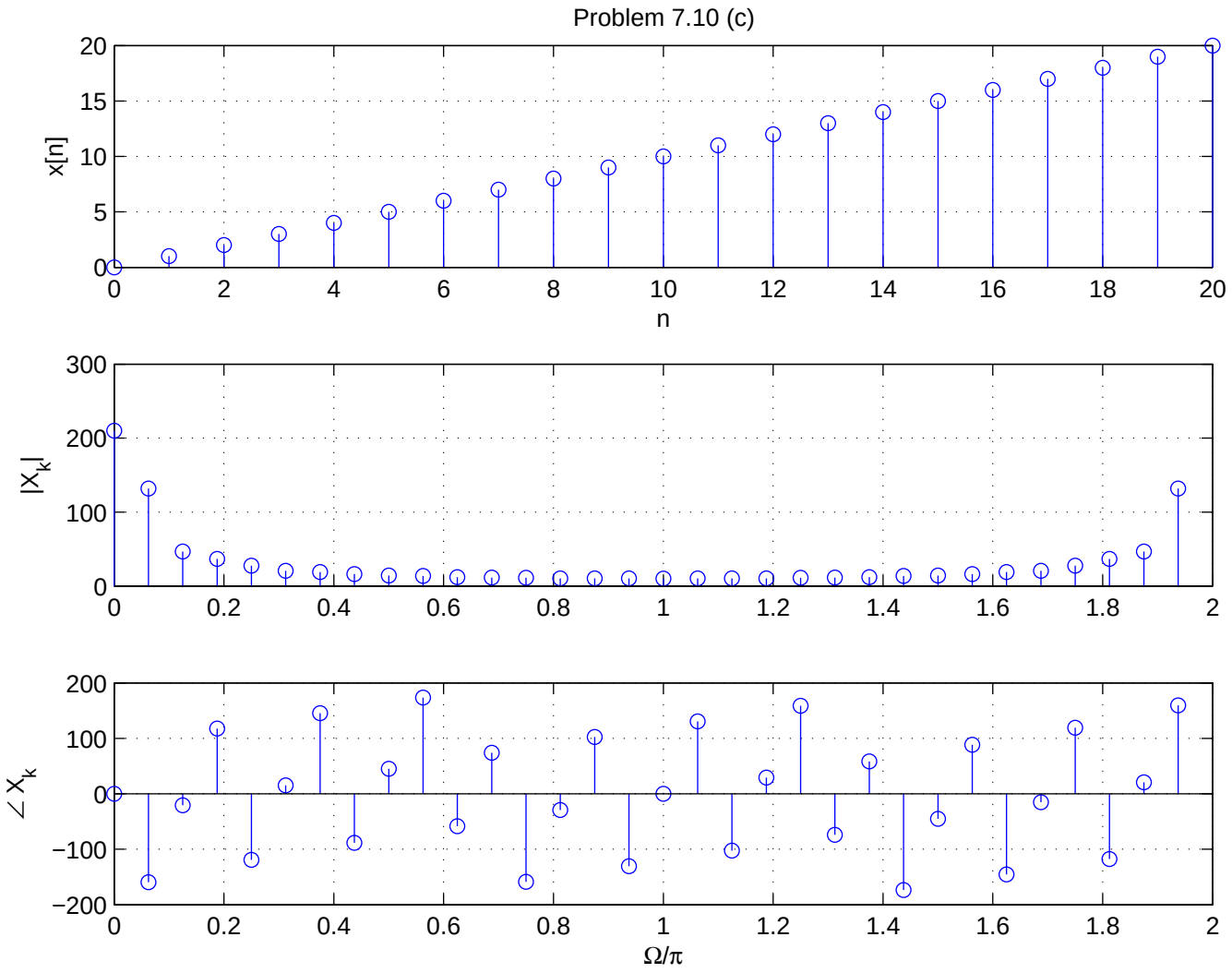
```
xlabel('\Omega/\pi')
print -dpsc2 p7_10_e.ps

% (f)
figure(6)
clf
n=0:10;
x = cos(9*pi*n/11);
subplot(311)
stem(n,x)
grid
xlabel('n')
ylabel('x[n]')
title('Problem 7.10 (f)')
Xk=fft(x,32);
k=0:31;
Wk = 2*pi*k/32;
subplot(312)
stem(Wk/pi,abs(Xk))
grid
ylabel('|X_k|')
subplot(313)
stem(Wk/pi,angle(Xk)*180/pi)
grid
ylabel('\angle X_k')
xlabel('\Omega/\pi')
print -dpsc2 p7_10_f.ps

% Problem 7.12
figure(7)
clf
N=32;
n=0:N-1;
x=rand(N,1)-0.5;
subplot(211)
stem(n,x);
grid
subplot(212)
Xk=fft(x,32);
k = 0:31;
Wk = 2*pi*k/32;
subplot(312)
stem(Wk/pi,abs(Xk))
grid
ylabel('|X_k|')
xlabel('\Omega/\pi')
print -dpsc2 p7_12.ps

% Problem 7.22
% (i)
figure(8)
clf
n = -20:20;
x = cos(pi*n/4);
y = zeros(size(n));
subplot(211)
stem(n,x)
grid
title('Problem 7.22 (i)')
ylabel('x[n]')
subplot(212)
stem(n,y)
grid
ylabel('y[n]')
```

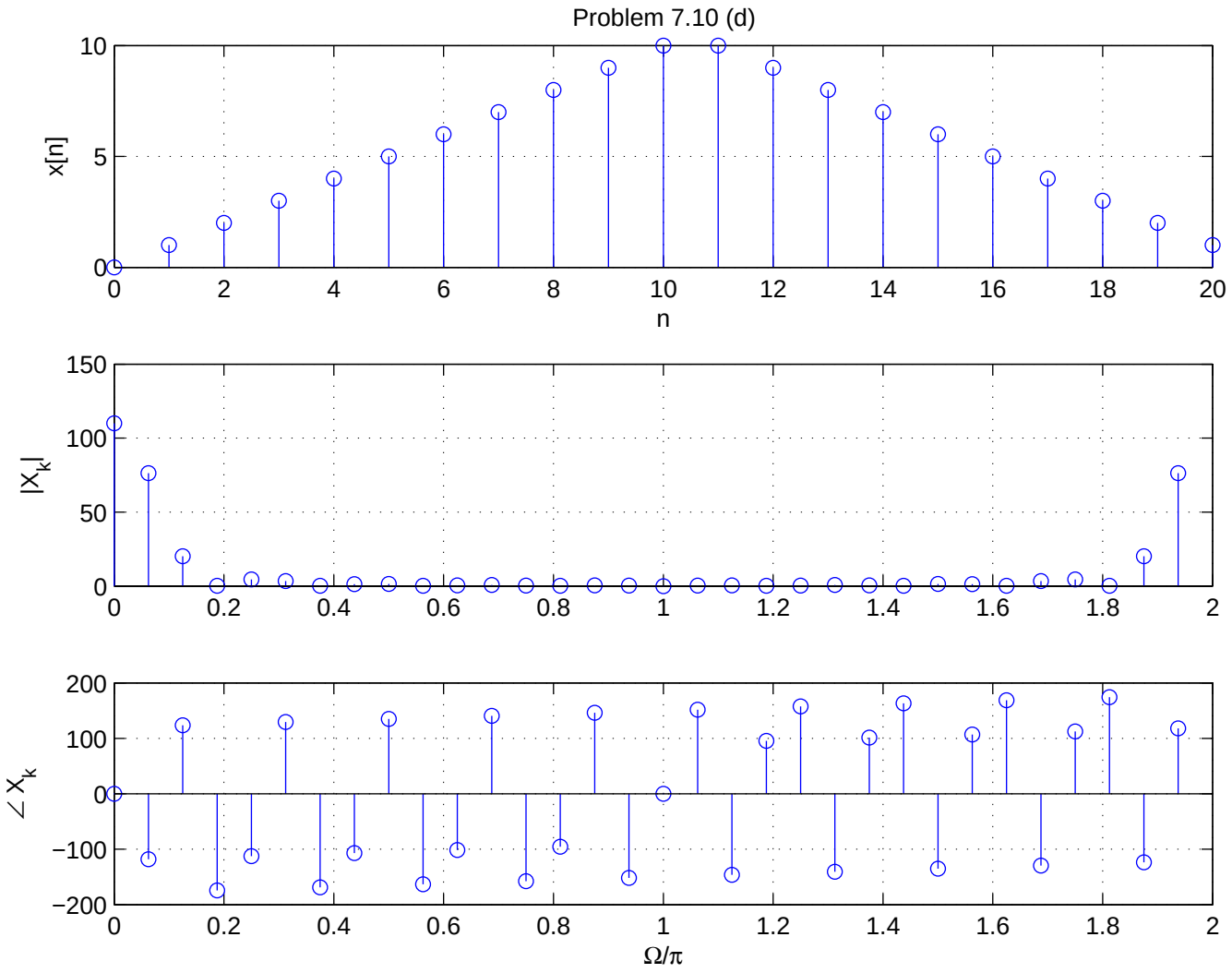
```
print -dpsc2 p7_22_i.ps

% (ii)
figure(9)
clf
n = -20:20;
x = cos(3*pi*n/4);
y = cos(3*pi*(n-3)/4);
subplot(211)
stem(n,x)
grid
title('Problem 7.22 (ii)')
ylabel('x[n]')
subplot(212)
stem(n,y)
grid
ylabel('y[n]')
print -dpsc2 p7_22_ii.ps

% (iii)
figure(10)
clf
n = -20:20;
x = sinc(n/2);
y = zeros(size(x));
subplot(211)
stem(n,x)
grid
title('Problem 7.22 (iii)')
ylabel('x[n]')
subplot(212)
stem(n,y)
grid
ylabel('y[n]')
print -dpsc2 p7_22_iii.ps

% (iv)
figure(11)
clf
n = -20:20;
x = sinc(n/4);
y = zeros(size(x));
subplot(211)
stem(n,x)
grid
title('Problem 7.22 (iv)')
ylabel('x[n]')
subplot(212)
stem(n,y)
grid
ylabel('y[n]')
print -dpsc2 p7_22_iv.ps

% (v)
figure(12)
clf
n = -20:20;
x = sinc(n/4).*cos(pi*n/8);
y = zeros(size(x));
subplot(211)
stem(n,x)
grid
title('Problem 7.22 (v)')
ylabel('x[n]')
```

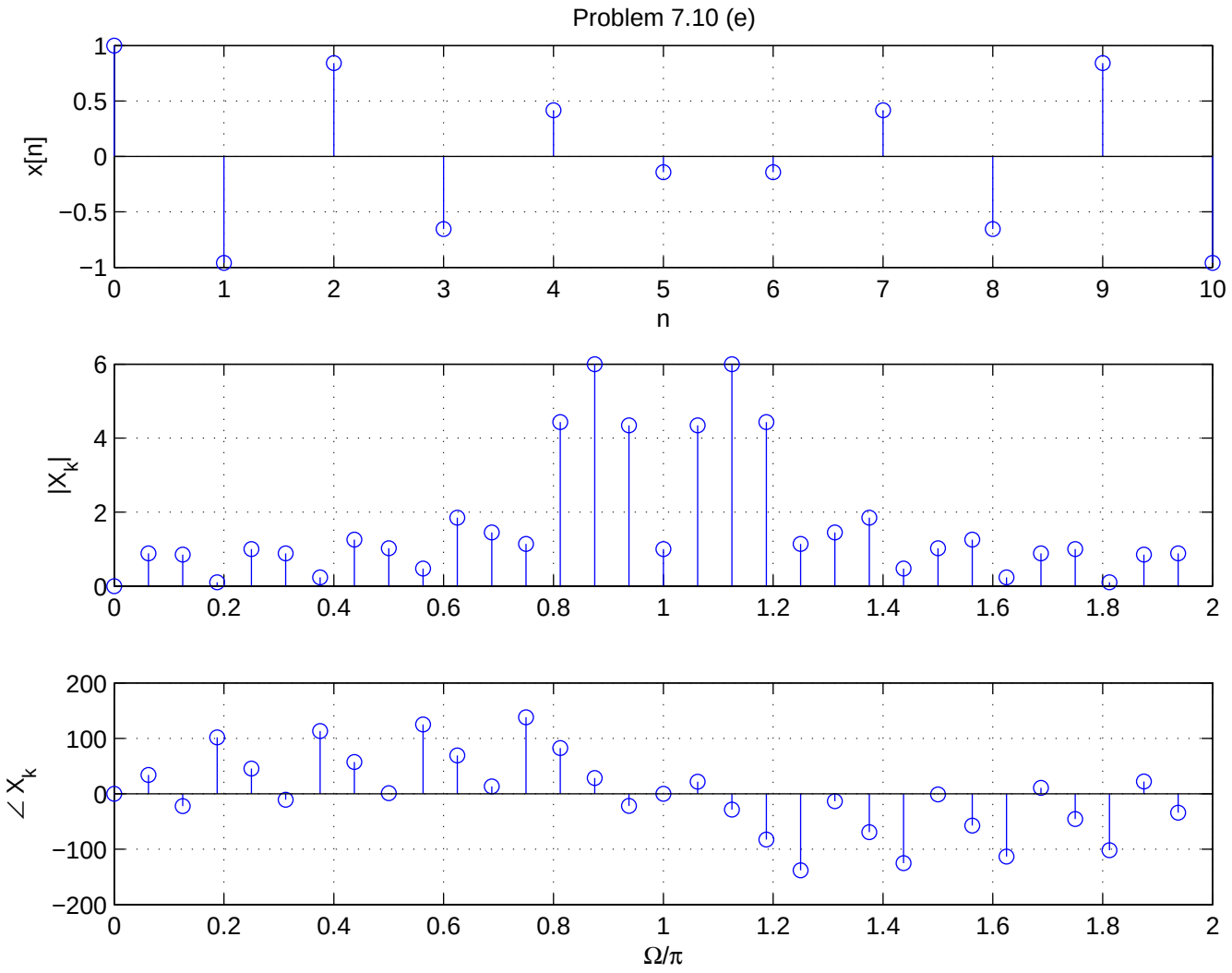
```
subplot(212)
stem(n,y)
grid
ylabel('y[n]')
print -dpsc2 p7_22_v.ps

% (vi)
figure(13)
clf
n = -20:20;
x = sinc(n/2).*cos(pi*n/8);
y = (5/4)*sinc(5*(n-3)/4) - (1/2)*sinc((n-3)/2);
subplot(211)
stem(n,x)
grid
title('Problem 7.22 (vi)')
ylabel('x[n]')
subplot(212)
stem(n,y)
grid
ylabel('y[n]')
print -dpsc2 p7_22_vi.ps

% Problem 7.30
% (d)
figure(14)
clf
n = 0:30;
x = (n>=0) - (n>=3);
y = 0.1*(0.9).^n.*(n>=0) + 0.1*(0.9).^(n-1).*(n>=1) + 0.1*(0.9).^(n-2).*(n>=2);
subplot(311)
stem(n,x);
grid
title('Problem 7.30 (e)')
ylabel('x[n]')
subplot(312)
stem(n,y);
grid
ylabel('y[n]')
h = 0.1*(0.9).^n.*(n>=0);
X1 = fft(x,31);
H1 = fft(h,31);
Y1 = X1.*H1;
y1 = ifft(Y1);
subplot(313)
stem(n,y1);
grid
ylabel('y[n] (fft)')
xlabel('n');
print -dpsc2 p7_30_e.ps
```

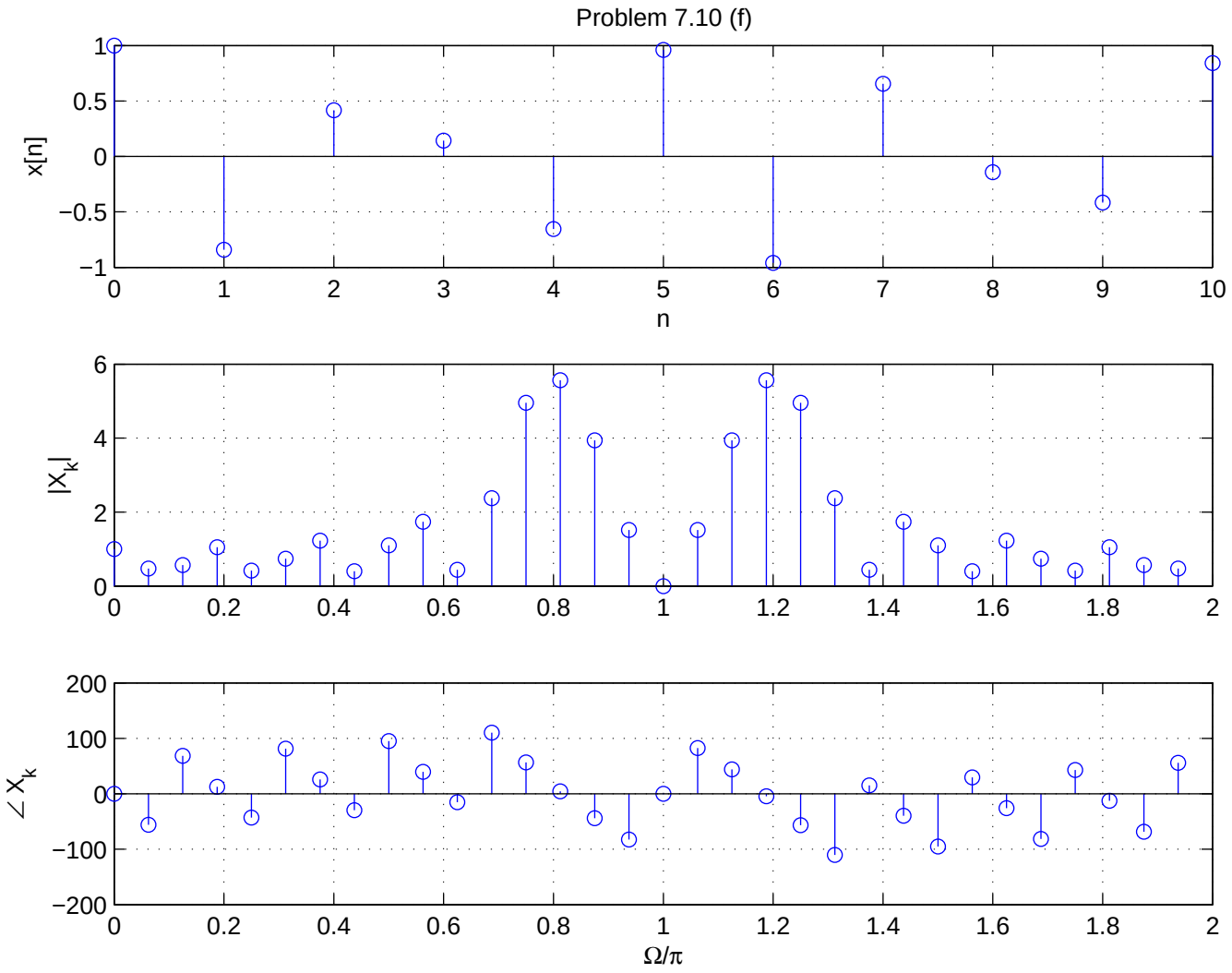

Problem 7.10 (a)
x[n]
n
|X_k|
∠X_k
Ω/π

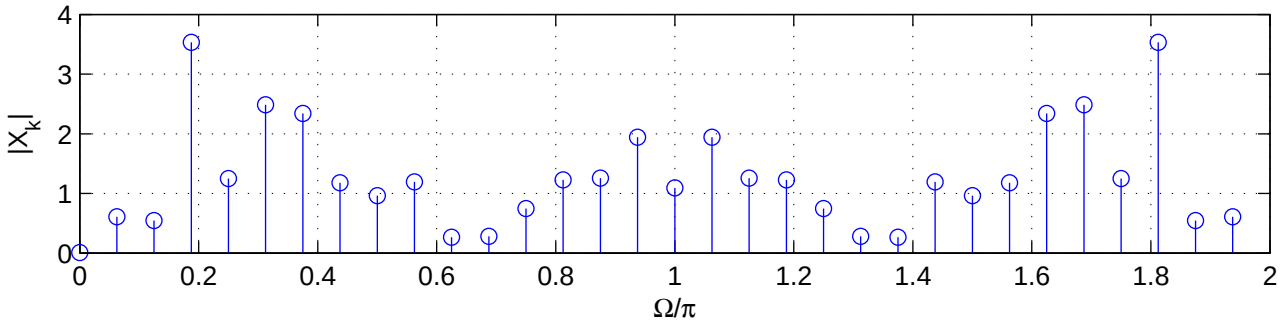

Problem 7.10 (b)
x[n]
n
|X_k|
∠X_k
Ω/π

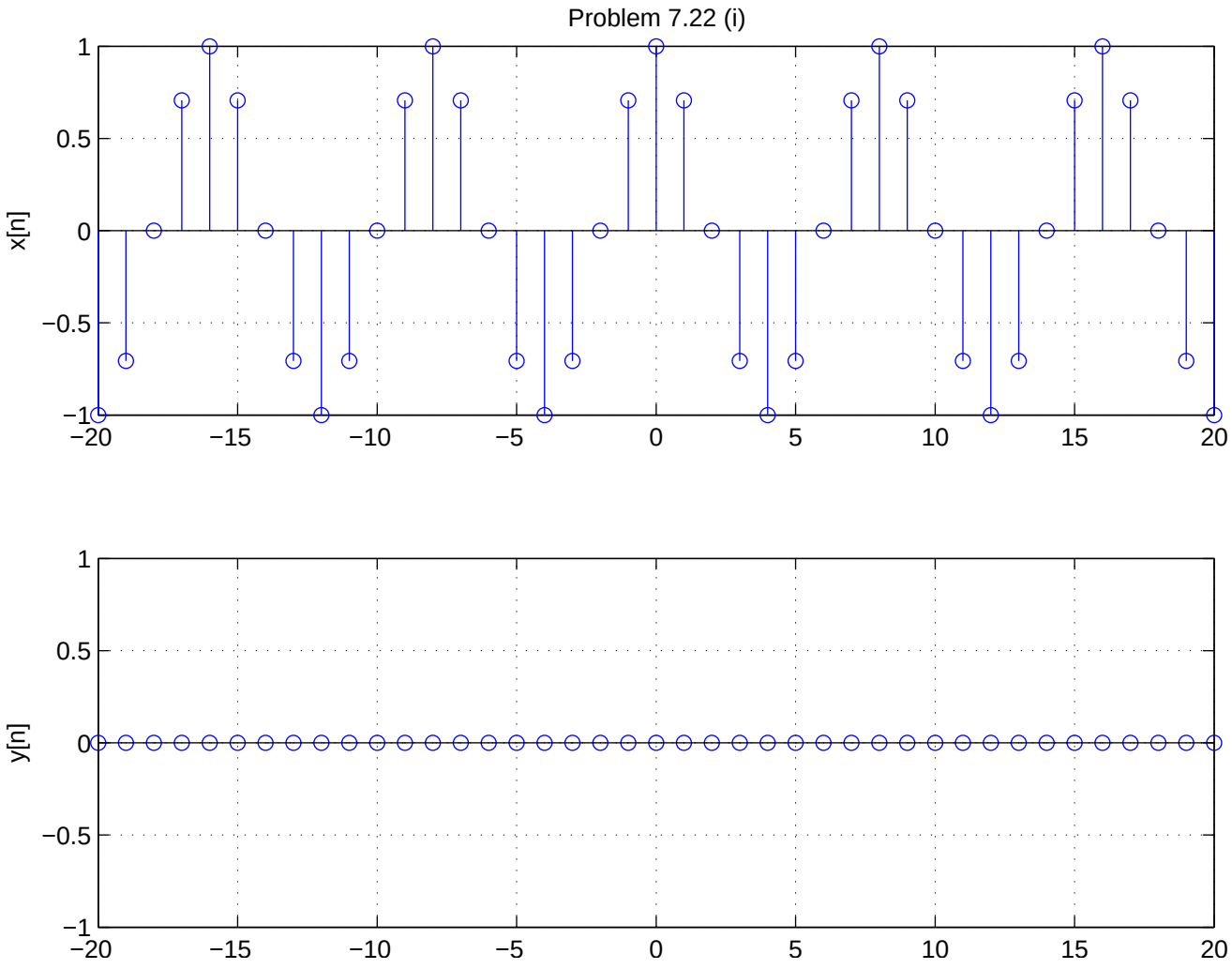

Problem 7.10 (c)
x[n]
n
|X_k|
∠X_k
Ω/π

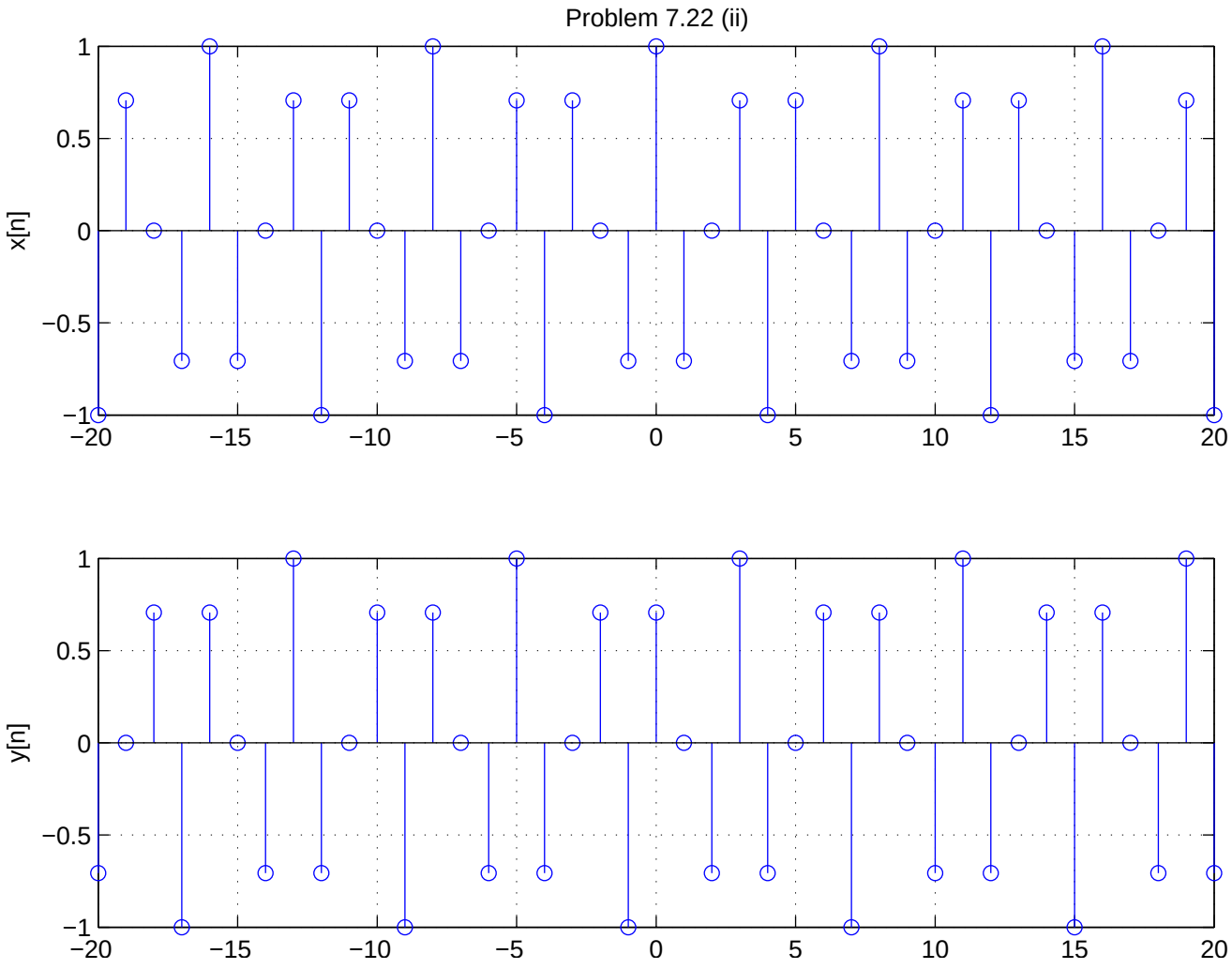

Problem 7.10 (d)
x[n]
n
|X_k|
∠X_k
Ω/π

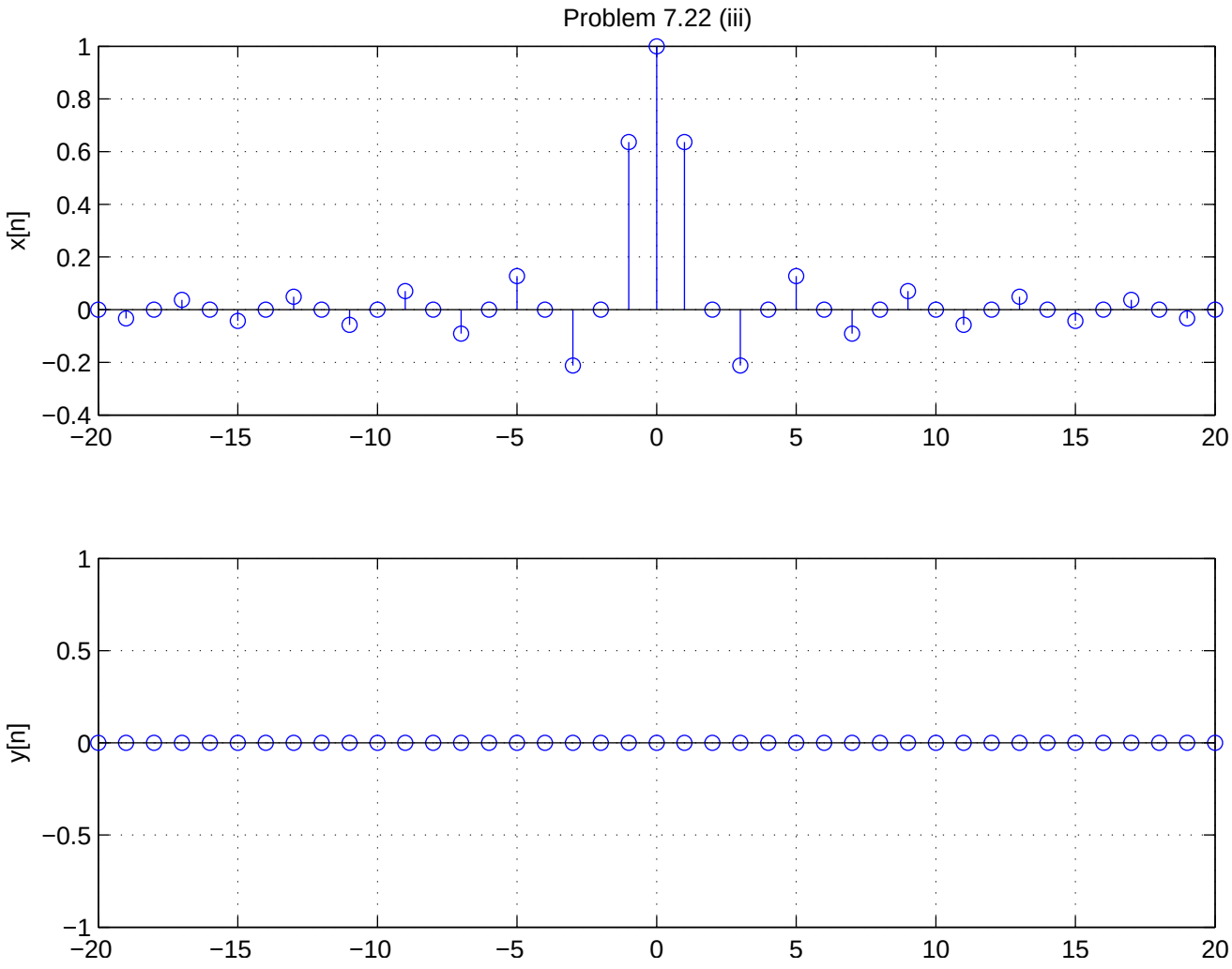

Problem 7.10 (e)
x[n]
n
|X_k|
∠X_k
Ω/π

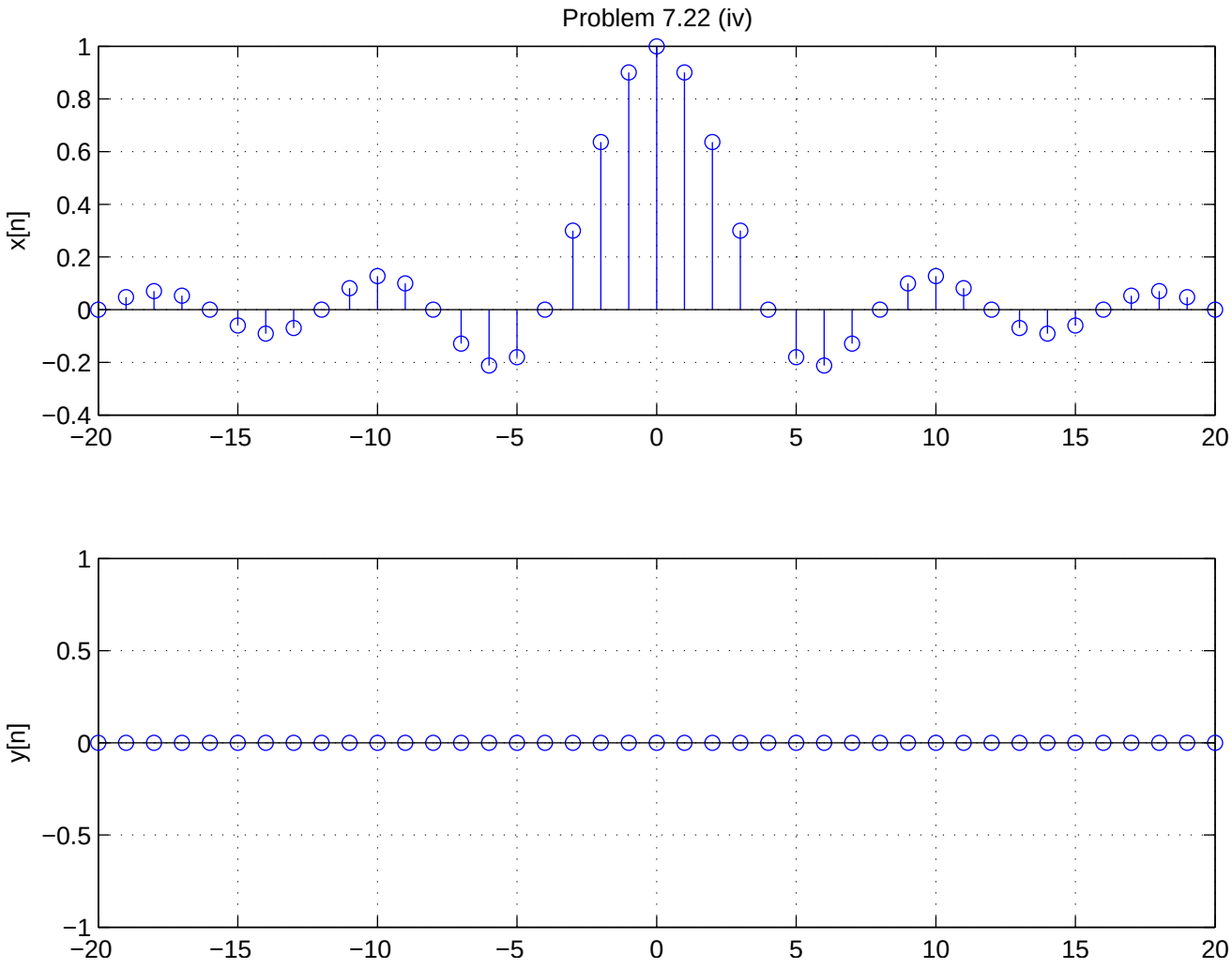

Problem 7.10 (f)
x[n]
n
|X_k|
∠X_k
Ω/π

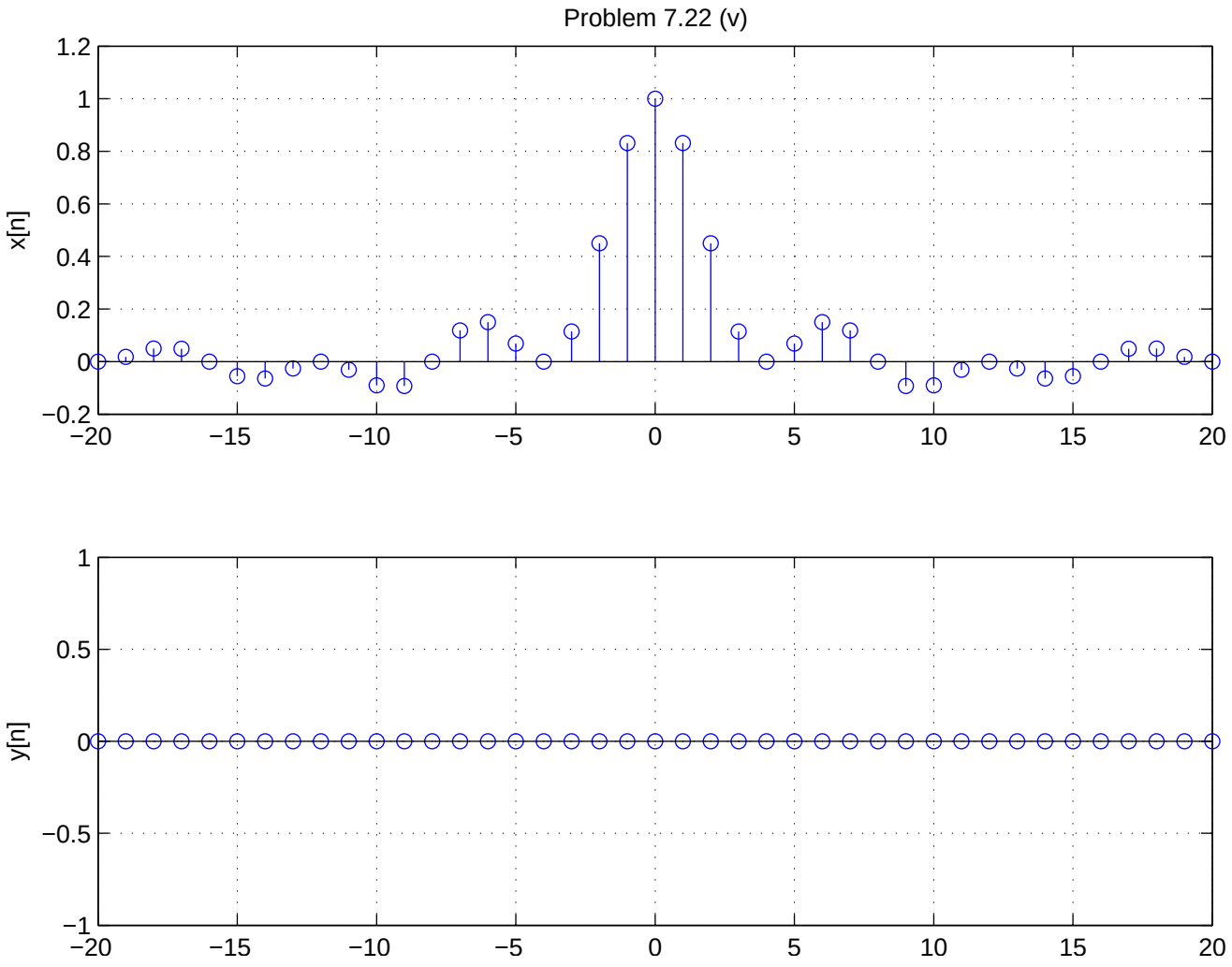

4
3
2
1
0
|X_k|
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Ω/π

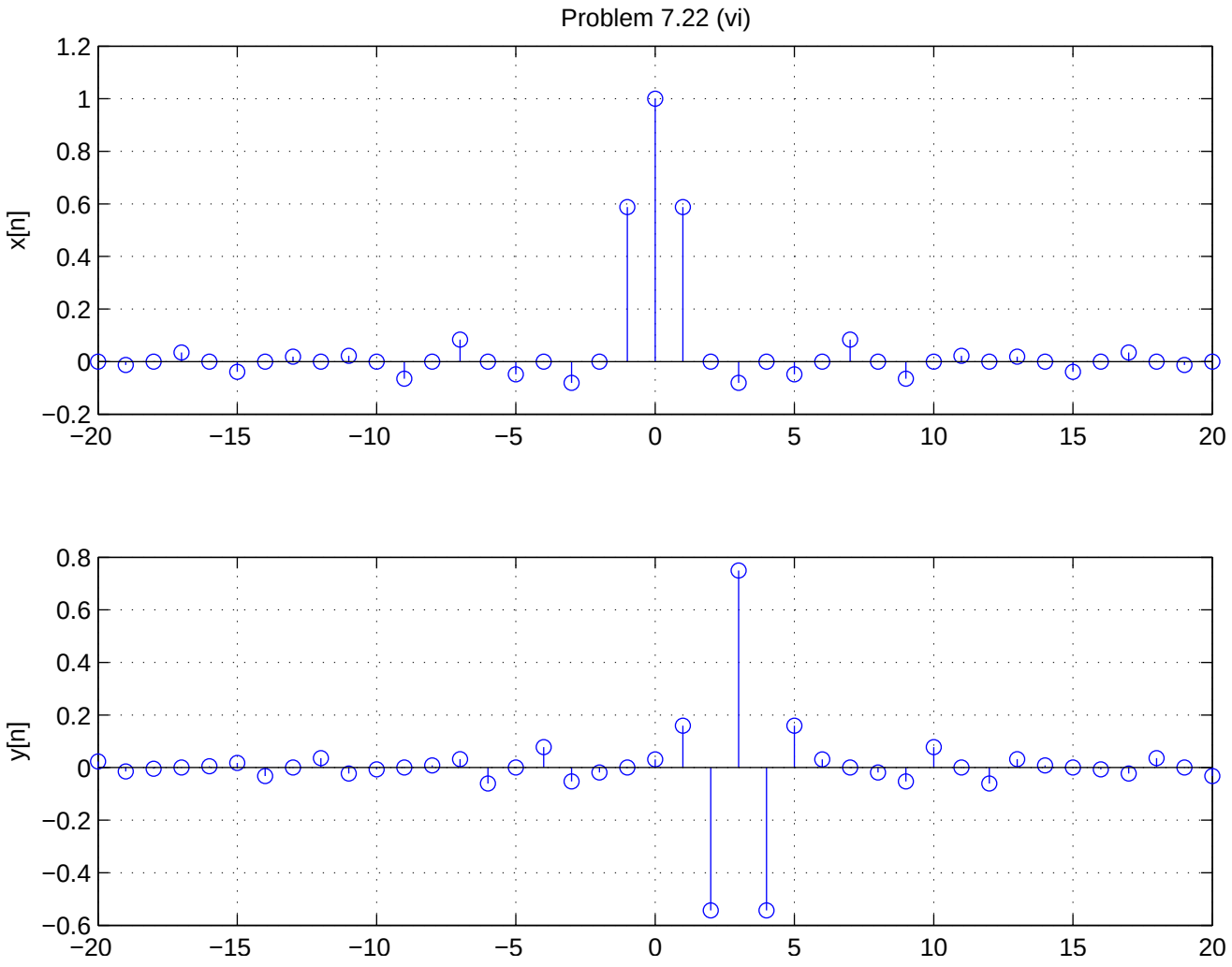

Problem 7.22 (i)
x[n]
y[n]
1
0.5
0
−0.5
−1
−20
−15
−10
−5
0
5
10
15
20

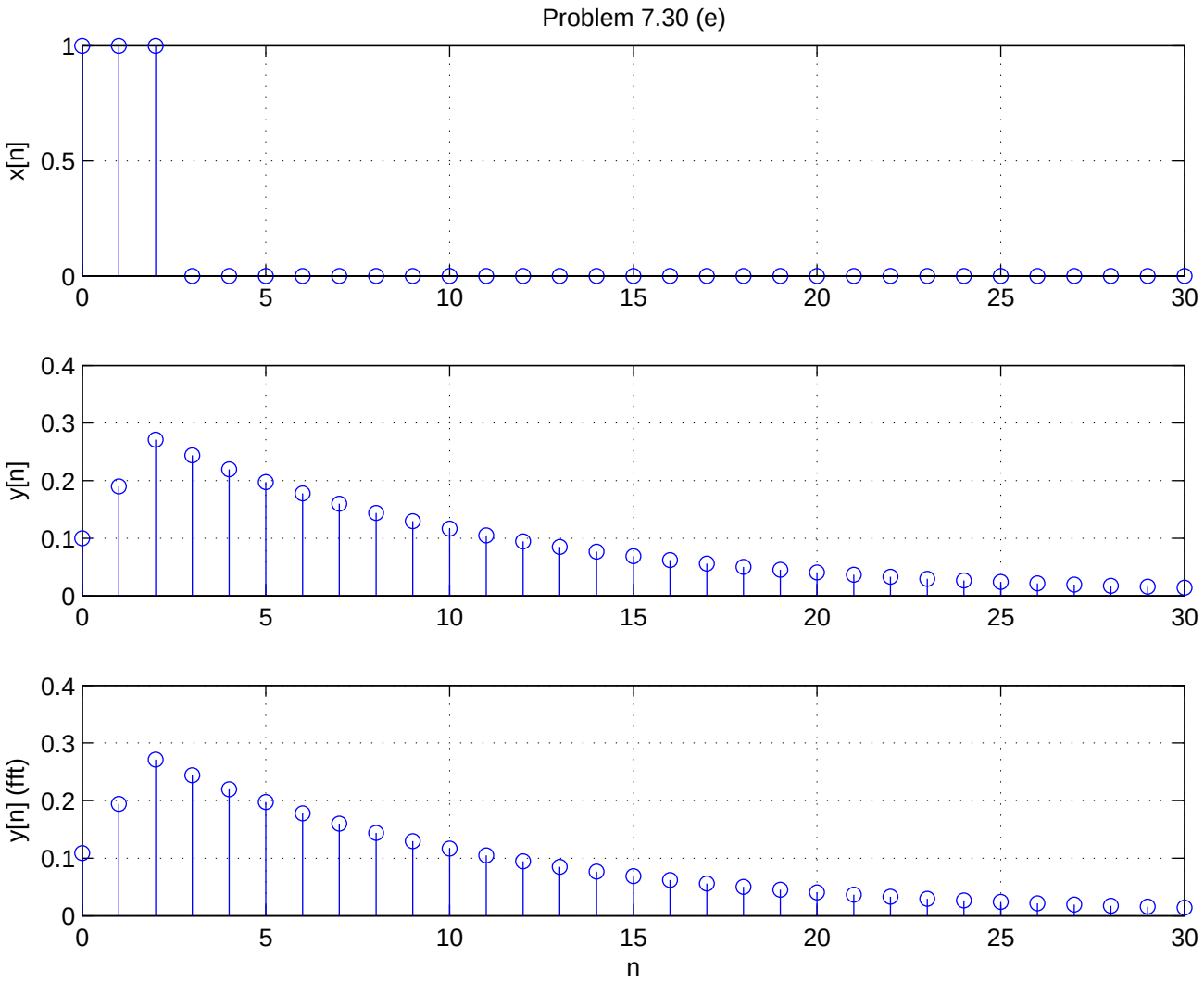

Problem 7.22 (ii)
x[n]
y[n]
1
0.5
0
−0.5
−1
−20
−15
−10
−5
0
5
10
15
20


Problem 7.22 (iii)
x[n]
y[n]
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
0.5
−0.5
−1
−20
−15
−10
−5
5
10
15
20


Problem 7.22 (iv)
x[n]
y[n]


Problem 7.22 (v)
x[n]
y[n]
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
0.5
−0.5
−1
−20
−15
−10
−5
5
10
15
20


Problem 7.22 (vi)
x[n]
y[n]
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−20
−15
−10
−5
0
5
10
15
20


Problem 7.30 (e)
x[n]
y[n]
y[n] (fft)
n
0
5
10
15
20
25
30
0
0.5
1
0.1
0.2
0.3
0.4