EE 341 - Homework 11

Due November 9, 2005

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots. Problems from the text tell you to use the M-file `dft`. You can use the built-in MATLAB function `fft` instead.

1. Problem 8.1 (a) (b) (c) (e)
2. Problem 8.2 (a) (b) (c) (f)
3. Problem 8.4 (b) (e)
4. Problem 8.5. When using the Final Value Theorem, make sure the final value exists.
5. Problem 8.6
6. Problem 8.10 (a) (b) (c) (d) (e)
7. Problem 8.12 (a) (c) (e) (f). You do not need to compute the partial frations by hand; use the MATLAB `residue` function if you prefer.

EE 341
HW #11

1. Problem 8.11

(a) $\cos(3t)u(t) \longleftrightarrow \dfrac{s}{s^2+9}$

(b) $e^{-10t}u(t) \longleftrightarrow \dfrac{1}{s+10}$

(c) $e^{-10t}\cos(3t)u(t) \longleftrightarrow \frac{1}{2}\left[\dfrac{1}{(s+j3)+10} + \dfrac{1}{(s-j3)+10}\right]$

$$= \frac{2(s+10)}{s^2+20s+91}$$

(e) $(2-2e^{-4t})u(t) = 2u(t) - 2e^{-4t}u(t)$

$$\longleftrightarrow \frac{2}{s} - \frac{2}{s+4} = \frac{8}{s(s+4)}$$

2. Problem 8.2

$$X(s) = \frac{s+1}{s^2+5s+7}$$

(a) $v(t) = x(3t-4)u(3t-4)$

$$= x(3(t-\tfrac{4}{3}))\,u(3(t-\tfrac{4}{3}))$$

$$\longleftrightarrow \frac{1}{3}X\left(\frac{s}{3}\right)e^{-\frac{4}{3}t} = \frac{1}{3}\,\frac{\frac{s}{3}+1}{(\frac{s}{3})^2+5(\frac{s}{3})+7}\,e^{-\frac{4}{3}t}$$

$$= \frac{s+3}{s^2+15s+63}e^{-\frac{4}{3}t}$$

(b) $v(t) = t\,x(t)$

$$V(s) = (-1)\frac{d}{ds}X(s) = -\frac{d}{ds}\frac{s+1}{s^2+5s+7} = -\frac{(s^2+5s+7)-(s+1)(2s+5)}{(s^2+5s+7)^2}$$

$$= \frac{s^2+2s-2}{(s^2+5s+7)^2}$$

(c) $v(t) = \dfrac{d^2x(t)}{dt^2}$

$$V(s) = s^2X(s) - s\,x(0) - \dot{x}(0)$$

$$x(0) = \lim_{s\to\infty} s\,X(s) = \lim_{s\to\infty}\frac{s(s+1)}{s^2+5s+7} = 1$$

$$\dot{x}(0) = \lim_{s\to\infty}\left[s^2X(s) - s\,x(0)\right] = \lim_{s\to\infty}\left[\frac{s^2(s+1)}{s^2+5s+7} - s\right]$$

$$= \lim_{s\to\infty}\left(\frac{s^3+s^2-s^3-5s^2-7s}{s^2+5s+7}\right) = \lim_{s\to\infty}\frac{-4s^2-7s}{s^2+5s+7} = -4$$

$$V(s) = \frac{s^2(s+1)}{s^2+5s+7} - s - 4 = \frac{-8s^2-27s-28}{s^2+5s+7}$$

(f) $v(t) = e^{-3t}x(t)$

$$V(s) = X(s+3) = \frac{(s+3)+1}{(s+3)^2+5(s+3)+7} = \frac{s+4}{s^2+11s+31}$$

3. Problem 8.4

(b) $x(t) = (e^{-bt}\sin^2\omega t)\,u(t)$

$$(\sin^2\omega t)\,u(t) \leftrightarrow \frac{2\omega^2}{s(s^2+4\omega^2)}$$

$$(e^{-bt}\sin^2\omega t)\,u(t) \leftrightarrow \frac{2\omega^2}{(s+b)((s+b)^2+4\omega^2)} = \frac{2\omega^2}{(s+3)(s^2+2bs+b^2+4\omega^2)}$$

(e) $x(t) = (\cos^3\omega t)\,u(t)$

$$(\cos^2\omega t)\,u(t) \leftrightarrow \frac{s^2+2\omega^2}{s(s^2+4\omega^2)}$$

$$(\cos^3\omega t)\,u(t) \leftrightarrow \frac{1}{2}\left[\frac{(s+j\omega)^2+2\omega^2}{(s+j\omega)((s+j\omega)^2+4\omega^2)} + \frac{(s-j\omega)^2+2\omega^2}{(s-j\omega)((s-j\omega)^2+4\omega^2)}\right]$$

$$=$$

4. Problem 8.5

(a) $X(s) = \frac{4}{s^2+s} = \frac{4}{s(s+1)}$

$\lim_{t\to\infty} x(t) = \lim_{s\to 0} \frac{4s}{s(s+1)} = \lim_{s\to 0} \frac{4}{s+1} = 4$

(b) $X(s) = \frac{3s+4}{s^2+s} = \frac{3s+4}{s(s+1)}$

$\lim_{t\to\infty} x(t) = \lim_{s\to 0} \frac{s(3s+4)}{s(s+1)} = \lim_{s\to 0} \frac{3s+4}{s+1} = 4$

(c) $X(s) = \frac{4}{s^2-s} = \frac{4}{s(s-1)}$

Pole at $s=+1 \Rightarrow x(t)$ blows up as $t\to\infty$

(d) $X(s) = \frac{3s^2+4s+1}{s^3+2s^2+s+2} = \frac{(3s+1)(s+1)}{(s+2)(s-j)(s+j)}$

Poles on imaginary axis — $x(t)$ oscillates forever

(e) $X(s) = \frac{3s^2+4s+1}{s^3+3s^2+3s+2} = \frac{(3s+1)(s+1)}{(s+2)(s+\frac{1}{2}-j\frac{\sqrt{3}}{2})(s+\frac{1}{2}+j\frac{\sqrt{3}}{2})}$

$\lim_{t\to\infty} x(t) = \lim_{s\to 0} \frac{s(3s^2+4s+1)}{s^3+3s^2+3s+2} = 0$

(f) $X(s) = \frac{3s^2+4s+1}{s^4+3s^3+3s^2+2s} = \frac{(3s+1)(s+1)}{s(s+2)(s+\frac{1}{2}-j\frac{\sqrt{3}}{2})(s+\frac{1}{2}+j\frac{\sqrt{3}}{2})}$

$\lim_{t\to\infty} x(t) = \lim_{s\to 0} \frac{s(3s^2+4s+1)}{s(s^3+3s^2+3s+2)} = \lim_{s\to 0} \frac{3s^2+4s+1}{s^3+3s^2+3s+2} = \frac{1}{2}$

5. Problem 8.1

(a) $X(s) = \dfrac{4}{s^2+s}$

$$x(0) = \lim_{s\to\infty} \frac{4s}{s^2+s} = \lim_{s\to\infty} \frac{4}{s+1} = 0$$

(b) $X(s) = \dfrac{3s+4}{s^2+s}$

$$x(0) = \lim_{s\to\infty} \frac{s(3s+4)}{s^2+s} = \lim_{s\to\infty} \frac{3s+4}{s+1} = 3$$

(c) $X(s) = \dfrac{4}{s^2-s}$

$$x(0) = \lim_{s\to\infty} \frac{4s}{s^2-s} = \lim_{s\to\infty} \frac{4}{s-1} = 0$$

(d) $X(s) = \dfrac{3s^2+4s+1}{s^3+2s^2+s+2}$

$$x(0) = \lim_{s\to\infty} \frac{s(3s^2+4s+1)}{s^3+2s^2+s+2} = \lim_{s\to\infty} \frac{3s^3+4s^2+s}{s^3+2s^2+s+2} = 3$$

(e) $X(s) = \dfrac{3s^2+4s+1}{s^3+3s^2+3s+2}$

$$x(0) = \lim_{s\to\infty} \frac{s(3s^2+4s+1)}{s^3+4s^2+3s+2} = 3$$

(f) $X(s) = \dfrac{3s^2+4s+1}{s^4+3s^3+3s^2+2s}$

$$x(0) = \lim_{s\to\infty} \frac{s(3s^2+4s+1)}{s^4+3s^3+3s^2+2s} = 0$$

6 Problem 8.10

(a) $X(s) = \dfrac{s+2}{s^2+7s+12} = \dfrac{s+2}{(s+3)(s+4)} = \dfrac{-1}{s+3} + \dfrac{2}{s+4}$

$x(t) = -e^{-3t}u(t) + 2e^{-4t}u(t)$

(b) $X(s) = \dfrac{s+1}{s^3+5s^2+7s} = \dfrac{s+1}{s(s+\frac{5}{2}-j\frac{\sqrt{3}}{2})(s+\frac{5}{2}+j\frac{\sqrt{3}}{2})}$

$= \dfrac{1/7}{s} + \dfrac{\frac{-1-j3\sqrt{3}}{14}}{s+\frac{5}{2}-j\frac{\sqrt{3}}{2}} \quad \dfrac{\frac{-1+j3\sqrt{3}}{14}}{s+\frac{5}{2}+j\frac{\sqrt{3}}{2}}$

$x(t) = \frac{1}{7}u(t) + 2\left|\frac{-1-j3\sqrt{3}}{14}\right| e^{-5/2t} \cos\left(\frac{\sqrt{3}}{2}t + \measuredangle\left(\frac{-1-j3\sqrt{3}}{14}\right)\right)$

$= \frac{1}{7}u(t) + 0.756e^{-\frac{5}{2}t}\cos\left(\frac{\sqrt{3}}{2} - 101^\circ\right)$

(c) $X(s) = \dfrac{2s^2-9s-35}{s^2+4s+2} = \dfrac{2s^2-9s-35}{(s+2-\sqrt{2})(s+2+\sqrt{2})} + 2$

$= \dfrac{2(-2+\sqrt{2})^2-9(-2-\sqrt{2})-35}{s+2+\sqrt{2}} + \dfrac{2(-2-\sqrt{2})^2-9(-2-\sqrt{2})-35}{s+2+\sqrt{2}} + 2$

$= \dfrac{-6.73}{s+3.414} + \dfrac{-10.27}{s+0.586} + 2$

$x(t) = -6.73e^{-3.414t}u(t) - 10.27e^{-0.586t}u(t) + 2\delta(t)$

(d) $X(s) = \dfrac{3s^2+2s+1}{s^3+5s^2+8s+4} = \dfrac{1}{s+2} + \dfrac{-9}{(s+2)^2} + \dfrac{2}{s+1}$

$x(t) = e^{-2t}u(t) - 9te^{-2t}u(t) + 2e^{-t}u(t)$

(e) $X(s) = \dfrac{s^2+1}{s^5+18s^3+81s} = \dfrac{-0.0062}{s-j3} + \dfrac{-0.0062}{s+j3} + \dfrac{-0.074j}{(s-j3)^2} + \dfrac{0.074j}{(s+j3)^2} + \dfrac{0.0123}{s}$

$x(t) = (0.0123\cos(3t+180^\circ) + 0.148\,t\cos(3t-90^\circ) + 0.123)u(t)$

7. Problem 8.11

See MATLAB. Note: you cannot do 8.11 (f) numerically using the techniques discussed in the book.

7. Problem 8.12

(a) $X(s) = \dfrac{s^2-2s+1}{s(s^2+4)} = \dfrac{\frac{3}{8}+j\frac{1}{2}}{s-j2} + \dfrac{\frac{3}{8}-j\frac{1}{2}}{s+j2} + \dfrac{1/4}{s}$

$x(t) = 2\left|\frac{3}{8}+j\frac{1}{2}\right| \cos(2t + \angle(\frac{3}{8}+j1/2))\, u(t) + 1/4\, u(t)$

$= 1.25 \cos(2t+53^\circ)\, u(t) + 0.25\, u(t)$

(c) $X(s) = \dfrac{s^2-2s+1}{s^2(s^2+4)} = \dfrac{\frac{1}{4}-j\frac{3}{16}}{s-j2} + \dfrac{\frac{1}{4}+j\frac{3}{16}}{s+j2} + \dfrac{-1/2}{s} + \dfrac{1/4}{s^2}$

$x(t) = 2\left|\frac{1}{4}-j\frac{3}{16}\right| \cos(2t + \angle(\frac{1}{4}-j\frac{3}{16})) - 1/2\, u(t) + 1/4\, t\, u(t)$

$= 0.625 \cos(2t-37^\circ) - \frac{1}{2} u(t) + \frac{1}{4} t\, u(t)$

(e) $X(s) = \dfrac{s^2-2s+1}{(s+2)^2+4} = \dfrac{-3-j\frac{5}{8}}{s+2-j2} + \dfrac{-3+j\frac{5}{8}}{s+2+j2} + 1$

$x(t) = 2\left|-3-j\frac{5}{8}\right| e^{-2t} \cos(2t + \angle(-3-j\frac{5}{8}))\, u(t) + \delta(t)$

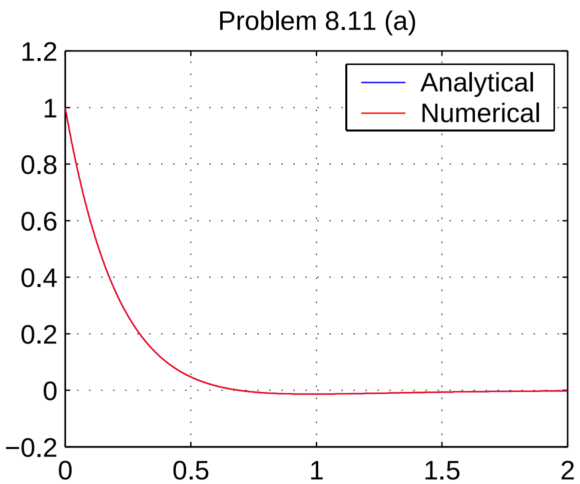
$= 6.5\, e^{-2t} \cos(2t-157^\circ)\, u(t) + \delta(t)$

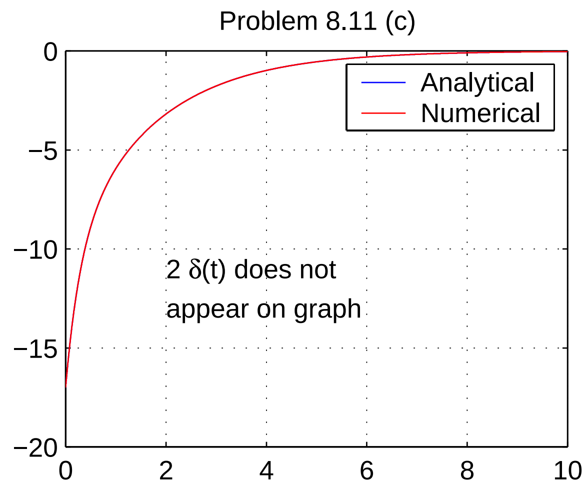
(f) $X(s) = \dfrac{s^2-2s+1}{s[(s+2)^2+4]} = \dfrac{\frac{7}{16}+j\frac{17}{16}}{s+2-j2} + \dfrac{\frac{7}{16}-j\frac{17}{16}}{s+2+j2} + \dfrac{1/8}{s}$

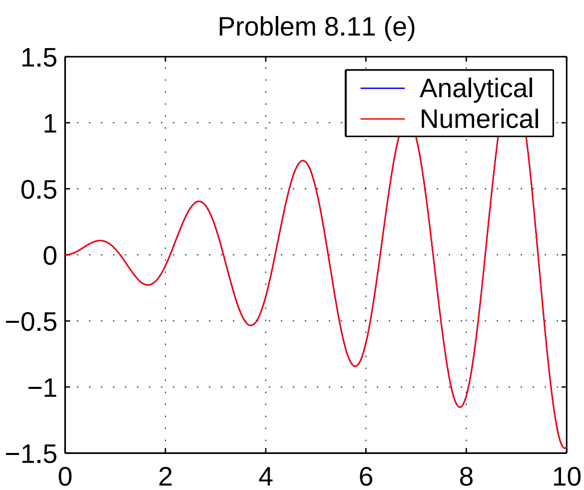
$x(t) = 2\left|\frac{7}{16}+j\frac{17}{16}\right| e^{-2t} \cos(2t + \angle(\frac{7}{16}+j\frac{17}{16}))\, u(t) + \frac{1}{8} u(t)$

$= 2.3\, e^{-2t} \cos(2t+67^\circ)\, u(t) + \frac{1}{8} u(t)$

```
% EE 341 Homework # 11

% Problem 8.11

% (a)
figure(1)
clf
subplot(221)
b=[1 2];
a=[1 7 12];
t=0:0.001:2;
% Analytical solution
x = -exp(-3*t)+2*exp(-4*t);
plot(t,x)
grid
hold on
% Numerical solution
xn=impulse(b,a,t);
plot(t,xn,'r');
legend('Analytical','Numerical')
title('Problem 8.11 (a)')

% (c)
subplot(222)
b=[2 -9 -35];
a=[1 4 2];
t=0:0.001:10;
% Analytical solution
[r,p,k]=residue(b,a);
x = r(1)*exp(p(1)*t) + r(2)*exp(p(2)*t);
plot(t,x)
grid
hold on
% Numerical solution
xn=impulse(b,a,t);
plot(t,xn,'r');
legend('Analytical','Numerical')
title('Problem 8.11 (c)')
text(2,-11,'2 \delta(t) does not')
text(2,-13,'appear on graph')

% (e)
subplot(223)
b=[1 0 1];
a=[1 0 18 0 81 0];
t=0:0.001:10;
% Analytical solution
[r,p,k]=residue(b,a);
x = 2*abs(r(1))*cos(imag(p(1))*t+angle(r(1))) + ...
    2*abs(r(2))*t.*cos(imag(p(2))*t+angle(r(2))) + r(5);
plot(t,x)
grid
hold on
% Numerical solution
xn=impulse(b,a,t);
plot(t,xn,'r');
legend('Analytical','Numerical')
title('Problem 8.11 (e)')
print -dpsc2 p8_11.ps
```


Problem 8.11 (a)
Analytical
Numerical
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
0
0.5
1
1.5
2


Problem 8.11 (c)
Analytical
Numerical
2 δ(t) does not
appear on graph
0
−5
−10
−15
−20
0
2
4
6
8
10


Problem 8.11 (e)
Analytical
Numerical
1.5
1
0.5
0
−0.5
−1
−1.5
0
2
4
6
8
10