EE 341 - Homework 12

Due November 16, 2005

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots.

1. Problem 8.14 (a) (b) (e) (g). Plot $y(t)$ for (e) and (g).
2. Problem 8.16 (a) (c) (d). Show that the output for (d) is the sum of the outputs for (a) and (b). Explain why.
3. Problem 8.25.
4. Problem 8.28.
5. Problem 8.31 (a) (b).
6. Problem 9.1. You may use MATLAB to find the poles of the signals.
7. Problem 9.16.
8. Problem 9.21.
9. Problem 9.35 (a) (c) (d) (e). You only need to sketch the magnitude response by hand.

EE 341
HW #12

1. Problem 8.14

(a) $\frac{dy}{dt} + 2y = u(t) \qquad y(0) = 0$

$sY(s) - \cancel{y(0)}^{0} + 2Y(s) = \frac{1}{s}$

$Y(s)(s+2) = \frac{1}{s} \qquad Y(s) = \frac{1}{s(s+2)} = \frac{1/2}{s} + \frac{-1/2}{s+2}$

$y(t) = \frac{1}{2}u(t) - \frac{1}{2}e^{-2t}u(t)$

(b) $\frac{dy}{dt} - 2y = u(t) \qquad y(0) = 1$

$sY(s) - \cancel{y(0)}^{1} - 2Y(s) = \frac{1}{s}$

$Y(s)(s-2) - 1 = 1/s \qquad Y(s) = \frac{1/s + 1}{s-2} = \frac{s+1}{s(s-2)}$

$Y(s) = \frac{-1/2}{s} + \frac{3/2}{s-2}$

$y(t) = -\frac{1}{2}u(t) + \frac{3}{2}e^{2t}u(t)$

(c) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = u(t) \qquad y(0) = 0 \qquad \dot{y}(0) = 1$

$s^2Y(s) - s\cancel{y(0)}^{0} - \cancel{\dot{y}(0)}^{1} + 6(sY(s) - \cancel{y(0)}^{0}) + 8Y(s) = 1/s$

$Y(s)(s^2+6s+8) - 1 = 1/s \qquad Y(s) = \frac{1/s + 1}{s^2+6s+8} = \frac{s+1}{s(s+2)(s+4)}$

$Y(s) = \frac{1/8}{s} + \frac{1/4}{s+2} + \frac{-3/8}{s+4}$

$y(t) = \frac{1}{8}u(t) + 1/4\, e^{-2t}u(t) - \frac{3}{8}e^{-4t}u(t)$

See MATLAB for plot

(g) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 13 = u(t) \qquad y(0) = 1 \qquad \dot{y}(0) = 1$

$s^2Y(s) - s\,y(0) - \dot{y}(0) + 6(sY(s) - y(0)) + 13 = 1/s$

$Y(s)(s^2 + 6s + 13) - s - 7 = 1/s \qquad Y(s)(s^2+6s+13) = \frac{1}{s} + s + 7 = \frac{s^2+7s+1}{s}$

$Y(s) = \frac{s^2+7s+1}{s(s+3-j2)(s+3+j2)} = \frac{1/13}{s} + \frac{\frac{6}{13} - j\frac{49}{52}}{s+3-j2} + \frac{\frac{6}{13} + j\frac{49}{52}}{s+3+j2}$

$y(t) = \frac{1}{13}u(t) + 2\left|\frac{6}{13} - j\frac{49}{52}\right| e^{-3t}\cos\left(2t + \measuredangle\left(\frac{6}{13} - j\frac{49}{52}\right)\right)$

$= \frac{1}{13}u(t) + 2.099\,e^{-3t}\cos(2t - 64°)$

See MATLAB for plot

2. Problem 8.16

$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 3y = 2\frac{d^2x}{dt^2} - 4\frac{dx}{dt} - x$

$s^2Y(s) - s\,y(0) - \dot{y}(0) + 4(sY(s) - y(0)) + 3Y(s) = 2s^2X(s) - 4sX(s) - X(s)$

$Y(s) = \frac{s\,y(0) + \dot{y}(0) + 4y(0)}{s^2+4s+3} + \frac{2s^2 - 4s - 1}{s^2+4s+3}X(s)$

(a) $y(0^-) = -2;\ \dot{y}(0^-) = 1,\ x(t) = 0 \Rightarrow X(s) = 0$

$Y(s) = \frac{-2s + 1 + 4(-2)}{s^2+4s+3} = \frac{-2s+5}{(s+3)(s+1)} = \frac{9}{s+2} + \frac{7}{s+1}$

$y_a(t) = 9e^{-2t}u(t) + 7e^{-t}u(t)$

(c) $y(0^-) = 0;\ \dot{y}(0) = 0,\ x(t) = u(t) \Rightarrow X(s) = \frac{1}{s}$

$Y(s) = \frac{2s^2-4s-1}{s^2+4s+5}\cdot\frac{1}{s} = \frac{2s^2-4s-1}{(s+2)(s+1)s} = \frac{15/2}{s+2} + \frac{-5}{s+1} + \frac{-1/3}{s}$

$y_c(t) = \frac{15}{2}e^{-2t}u(t) - 5e^{-t}u(t) - \frac{1}{3}u(t)$

(d) $y(0)=-4 \quad \dot{y}(0)=1, \quad x(t)=u(t)$

$$Y(s)= \frac{-2s+1+4(1)}{s^2+4s+3} + \frac{2s^2-4s-1}{s^2+4s+3}\,\frac{1}{s} = \frac{-2s+5}{s^2+4s+3} + \frac{2s^2-4s-1}{s^2+4s+3}\,\frac{1}{s}$$

$$= \frac{9}{s+2} + \frac{7}{s+1} + \frac{15/2}{s+2} + \frac{-5}{s+1} + \frac{-1/3}{s}$$

① $y_d(t) = 9e^{-2t}u(t) + 7e^{-t}u(t) + \frac{15}{2}e^{-2t}u(t) - 5e^{-t}u(t) - \frac{1}{3}u(t)$

$y_d(t) = \frac{33}{2}e^{-2t}u(t) + 2e^{-t}u(t) - \frac{1}{3}u(t)$

① Shows that $y_d(t) = y_a(t) + y_c(t)$

This is due to superposition

3. Problem 8.25

(a) No transfer function — system not time invariant

(b) $Y(s) + V(s)\,Y(s) = X(s) \qquad V(s) = \frac{1}{s^2+1}$

$Y(s) = \dfrac{X(s)}{1+V(s)}$

$H(s) = \dfrac{1}{1+1/s^2+1} = \dfrac{s^2+1}{s^2+2}$

(c) $s^2 Y(s) + \frac{1}{s} Y(s) = s\,X(s) - X(s)$

$Y(s) = \dfrac{s-1}{s^2+1/s}\,X(s)$

$H(s) = \dfrac{s^2-s}{s^3+1}$

(d) No transfer function - system not linear

(e) No transfer function — system not time invariant

4. Problem 8.28

$x_1(t) = e^{-t}u(t) \quad X_1(s) = \frac{1}{s+1} \qquad y_1(t) = 3t + 2 - e^{-t} \Rightarrow Y_1(s) = \frac{3}{s^2} + \frac{2}{s} - \frac{1}{s+1}$

$= \frac{s^2+5s+3}{s^2(s+1)}$

$x_2(t) = e^{-2t}u(t) \Rightarrow X_2(s) = \frac{1}{s+2}$

$y_2(t) = 2t + 2 - e^{-2t} \Rightarrow Y_2(s) = \frac{2}{s^2} + \frac{2}{s} - \frac{1}{s+2}$

$= \frac{s^2+6s+4}{s^2(s+2)}$

$$Y_1(s) = \frac{C(s)}{A(s)} + \frac{B(s)}{A(s)} X_1(s) = \frac{C(s)}{A(s)} + \frac{B(s)}{A(s)} \frac{1}{(s+1)}$$

$$= \frac{C(s)(s+1) + B(s)}{A(s)(s+1)} = \frac{s^2+5s+3}{s^2(s+1)} \quad (1)$$

$$Y_2(s) = \frac{C(s)(s+2) + B(s)}{A(s)(s+2)} = \frac{s^2+6s+4}{s^2(s+2)} \quad (2)$$

We see that $A(s) = s^2$

For a 2^{nd} order system, $C(s) = y(0^-)s + \dot{y}(0^-) + a_1 y(0^-)$

$B(s) = b_2 s^2 + b_1 s + b_0$

But $A(s) = s^2 + a_1 s + a_0 = s^2 \Rightarrow a_1 = 0$, so $C(s) = y(0^-)s + \dot{y}(0^-)$

(1). $(y(0^-)s + \dot{y}(0^-))(s+1) + b_0 s^2 + b_1 s + b_0 = s^2 + 5s + 3$

$(y(0^-) + b_0)s^2 + (\dot{y}(0^-) + y(0^-) + b_1)s + (\dot{y}(0^-) + b_0) = s^2 + 5s + 3$

(2) $(y(0^-)s + \dot{y}(0^-))(s+2) + b_0 s^2 + b_1 s + b_0 = s^2 + 6s + 4$

$(y(0^-) + b_0)s^2 + (\dot{y}(0^-) + 2y(0^-) + b_1)s + (2\dot{y}(0^-) + b_0) = s^2 + 6s + 4$

Coef of s^0: $\dot{y}(0^-) + b_0 = 3$, $2\dot{y}(0^-) + b_0 = 4 \Rightarrow \dot{y}(0^-) = 1$, $b_0 = 2$

Coef of s^1: $\dot{y}(0^-) + y(0^-) + b_1 = 5$; $\dot{y}(0^-) + 2y(0^-) + b_1 = 6 \Rightarrow y(0^-) = 1$, $b_1 = 3$

Coef of s^2: $y(0^-) + b_2 = 1$, $y(0^-) + b_2 = 1 \Rightarrow b_2 = 0$

$B(s) = b_2 s^2 + b_1 s + b_0 = 3s + 2$

$A(s) = s^2$

$$H(s) = \frac{B(s)}{A(s)} = \frac{3s+2}{s^2} = \frac{3}{s} + \frac{2}{s^2}$$

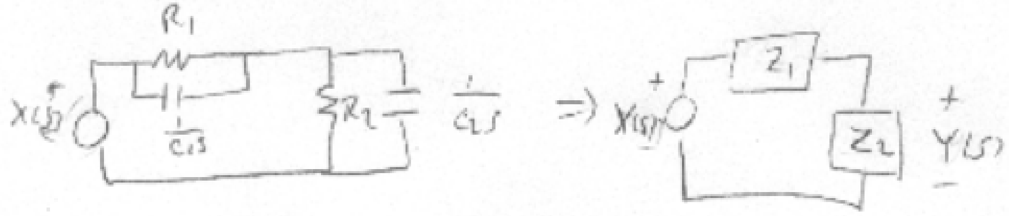
$h(t) = 3u(t) + 2t\,u(t)$

$y(0^-) = 1$

$\dot{y}(0^-) = -1$

5. Problem 8.31

(a)



$$Z_1 = \frac{R_1\left(\frac{1}{C_1 s}\right)}{R_1 + \frac{1}{C_1 s}} = \frac{R_1}{R_1 C_1 s + 1}$$

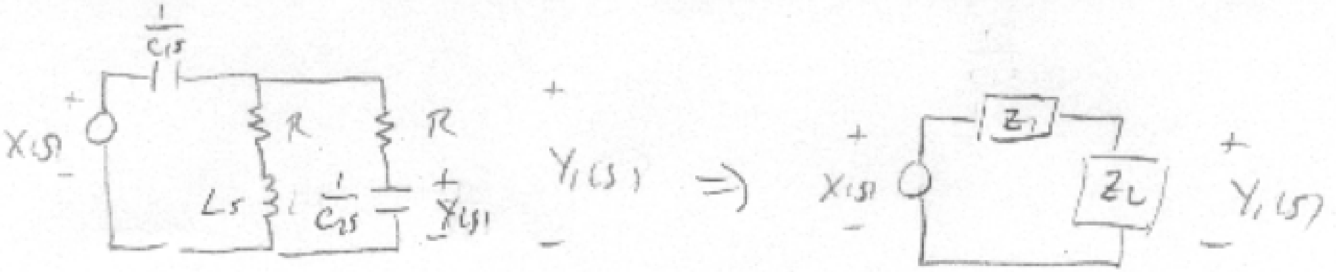
$$Z_2 = \frac{R_2\left(\frac{1}{C_2 s}\right)}{R_2 + \frac{1}{C_2 s}} = \frac{R_2}{R_2 C_2 s + 1}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{R_2}{R_2 C_2 s + 1}}{\frac{R_1}{R_1 C_1 s + 1} + \frac{R_2}{R_2 C_2 s + 1}}$$

$$= \frac{R_2(R_1 C_1 s + 1)}{R_1(R_2 C_2 s + 1) + R_2(R_1 C_1 s + 1)} = \frac{R_1 R_2 C_1 s + R_2}{R_1 R_2 (C_1 + C_2) s + (R_1 + R_2)}$$

$$= \frac{\left(\frac{C_1}{C_1 + C_2}\right) s + \frac{1}{R_1 (C_1 + C_2)}}{s + \frac{R_1 + R_2}{R_1 R_2 (C_1 + C_2)}}$$

(b)



$$Z_1 = \frac{1}{C_1 s}$$

$$Z_2 = \frac{\left(R + \frac{1}{C_2 s}\right)(R + Ls)}{\left(R + \frac{1}{C_2 s}\right) + (R + Ls)} = \frac{RLC_2 s^2 + (R^2 C_2 + L)s + R}{LC_2 s^2 + 2RC_2 s + 1}$$

$$Y_1(s) = \frac{Z_2}{Z_1 + Z_2} X(s) = \frac{RLC_1C_2 s^3 + (R^2C_2 + L)C_1 s^2 + RC_1 s}{RLC_1C_2 s^3 + (R^2C_1C_2 + L(C_1+C_2))s^2 + (C_1 + 2C_2)Rs + 1} X(s)$$

$$Y(s) = \frac{\frac{1}{C_2 s}}{R + \frac{1}{C_2 s}} Y_1(s) = \frac{C_1 L s^2 + C_1 R s}{RLC_1C_2 s^3 + (R^2C_1C_2 + L(C_1+C_2))s^2 + (C_1+2C_2)Rs + 1} X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\frac{1}{RC_2}s^2 + \frac{1}{LC_2}s}{s^3 + \left(\frac{R}{L} + \frac{1}{RC_2} + \frac{1}{RC_1}\right)s^2 + \left(\frac{1}{LC_2} + \frac{2}{LC_1}\right)s + \frac{1}{RLC_1C_2}}$$

6. Problem 9.1

(a) $H(s) = \frac{s-4}{s(s+7)}$ Marginally stable - pole on imaginary axis

Poles: $0, -7$

(b) $H(s) = \frac{s+3}{s^2+3} = \frac{s+3}{(s+j\sqrt{3})(s-j\sqrt{3})}$ Poles at $j\sqrt{3}$, $-j\sqrt{3}$

Marginally stable, poles on imaginary axis

(c) $H(s) = \frac{2s^2+3s+1}{s^2+2s^2+4} = \frac{2s^2+3+1}{(s+1-j\sqrt{3})(s+1+j\sqrt{3})}$ Poles at $-1 \pm j\sqrt{3}$

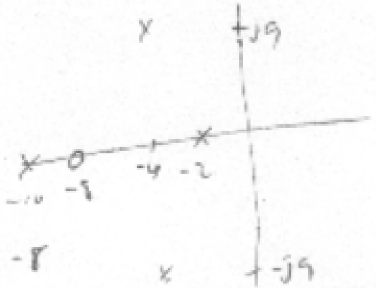
Stable

(d) $H(s) = \frac{3s^3-2s+6}{s^3+s^2+s+1} = \frac{3s^3-2s+6}{(s+1)(s-j)(s+j)}$ Poles at $-1, j, -j$

Marginally stable

7. Problem 9.16

(i) $H(s) = \dfrac{242.5(s+8)}{(s+2)[(s+4)^2+81](s+10)}$



(a) Poles at -2, $-4+j9$, $-4-j9$, -10; zero at -8

(b) $G(s) = \dfrac{242.5(s+8)}{s(s+2)[(s+4)^2+81](s+10)} = \dfrac{r_0}{s} + \dfrac{r_1}{s+2} + \dfrac{r_2}{s+4-j9} + \dfrac{r_2^*}{s+4+j9} + \dfrac{r_4}{s+10}$

$g(t) = r_1 u(t) + r_2 e^{-2t}u(t) + 2|r_2|e^{-4t}\cos(9t + \measuredangle r_2)u(t) + r_4 e^{-10t}u(t)$

(c) $g(\infty) = \lim\limits_{s\to 0} s\,G(s) = \lim\limits_{s\to 0} \dfrac{242.5(s+8)}{(s+2)[(s+4)^2+81](s+10)} = \dfrac{(242.5)(8)}{2(4^2+81)(10)} = 1$

(d) Dominant pole: -2

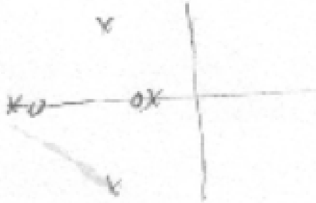
(e) See MATLAB

(b) Step response shows several exponential signals

(c) Final value is 1

(d) It takes a few seconds to get to final value, consistent with dominant pole at -2 ($\frac{1}{2}$ second time constant)

(ii) $H(s) = \dfrac{115.5(s+8)(s+2.1)}{(s+2)[(s+4)^2+81](s+10)}$



(a) Poles at -2, $-4+j9$, $-4-j9$, -10; zeros at -8, -2.1

(b) Same as (i)

(c) $g(\infty) = \lim\limits_{s\to 0} s\,G(s) = \lim\limits_{s\to 0} \dfrac{115.5(s+8)(s+2.1)}{(s+2)[(s+4)^2+81](s+10)} = \dfrac{115.5(8)(2.1)}{(2)(4^2+81)(10)} = 1.0002$

(d) Dominant pole: -2

(e) See MATLAB

(b) Step response shows several exponential signals; damped sinusoid very evident

(c) Final value is about 1.0002

(d) Dominant pole at -2 less prominent because it is nearly canceled by zero at -2.1

8. Problem 9.21

$$H(s) = \frac{s^2+16}{s^2+7s+12}$$

$$x(t) = 2\cos 4t \quad t \geq 0 \implies X(s) = \frac{s}{s^2+16}$$

$$Y(s) = H(s)X(s) = \frac{s^2+16}{s^2+7s+12}\,\frac{s}{s^2+16} = \frac{s}{(s+3)(s+4)} = \frac{-3}{s+3} + \frac{4}{s+4}$$

$$y(t) = \underbrace{-3e^{-3t}u(t) + 4e^{-4t}u(t)}_{\text{Transient response}} \qquad \underbrace{\qquad\qquad}_{\text{steady state response}}$$

There is no steady state response for the input $2\cos(4t)$ because the system zeros at $\pm j4$ cancel out the input poles at $\pm j4$

9. Problem 9.35

(a) $$H(s) = \frac{16}{(s+1)(s+8)}$$

At $\omega = 0.1$, $|H(.1)| \approx \frac{16}{(1)(8)} = 2 = 6\text{ dB}$

Signal stays flat from 0.1 to 1; drops at -20 dB/decade to 8 rad/s, then falls off at -40 dB/decade

See plot which follows

(b) $H(s) = \dfrac{10(s+4)}{(s+1)(s+10)}$

Start at $\omega = 0.1$ rad/sec. $H(0.1) \simeq \dfrac{10(4)}{(1)(10)} = 4 = 12$ dB

Gain flat to 1 rad/sec, falls at -20 dB/decade to 4 rad/sec, flat to 10 rad/sec, falls at -20 dB/decade from then on

See sketch

(e) $H(s) = \dfrac{10}{(s+1)(s^2+s+16)} = \dfrac{10}{(s+1)(s^2+2(\frac{1}{4})4s+4^2)}$

$\uparrow \zeta \quad \uparrow \omega_n \quad \uparrow \omega_n^2$

Start with $\omega = 0.1$ rad/s

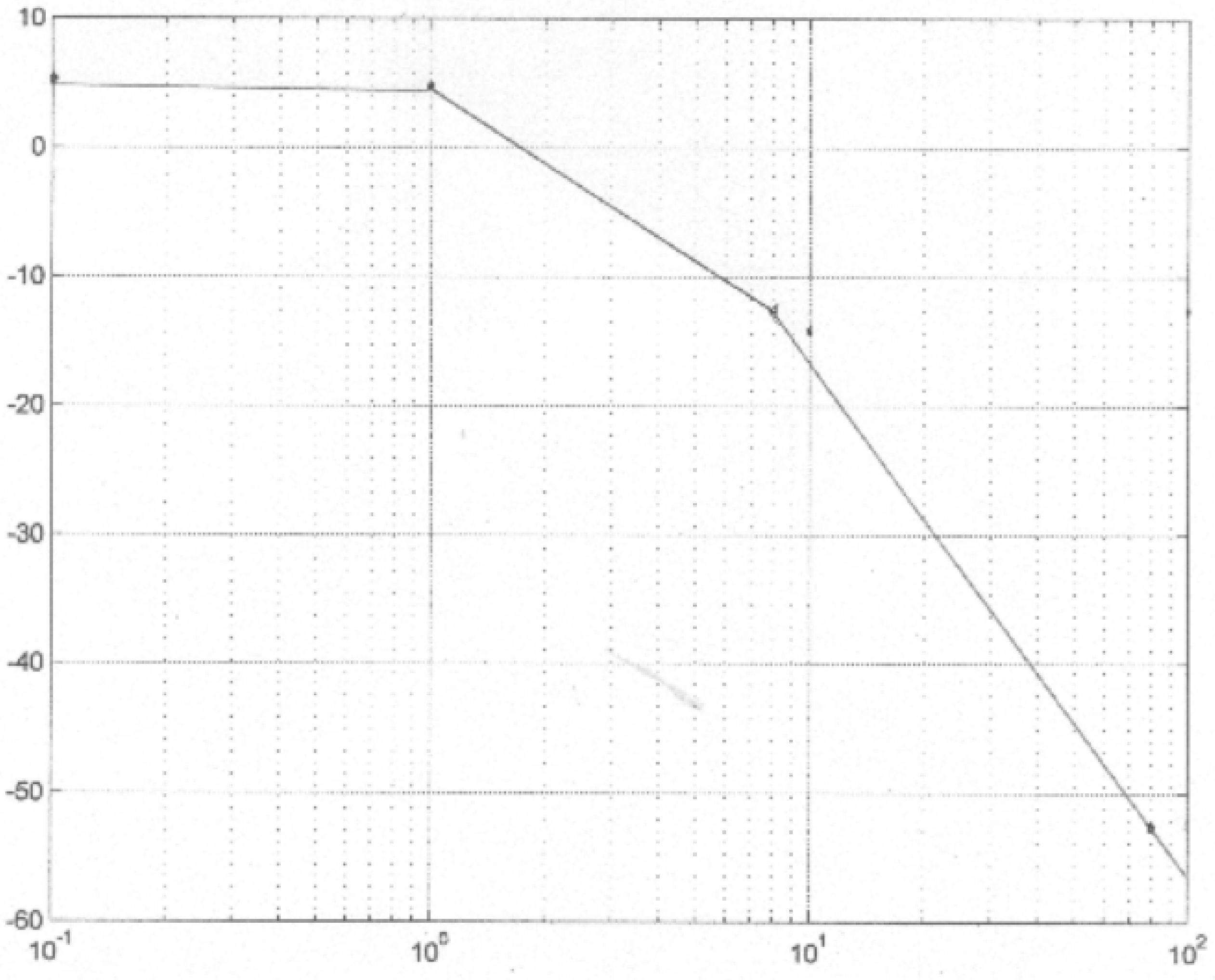
$H(0.1) \simeq \dfrac{10}{(1)(16)} = 0.625 = -4$ dB

Gain flat until 1 rad/sec, falls at -20 dB/decade to 4 rad/sec, falls at -60 dB/decade from then on.

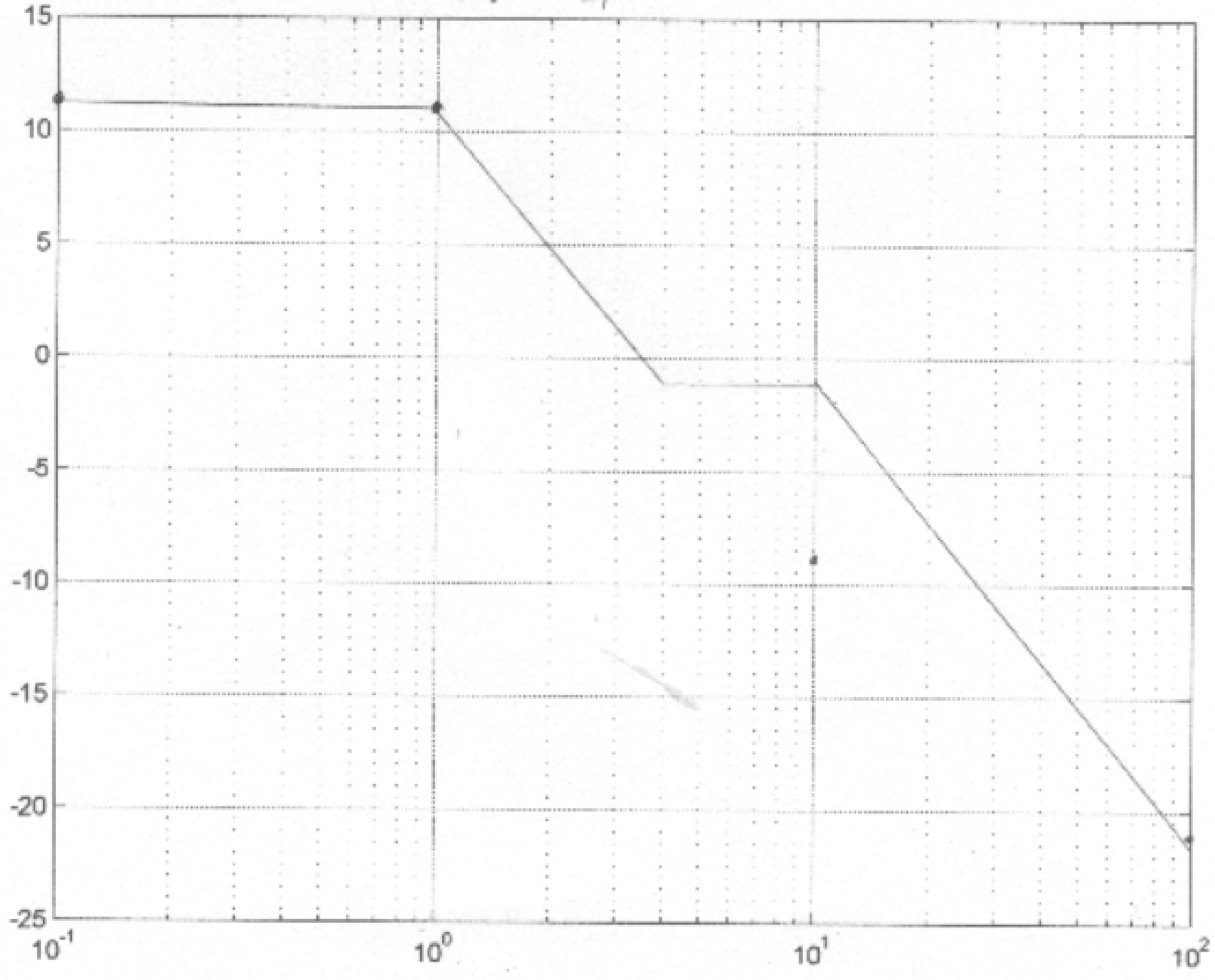
Because $\zeta = 0.25$, there is a bump a few dB high at 4 rad/sec

See sketch which follows

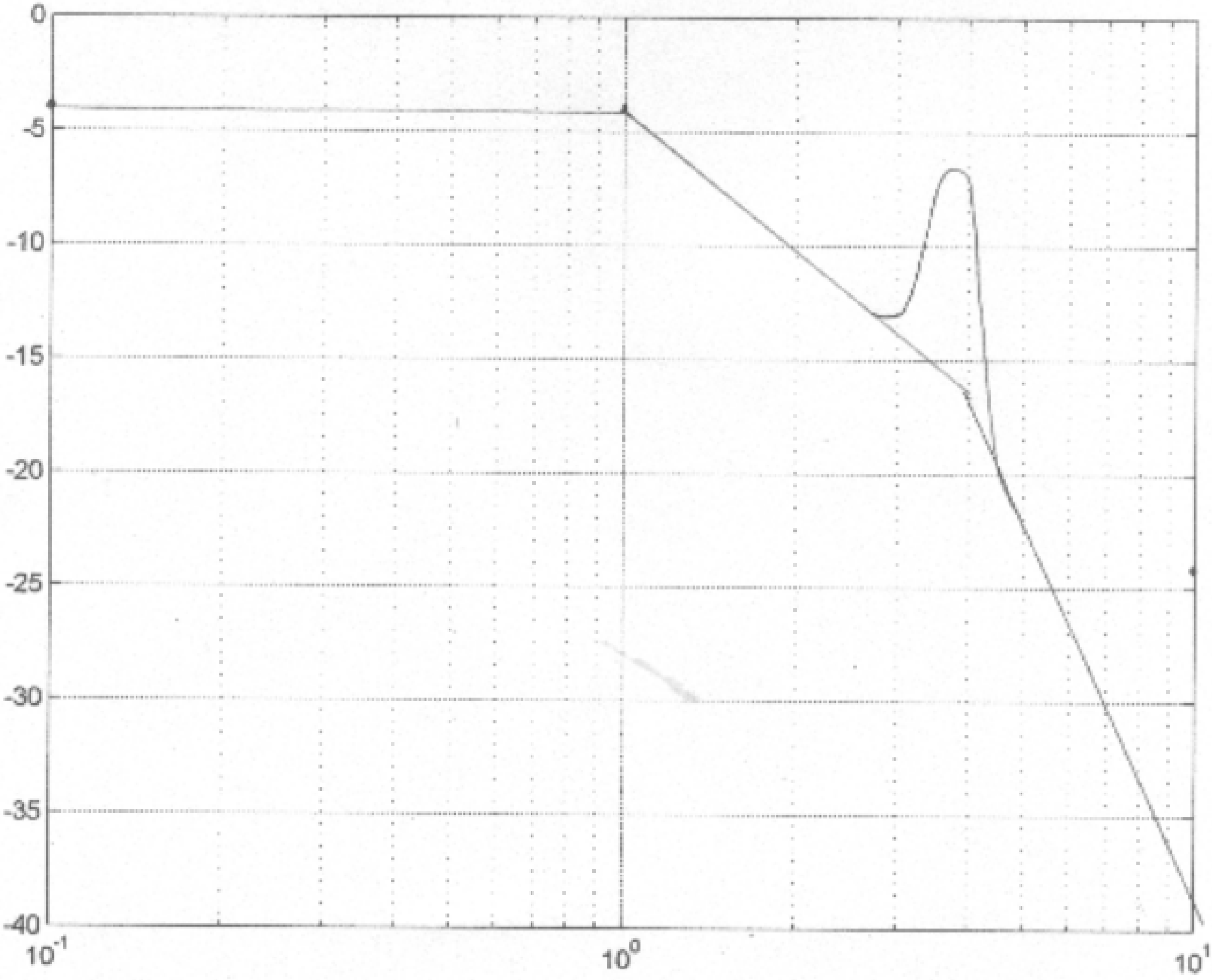
9.35 (a)



9.35 (b)



9 35 (e)



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% EE 341 HW #12

% Problem 8.14
figure(1)
clf
% (e)
t=0:0.001:3;
ye=(1/8) + (1/4)*exp(-2*t)-(3/8)*exp(-4*t);
subplot(211)
plot(t,ye)
grid
title('Problem 8.14 (e) (g)')
ylabel('y_e(t)')

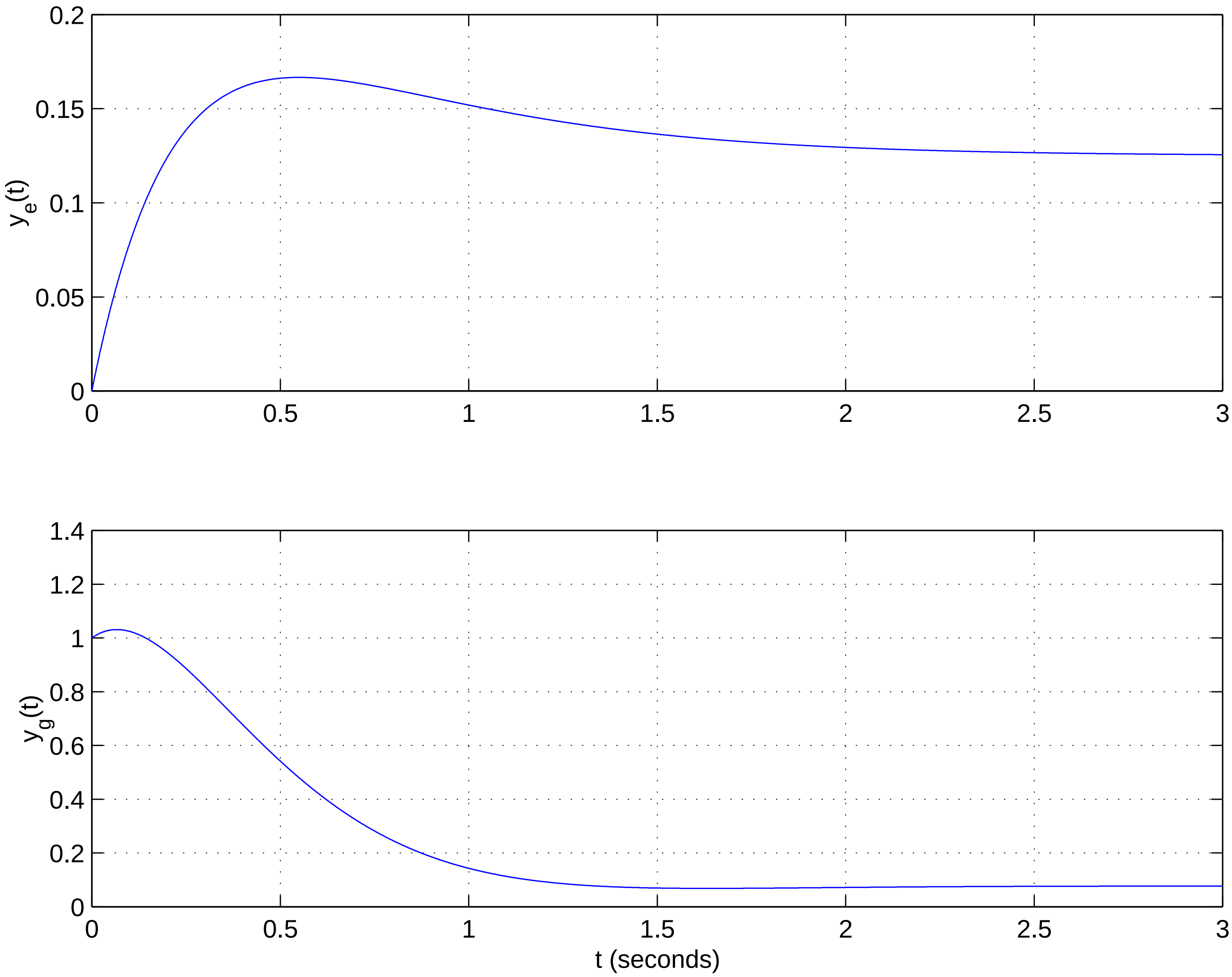
% (g)
b = [1 7 1];a = conv([1 0],[1 6 13]);
[r,p,k]=residue(b,a);
yg = r(3) + 2*abs(r(1))*exp(real(p(1))*t).*cos(imag(p(1))*t + angle(r(1)));
subplot(212)
plot(t,yg)
grid
ylabel('y_g(t)')
xlabel('t (seconds)')
print -dpsc2 'p8_14.ps'

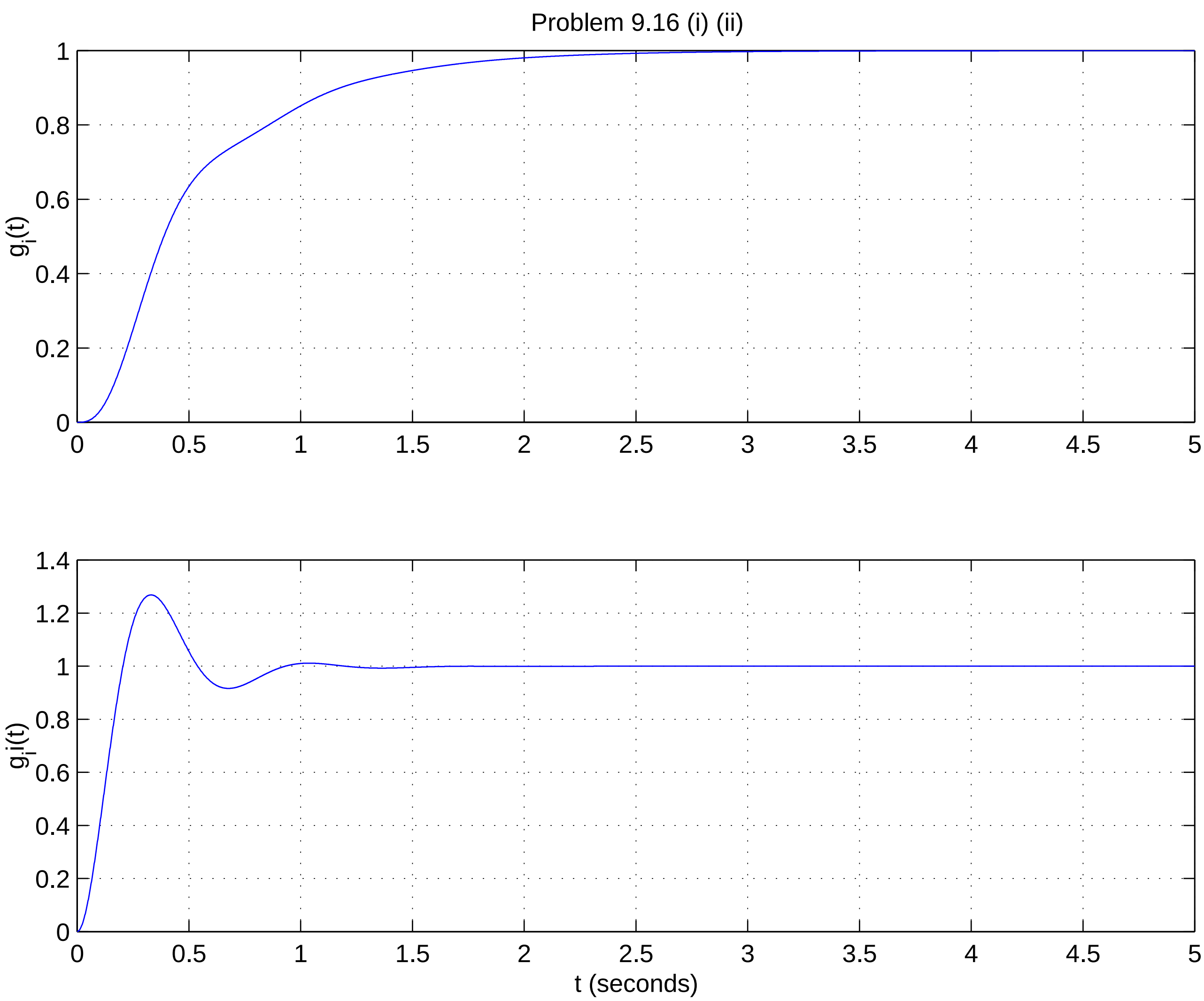
% Problem 9.16
figure(2)
clf

% (i)
b = 242.5*[1 8];
a=conv([1 2],conv([1 8 16+81],[1 10]));
t=0:0.001:5;
gi = step(b,a,t);
subplot(211)
plot(t,gi);
grid
title('Problem 9.16 (i) (ii)')
ylabel('g_i(t)')

% (i)
b = 115.5*conv([1 8],[1 2.1]);
a=conv([1 2],conv([1 8 16+81],[1 10]));
t=0:0.001:5;
gii = step(b,a,t);
subplot(212)
plot(t,gii);
grid
ylabel('g_ii(t)')
xlabel('t (seconds)')
print -dpsc2 'p9_16.ps'
```

Problem 8.14 (e) (g)





Problem 9.16 (i) (ii)
$g_i(t)$
1
0.8
0.6
0.4
0.2
0
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
$g_ii(t)$
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5
t (seconds)