

EE 341 - Homework 13**Due November 30, 2005**

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots.

1. Problem 11.3 (a) (b) (c) (g) (h) (i)
2. Problem 11.4 (a) (c) (e)
3. Problem 11.9 (a) (b) (c) (d) (f). Hint: You can do (a) through (d) with the table of transforms and table of properties – you do not need to do a partial fraction expansion.
4. Problem 11.11 (a) (b) (c) (d) (e)

EE 341

HW # 13

1. Problem 11.3

$$(a) x(n) = \delta(n) + 2\delta(n-1)$$

$$X(z) = 1 + 2z^{-1} = \frac{z+2}{z}$$

$$(b) x(n) = 2u(n) - \delta(n) - \delta(n-1)$$

$$X(z) = \frac{2z}{z-1} - 1 - z^{-1} = \frac{z^2+1}{z(z-1)}$$

$$(c) x(n) = e^{0.5n} u(n) + u(n-2) = (e^{0.5})^n u(n) + u(n-2)$$

$$X(z) = \frac{z}{z-e^{0.5}} + z^{-2} \frac{z}{z-1} = \frac{z^3 - z^2 + z - e^{0.5}}{z(z-1)(z-e^{0.5})}$$

$$(g) x(n) = u(n) - nu(n-1) + \left(\frac{1}{3}\right)^n u(n-2)$$

$$= u(n) - (n-1)u(n-1) - u(n-1) + \frac{1}{9} \left(\frac{1}{3}\right)^{n-2} u(n-2)$$

$$X(z) = \frac{z}{z-1} - z^{-1} \frac{z}{(z-1)^2} - z^{-1} \frac{z}{z-1} + \frac{1}{9} z^{-2} \frac{z}{z-1/3}$$

$$= \frac{z^2(z-1)(z-\frac{1}{3}) - z(z-1/3) - z(z-1)(z-\frac{1}{3}) + \frac{1}{9}(z-1)^2}{z(z-1)^2(z-\frac{1}{3})}$$

$$= \frac{z^4 - 7z^3 + \frac{7}{9}z^2 - \frac{2}{9}z + 1}{z(z-1)^2(z-\frac{1}{3})}$$

$$(h) x(n) = 0\delta(n) + 1\delta(n-1) + 2\delta(n-2) - nu(n-3)$$

$$= \delta(n-1) + 2\delta(n-2) - (n-3)u(n-3) - 3u(n-3)$$

$$X(z) = z^{-1} + 2z^{-2} - z^{-3} \frac{z}{(z-1)^2} - 3z^{-3} \frac{z}{z-1}$$

$$= \frac{z(z-1)^2 + 2(z-1)^2 - 1 - 3(z-1)}{z^2(z-1)^2}$$

$$= \frac{z^3 - 6z + 4}{z^2(z-1)^2}$$

$$(i) x(n) = (n-1)u(n) - n u(n-3)$$

$$= n u(n) - u(n) - (n-3)u(n-3) - 3u(n-3)$$

$$X(z) = \frac{z}{(z-1)^2} - \frac{z}{z-1} - z^{-3} \frac{z}{(z-1)^2} - 3z^{-3} \frac{z}{z-1}$$

$$= \frac{z^3 - z^3(z-1) - 1 - 3(z-1)}{z^2(z-1)^2}$$

$$= \frac{z^3 - z^2 - 2 + 2}{z^2(z-1)^2}$$

2. Problem 11.4

$$(a) x(n) = (\cos^2 \omega n) u(n) = \left[\frac{1}{2} + \frac{1}{2} \cos(2\omega n) \right] u(n)$$

$$X(z) = \frac{\frac{1}{2}z}{z-1} + \frac{1}{2} \frac{z^2 - \cos(2\omega)z}{z^2 - 2\cos(2\omega)z + 1}$$

$$(c) x(n) = n (\cos \omega n) u(n)$$

$$(\cos \omega n) u(n) \leftrightarrow \frac{z^2 - (\cos \omega)z}{z^2 - 2(\cos \omega)z + 1}$$

$$n (\cos \omega n) u(n) \leftrightarrow z \frac{d}{dz} \frac{z^2 - (\cos \omega)z}{z^2 - 2(\cos \omega)z + 1}$$

$$X(z) = z \left[\frac{(z^2 - 2(\cos \omega)z + 1)(2z - (\cos \omega)) - (z^2 - (\cos \omega)z)(2z - 2(\cos \omega))}{(z^2 - 2(\cos \omega)z + 1)^2} \right]$$

$$= \frac{z(z^2(\cos \omega) - 2z + \cos \omega)}{z^2 - 2(\cos \omega)z + 1}$$

$$(e) x(n) = n e^{-bn} (\cos \omega n) u(n)$$

$$(e^{-b})^n (\cos \omega n) u(n) \leftrightarrow \frac{z^2 - (e^{-b} \cos \omega)z}{z^2 - (2e^{-b} \cos \omega)z + e^{-2b}}$$

$$n e^{-bn} (\cos \omega n) u(n) \leftrightarrow z \frac{d}{dz} \left[\right]$$

$$X(z) = z \left[\frac{(z^2 - (2e^{-b} \cos \omega)z + e^{-2b})(2z - e^{-b} \cos \omega) - (z^2 - (e^{-b} \cos \omega)z)(2z - 2e^{-b} \cos \omega)}{(z^2 - (2e^{-b} \cos \omega)z + e^{-2b})^2} \right]$$

$$= \frac{z e^{-b} (\cos \omega z^2 - 2e^{-b} z + e^{-2b} \cos \omega)}{(z^2 - 2e^{-b} \cos \omega z + e^{-2b})^2}$$

3. Problem 11.9

$$(a) X(z) = \frac{z}{z^2 + 1} = \frac{\sin(\frac{\pi}{2})z}{z^2 - (2\cos\frac{\pi}{2})z + 1}$$

$$x(n) = (\sin(\frac{\pi}{2}n))u(n)$$

$$(b) X(z) = \frac{z^2}{z^2 + 1} = \frac{z^2 - (\cos\frac{\pi}{2})z}{z^2 - (2\cos\frac{\pi}{2})z + 1}$$

$$x(n) = (\cos(\frac{\pi}{2}n))u(n)$$

$$(c) X(z) = \frac{1}{z^2 + 1} + \frac{1}{z^2 - 1} = z^{-1} \frac{z}{z^2 + 1} + \frac{1}{(z+1)(z-1)}$$

$$\frac{X(z)}{z} = \frac{1}{z(z+j)(z-j)} + \frac{1}{z(z+1)(z-1)}$$

$$= \frac{1}{z} + \frac{-1/2}{z-j} + \frac{-1/2}{z+j} + \frac{-1}{z} + \frac{1/2}{z-1} + \frac{1/2}{z+1}$$

$$X(z) = -\frac{1}{2} \frac{z}{z-j} - \frac{1}{2} \frac{z}{z+j} + \frac{1}{2} \frac{z}{z-1} + \frac{1}{2} \frac{z}{z+1}$$

$$x(n) = 2|-\frac{1}{2}| \cos(\frac{\pi}{2}n + \angle(-\frac{1}{2})) + \frac{1}{2}u(n) + \frac{1}{2}(-1)^n u(n)$$

$$= -\cos(\frac{\pi}{2}n) + \frac{1}{2}u(n) + \frac{1}{2}(-1)^n u(n)$$

$$(d) X(z) = \frac{z^2}{z^2 + 1} + \frac{z}{z^2 - 1} \quad H_1(z) = \frac{1}{z} = \frac{1}{(z+1)(z-1)} = \frac{-1/2}{z-1} + \frac{-1/2}{z+1}$$

Same as (b)

$$x(n) = (\cos(\frac{\pi}{2}n))u(n) + \frac{1}{2}u(n) - \frac{1}{2}(-1)^n u(n)$$

$$(e) X(z) = \frac{z^2 - 1}{z^2 + 1} \quad \frac{X(z)}{z} = \frac{z^2 - 1}{z(z^2 + 1)} = \frac{-1}{z} + \frac{1}{z-j} + \frac{1}{z+j}$$

$$x(n) = -\delta(n) + 2 \cos(\frac{\pi}{2}n) u(n)$$

$$= -\delta(n) + 2 \cos(\frac{\pi}{2}n) u(n)$$

4. Problem 11.11

(4)

$$(a) X(z) = \frac{z+0.3}{z^2+0.75z+0.125}$$

$$\frac{X(z)}{z} = \frac{z+0.3}{z(z+0.5)(z+0.25)} = \frac{-1.6}{z+0.5} + \frac{-0.8}{z+0.25} + \frac{2.4}{z}$$

$$X(z) = -1.6 \frac{z}{z+0.5} - 0.8 \frac{z}{z+0.25} + 2.4$$

$$x(n) = (-1.6(0.5)^n - 0.8(0.25)^n)u(n) + 2.4\delta(n)$$

$$(b) X(z) = \frac{5z+1}{4z^2+4z+1}$$

$$\frac{X(z)}{z} = \frac{\frac{5}{4}z + \frac{1}{4}}{z(z+0.5)^2} = \frac{-1}{z+0.5} + \frac{0.75}{(z+0.5)^2} + \frac{1}{z}$$

$$X(z) = -1 \frac{z}{z+0.5} + 1.5 \frac{0.5z}{(z+0.5)^2} + 1$$

$$x(n) = -\left(-\frac{1}{2}\right)^n u(n) + \frac{3}{2} n \left(-\frac{1}{2}\right)^{n-1} u(n) + \delta(n)$$

$$(c) X(z) = \frac{4z+1}{z^2-z+0.5}$$

$$\frac{X(z)}{z} = \frac{4z+1}{z(z-0.5-j0.5)(z-0.5+j0.5)} = \frac{-1-j5}{z-0.5-j0.5} + \frac{-1+j5}{z-0.5+j0.5} + \frac{2}{z}$$

$$X(z) = (-1-j5) \frac{z}{z-0.5-j0.5} + (-1+j5) \frac{z}{z-0.5+j0.5} + 2$$

$$x(n) = 2|-1-j5| |0.5+j0.5|^n \cos(\angle(0.5+j0.5)n + \angle(-1-j5)) u(n) + 2\delta(n)$$

$$= 2\sqrt{26} \left(\frac{\sqrt{2}}{2}\right)^n \cos\left(\frac{\pi}{4}n - 1.77\right) u(n) + 2\delta(n)$$

(d) $X(z) = \frac{z}{16z^2 + 1}$

$$\frac{X(z)}{z} = \frac{\frac{1}{4} (\sin \frac{\pi}{2}) z}{z^2 - (\frac{1}{4} \cos \frac{\pi}{2}) z + (\frac{1}{4})^2}$$

$$x[n] = \frac{1}{4} \left(\frac{1}{4}\right)^n \sin\left(\frac{\pi}{2} n\right) u[n]$$

(e) $X(z) = \frac{2z+1}{z(10z^2 - z - 2)}$

$$\frac{X(z)}{z} = \frac{\frac{1}{5}z + \frac{1}{10}}{z^2(z^2 - \frac{1}{10}z - \frac{1}{5})} = \frac{\frac{1}{5}z + \frac{1}{10}}{z^2(z - \frac{1}{2})(z + \frac{2}{5})} = \frac{8/9}{z - \frac{1}{2}} + \frac{-5/36}{z + \frac{2}{5}} + \frac{-3/4}{z} + \frac{-1/2}{z^2}$$

$$X(z) = \frac{8}{9} \frac{z}{z - \frac{1}{2}} - \frac{5}{36} \frac{z}{z + \frac{2}{5}} - \frac{3}{4} - \frac{1}{2} z^{-1}$$

$$x[n] = \frac{8}{9} \left(\frac{1}{2}\right)^n u[n] - \frac{5}{36} \left(-\frac{2}{5}\right)^n u[n] - \frac{3}{4} \delta[n] - \frac{1}{2} \delta[n-1]$$