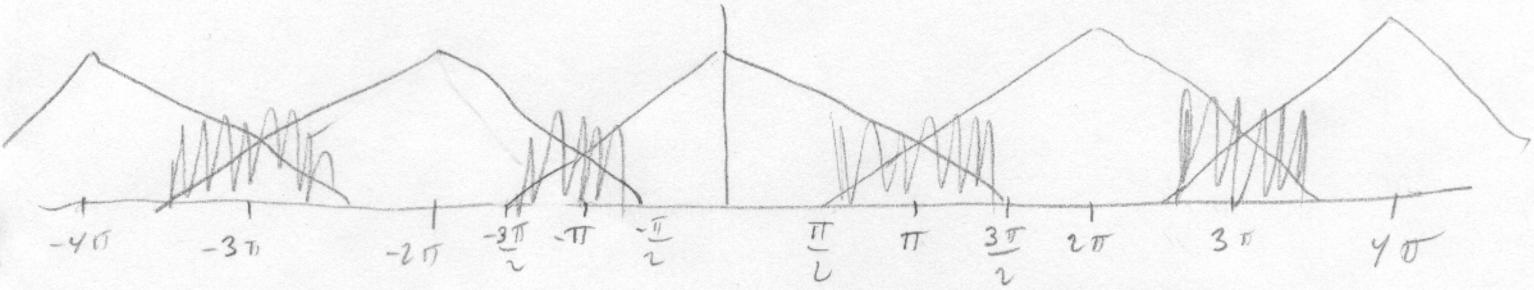
EE 342
HW #2

1. $f_s = 400\,Hz \qquad \Omega = 2\pi \frac{f}{f_s} = 2\pi \frac{300\,Hz}{400\,Hz} = \frac{3\pi}{2}$



Frequencies from $\frac{\pi}{2}$ to π are corrupted by aliasing

2. Problem 8.5

(a) $X(s) = \frac{4}{s^2+s} = \frac{4}{s(s+1)}$ Poles: 0, -1 $\lim_{t\to\infty} x(t)$ exists

$\lim_{x\to\infty} x(t) = \frac{b_0}{a_1} = \frac{4}{1} = 4$

(b) $X(s) = \frac{3s+4}{s^2+s} = \frac{3s+4}{s(s+1)}$ Poles: 0, -1 $\lim_{t\to\infty} x(t)$ exists

$\lim_{x\to\infty} x(t) = \frac{b_0}{a_1} = \frac{4}{1} = 4$

(c) $X(s) = \frac{4}{s^2-s} = \frac{4}{s(s-1)}$ Poles: 0, +1 $\lim_{t\to\infty} x(t)$ does not exist

(d) $X(s) = \frac{3s^2+4s+1}{s^3+2s^2+s+2} = \frac{3s^2+4s+1}{(s+j)(s-j)(s+2)}$ Poles: $j, -j, -2$

$\lim_{t\to\infty} x(t)$ does not exist

(e) $X(s) = \frac{3s^2+4s+1}{s^3+3s^2+3s+2} = \frac{3s^2+4s+1}{(s+2)(s+0.5+j0.866)(s+0.5-j0.866)}$

Poles: $-2, -0.5+j0.866, -0.5-j0.866$ $\lim_{t\to\infty} x(t)$ exists

$\lim_{t\to\infty} x(t) = 0$ because $a_0 \neq 0$

(f) $X(s) = \frac{3s^2+4s+1}{s^4+3s^3+3s^2+2s} = \frac{3s^2+4s+1}{s(s+2)(s+0.5+j0.866)(s+0.5-j0.866)}$

Poles: $0, -2, -0.5+j0.866, -0.5-j0.866$ $\lim_{t\to\infty} x(t)$ exists

$$\lim_{t\to\infty} x(t) = \frac{b_0}{a_1} = \frac{1}{2}$$

3. Problem 8.6

(a) $N=2, M=0$, $m=N-2 \Rightarrow x(0)=0$

(b) $N=2, m=1$, $m=N-1 \Rightarrow x(0) = \frac{b_M}{a_N} = \frac{3}{1} = 3$

(c) $N=2, m=0$, $m=N-1 \Rightarrow x(0)=0$

(d) $N=3$ $M=2$, $M=N-1 \Rightarrow x(0) = \frac{b_M}{a_N} = \frac{3}{1} = 3$

(e) $N=3$, $M=2$ $M=N-1 \Rightarrow x(0) = \frac{b_M}{b_N} = \frac{3}{1} = 3$

(f) $N=4$, $M=2$, $M=N-2 \Rightarrow x(0)=0$

4. Problem 8.8

(a) $x(t) = e^{-t}u(t)$ $X(s) = \frac{1}{s+1}$

$v(t) = \sin(t)\,u(t)$ $V(s) = \frac{1}{s^2+1}$

$$Y(s) = X(s)V(s) = \frac{1}{(s+1)(s^2+1)} = \frac{1}{(s+1)(s+j)(s-j)}$$

$$= \frac{1}{s+1} + \frac{(1+j)/4}{s+j} + \frac{(1-j)/4}{s-j}$$

$$x(t) = e^{-t}u(t) + 2|r_1|\cos(t+\measuredangle r_1)\,u(t)$$

$$= e^{-t}u(t) + \frac{\sqrt{2}}{2}\cos(t+45^\circ)$$

(b) $x(t) = \cos t \, u(t)$ $\quad X(s) = \dfrac{s}{(s^2+1)}$

$v(t) = \sin t \, u(t)$ $\quad V(s) = \dfrac{1}{(s^2+1)}$

$$Y(s) = X(s)V(s) = \frac{s}{(s^2+1)^2} = \frac{s}{(s+j)^2(s-j)^2}$$

$$= \frac{1}{4}\left(\frac{-j}{s-j} + \frac{j}{s+j} - \frac{1}{(s-j)^2} - \frac{1}{(s+j)^2} \right)$$

$$y(t) = \frac{1}{4}\left(-je^{jt} + je^{-jt} - te^{jt} - te^{-jt}\right)u(t)$$

$$= \frac{1}{4}\left[-j(e^{jt} - e^{-jt}) - t(e^{jt} + e^{-jt})\right]u(t)$$

$$= \frac{1}{4}\left[-j(2j \sin t) - t(2\cos t)\right]u(t)$$

$$= \left(\frac{1}{2}\sin t - \frac{1}{2}t\cos t\right)u(t)$$

(d) $x(t) = \sin^2 t \, u(t)$ $\quad X(s) = \dfrac{2}{s(s^2+4)}$

$v(t) = t\,u(t)$ $\quad X(s) = \dfrac{1}{s^2}$

$$Y(s) = X(s)V(s) = \frac{2}{s^3(s^2+4)} = \frac{2}{s^3(s+j2)(s-j2)}$$

$$= \frac{-\frac{1}{8}}{s} + \frac{\frac{1}{2}}{s^3} + \frac{1/16}{s-j2} + \frac{1/16}{s+j2}$$

$$y(t) = -\frac{1}{8}u(t) + \frac{1}{4}t^2u(t) + \left(\frac{1}{16}e^{j2t} + \frac{1}{16}e^{-j2t}\right)u(t)$$

$$= -\frac{1}{8}u(t) + \frac{1}{4}t^2u(t) + \frac{1}{8}\cos 2t \, u(t)$$

5. Problem 8.10

(a) $X(s) = \dfrac{s+2}{s^2+7s+12} = \dfrac{s+2}{(s+3)(s+4)} = \dfrac{-1}{s+3} + \dfrac{2}{s+4}$

$x(t) = -e^{-3t}u(t) + 2e^{-4t}u(t)$

See MATLAB for plots

(b) $X(s) = \dfrac{s+1}{s^3+5s^2+7s} = \dfrac{s+1}{s(s+2.5+j\sqrt{3}/2)(s+2.5-j\sqrt{3}/2)}$

$= \dfrac{1/7}{s} + \dfrac{-1/14 + j\frac{3}{14}\sqrt{3}}{s+2.5+j\sqrt{3}/2} + \dfrac{-\frac{1}{14} - j\frac{3}{14}\sqrt{3}}{s+2.5-j\sqrt{3}/2}$

$= \dfrac{1/7}{s} + \dfrac{r_1}{s-p_1} + \dfrac{r_1^*}{s-p_1^*}$

$x(t) = \frac{1}{7}u(t) + 2|r_1| e^{\mathrm{Re}(p_1)} \cos(\mathrm{Im}(p_1)t + \angle r_1)\, u(t)$

$= \frac{1}{7} + \frac{2}{7}\sqrt{7}\, e^{-2.5t} \cos\left(\frac{\sqrt{3}}{2}t - 101^\circ\right) u(t)$

(c) $X(s) = \dfrac{2s^2-9s-35}{s^2+4s+2} = \dfrac{2(s^2+4s+2)-17s-43}{s^2+4s+4} = 2 + \dfrac{-17s-39}{(s+2+\sqrt{2})(s+2-\sqrt{2})}$

$= 2 + \dfrac{-17/2 + \frac{5}{4}\sqrt{2}}{s+2+\sqrt{2}} + \dfrac{-\frac{17}{2} - \frac{5}{4}\sqrt{2}}{s+2-\sqrt{2}}$

$x(t) = 2\delta(t) + \left(-\frac{17}{2} + \frac{5}{4}\sqrt{2}\right) e^{-(2+\sqrt{2})t} u(t) + \left(-\frac{17}{2} - \frac{5}{4}\sqrt{2}\, e^{-(2-\sqrt{2})t}\right) u(t)$

(d) $X(s) = \dfrac{3s^2+2s+1}{s^3+5s^2+8s+4} = \dfrac{3s^2+2s+1}{(s+1)(s+2)^2}$

$= \dfrac{r_1}{s+2} + \dfrac{r_2}{(s+2)^2} + \dfrac{r_3}{s+1}$

$r_1 = \dfrac{d}{ds}\left[(s+2)^2 X(s)\right]\Big|_{s=-2} = \dfrac{d}{ds}\left(\dfrac{3s^2+2s+1}{s+1}\right)\Big|_{s=-2} = \dfrac{3s^2+6s+1}{(s+1)^2}\Big|_{s=-2}$

$= 1$

$r_2 = \dfrac{3s^2+2s+1}{s+1}\Big|_{s=-2} = -9$

$r_3 = \dfrac{3s^2+2s+1}{(s+2)^2}\Big|_{s=-1} = 2$

$$X(s) = \frac{1}{s+2} - \frac{9}{(s+2)^2} + \frac{2}{s+1}$$

$$x(t) = e^{-2t}u(t) - 9te^{-2t}u(t) + 2e^{-t}u(t)$$

(e) $$X(s) = \frac{s^2+1}{s^5+18s^3+81s} = \frac{s^2+1}{s(s+j3)^2(s-j3)^2}$$

$$= \frac{r_1}{s+j3} + \frac{r_2}{(s+j3)^2} + \frac{r_3}{s-j3} + \frac{r_4}{(s-j3)^2} + \frac{r_5}{s}$$

$$= \frac{-1/162}{s+j3} + \frac{j\frac{2}{27}}{(s+j3)^2} + \frac{-1/162}{s-j3} + \frac{-j\frac{2}{27}}{(s-j3)^2} + \frac{1/81}{s}$$

$$x(t) = \left[r_1 e^{j3t} + r_3 e^{-j3t} + r_2 t e^{j3t} + r_4 t e^{-j3t} + r_5\right]u(t)$$

$$= -\frac{1}{81}\cos(3t)u(t) + \frac{4}{27}t\sin(3t)u(t) + \frac{1}{81}u(t)$$

(f) $$X(s) = \frac{s+e^{-s}}{s^2+s+1} = \frac{s}{(s+\frac{1}{2}+j\frac{\sqrt{3}}{2})(s+\frac{1}{2}-j\frac{\sqrt{3}}{2})} + \frac{e^{-s}}{(s+\frac{1}{2}+j\frac{\sqrt{3}}{2})(s+\frac{1}{2}-j\frac{\sqrt{3}}{2})}$$

$$= \frac{\frac{1}{2}-j/\sqrt{12}}{s+\frac{1}{2}+j\frac{\sqrt{3}}{2}} + \frac{\frac{1}{2}+j/\sqrt{12}}{s+\frac{1}{2}-j\frac{\sqrt{3}}{2}} + e^{-s}\left(\frac{j/\sqrt{3}}{s+\frac{1}{2}+j\frac{\sqrt{3}}{2}} - \frac{j/\sqrt{3}}{s+\frac{1}{2}-j\sqrt{3}/2}\right)$$

$$= 2\frac{\sqrt{3}}{3}e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t - \frac{\pi}{6}\right)u(t) + 2\frac{\sqrt{3}}{3}e^{-\frac{1}{2}(t-1)}\sin\left(\frac{\sqrt{3}}{2}(t-1)\right)$$

(g) $$X(s) = \frac{s}{s+1} + \frac{s\,e^{-s}}{s^2+2s+1} + \frac{e^{-2s}}{s^2+2s+1}$$

$$= \frac{s}{s+1} + \frac{s\,e^{-s}}{(s+1)^2} + \frac{e^{-2s}}{(s+1)^2}$$

$$= 1 - \frac{1}{s+1} + \left(\frac{1}{s+1} - \frac{1}{(s+1)^2}\right)e^{-s} + \frac{1}{(s+1)^2}e^{-2s}$$

$$x(t) = \delta(t) - e^{-t}u(t) + \left[e^{-(t-1)} - (t-1)e^{-(t-1)}\right]u(t-1) + (t-2)e^{-(t-2)}u(t-2)$$

$$= \delta(t) - e^{-t}u(t) + (t-2)e^{-(t-1)}u(t-1) + (t-2)e^{-(t-2)}u(t-2)$$

6. Problem 8.11 See MATLAB

7. Problem 8.12

(a) $X(s) = \dfrac{s^2-2s+1}{s(s^2+4)} = \dfrac{\frac{3}{8}+j\frac{1}{2}}{s-j2} + \dfrac{\frac{3}{8}-j\frac{1}{2}}{s+j2} + \dfrac{1/4}{s}$

$x(t) = \frac{5}{4}\cos(2t+53^\circ)u(t) + \frac{1}{4}u(t)$

See MATLAB for plots

(b) $X(s) = \dfrac{s^2-2s+1}{s(s^2+4)^2} = \dfrac{s^2-2s+1}{s(s-j2)^2(s+j2)^2}$

$= \dfrac{-0.0312+j0.0625}{s-j2} + \dfrac{-0.0312-j0.0625}{s+j2}$

$+ \dfrac{0.125-j0.0938}{(s-j2)^2} + \dfrac{0.125+j0.0938}{(s+j2)^2} + \dfrac{0.0625}{s}$

$x(t) = [0.1398\cos(2t+117^\circ) + 0.3125\, t\cos(2t-37^\circ) + 0.0625]u(t)$

(c) $X(s) = \dfrac{s^2-2s+1}{s^2(s^2+4)} = \dfrac{s^2+2s+1}{s^2(s-j2)(s+j2)}$

$= \dfrac{0.25-j0.1875}{s-j2} + \dfrac{0.25+j0.1875}{s+j2} - \dfrac{0.5}{s} + \dfrac{0.25}{s^2}$

$x(t) = 0.625\cos(2t-37^\circ)u(t) - 0.5u(t) + 0.25t\,u(t)$

(d) $X(s) = \dfrac{s^2-2s+1}{s^2(s^2+4)^2} = \dfrac{s^2-2s+1}{s^2(s-j2)^2(s+j2)^2}$

$= \dfrac{0.0625-j0.0078}{s-j2} + \dfrac{0.0625+j0.0078}{s+j2}$

$+ \dfrac{-0.0469-j0.0625}{(s-j2)^2} + \dfrac{-0.0469+j0.0625}{(s+j2)^2} - \dfrac{0.125}{s} + \dfrac{0.0625}{s^2}$

$x(t) = 0.126\cos(2t-7^\circ)u(t) + 0.1562\, t\cos(2t-127^\circ) - 0.125u(t) + 0.0625t\,u(t)$

(e) $X(s) = \dfrac{s^2 - 2s + 1}{(s+2)^2 + 4} = \dfrac{s^2 + 4s + 8 - 6s - 7}{s^2 + 4s + 8} = 1 + \dfrac{-6s - 7}{(s+2-j2)(s+2+j2)}$

$= 1 + \dfrac{-3 - j\frac{5}{4}}{s+2-j2} + \dfrac{-3 + j\frac{5}{4}}{s+2+j2}$

$x(t) = \delta(t) + 6.5e^{-2t}\cos(2t - 157^\circ)$

8. Problem 8.13

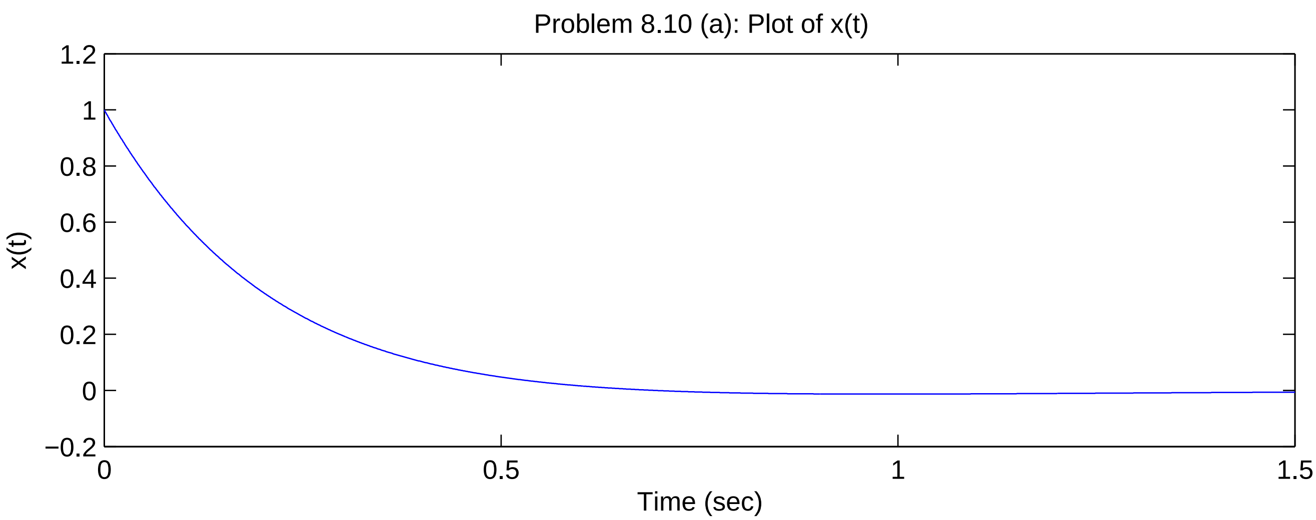
See MATLAB

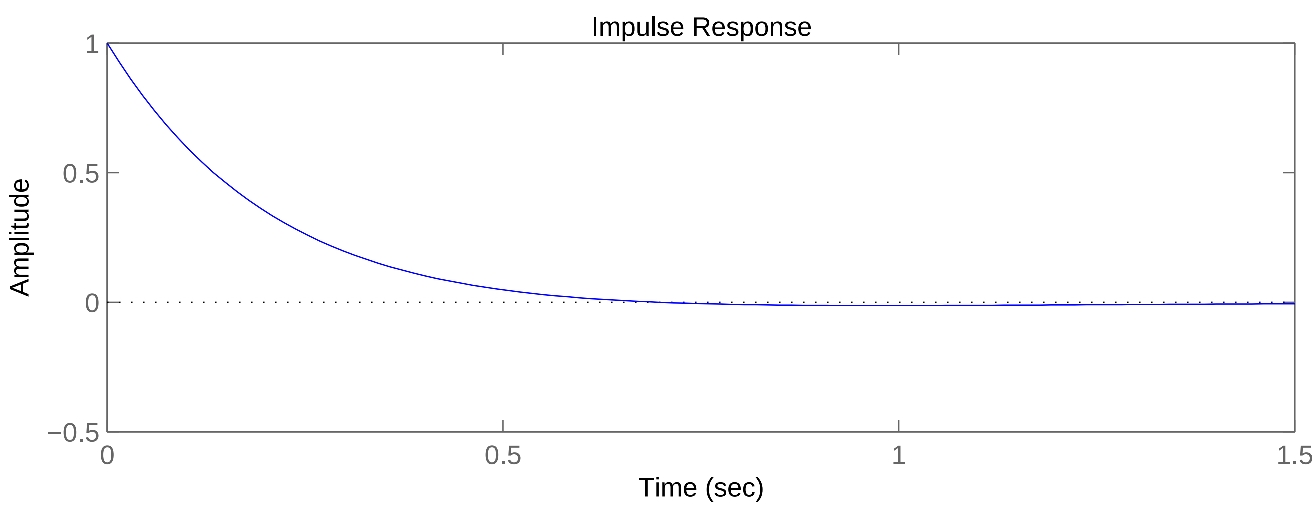
```
clear

% Problem 8.10 and 8.11

% (a)
figure(1)
subplot(211)
t = 0:0.001:1.5;
x = -exp(-3*t)+2*exp(-4*t);
plot(t,x)
title('Problem 8.10 (a): Plot of x(t)')
xlabel('Time (sec)')
ylabel('x(t)')
subplot(212)
b = [1 2];
a = [1 7 12];
impulse(b,a)
print -depsc2 p8_10_a.eps

% (b)
figure(2)
subplot(211)
t = 0:0.001:4;
x = 1/7 + 2*sqrt(7)/7*exp(-2.5*t).*cos(sqrt(3)*t/2-101*pi/180);
plot(t,x)
axis([0 4 0 0.2])
title('Problem 8.10 (b): Plot of x(t)')
xlabel('Time (sec)')
ylabel('x(t)')
subplot(212)
b = [1 1];
a = [1 5 7 0];
impulse(b,a)
print -depsc2 p8_10_b.eps


% (c)
figure(3)
subplot(211)
t = 0:0.001:10;
x = (-17/2+5*sqrt(2)/4)*exp(-(2+sqrt(2))*t) + ...
    (-17/2-5*sqrt(2)/4)*exp(-(2-sqrt(2))*t);
plot(t,x)
axis([0 10 -20 0])
title('Problem 8.10 (c): Plot of x(t)')
xlabel('Time (sec)')
ylabel('x(t)')
text(1,-15,'Figure does not show 2 \delta(t)');
subplot(212)
b = [2 -9 -35];
a = [1 4 2];
impulse(b,a)
title('Output of MATLAB impulse          function  ')
print -depsc2 p8_10_c.eps


% (e)
figure(4)
subplot(211)
t = 0:0.001:20;
x = (1/81)*cos(3*t) + (4/27)*t.*sin(3*t) + (1/81);
plot(t,x)
axis([0 20 -4 4])
title('Problem 8.10 (d): Plot of x(t)')
xlabel('Time (sec)')
ylabel('x(t)')
subplot(212)
b = [1 0 1];
a = [1 0 18 0 81 0];
```

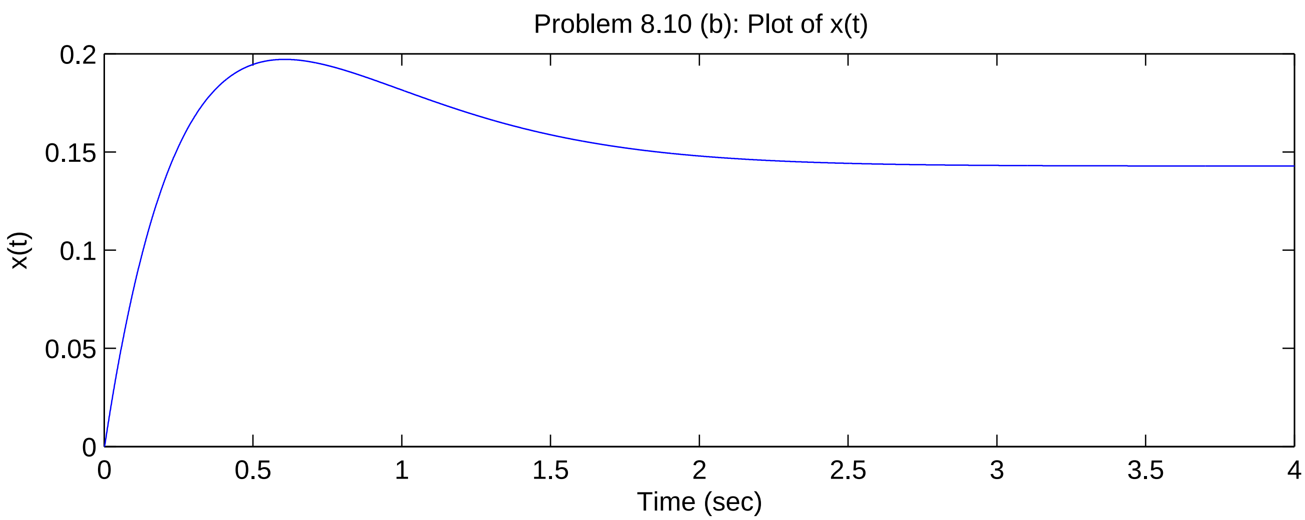


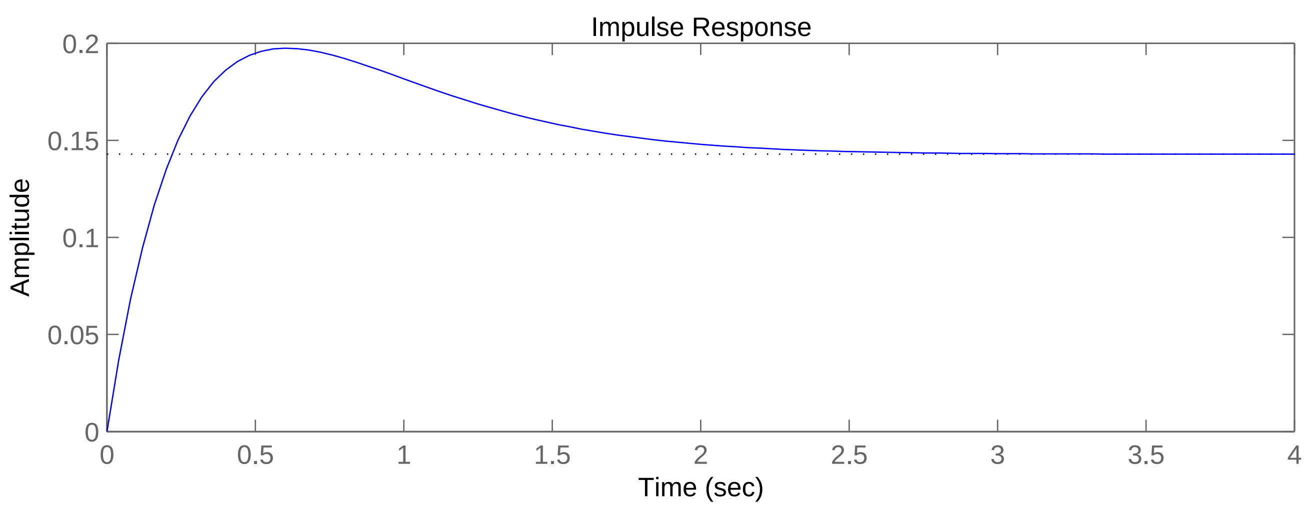
```
impulse(b,a)
title('Output of MATLAB impulse          function  ')
print -depsc2 p8_10_d.eps

% (g)
figure(5)
subplot(211)
t = 0:0.001:10;
x = -exp(-t) + (t-2).*exp(-(t-1)).*(t >= 1) + ...
    (t-2).*exp(-(t-2)).*(t >= 2);
plot(t,x);
title('Problem 8.10 (g): Plot of x(t)')
xlabel('Time (sec)')
ylabel('x(t)')
print -depsc2 p8_10_g.eps

% Problems 8.12 and 8.13
% (a)
figure(6)
subplot(211)
t = 0:0.001:25;
x = 1.25*cos(2*t + 53*pi/180) + 0.25;
plot(t,x)
axis([0 25 -1 2])
title('Problem 8.12 (a): Plot of x(t)')
xlabel('Time (sec)')
ylabel('x(t)')
subplot(212)
b = [1 -2 1];
a = conv([1 0],[1 0 4]);
impulse(b,a)
title('Output of MATLAB impulse          function  ')
print -depsc2 p8_12_a.eps

% (b)
figure(7)
t = 0:0.001:25;
x = 0.1298*cos(2*t+117*pi/180)+ 0.3125*t.*cos(2*t-37*pi/180) + 0.0625;
subplot(211)
plot(t,x)
axis([0 25 -10 10])
title('Problem 8.12 (b): Plot of x(t)')
xlabel('Time (sec)')
ylabel('x(t)')
subplot(212)
b = [1 -2 1];
a = conv([1,0],conv([1 0 4],[1 0 4]));
impulse(b,a)
print -depsc2 p8_12_b.eps

% (c)
figure(8)
t = 0:0.001:25;
x = 0.625*cos(2*t-37*pi/180) - 0.5 + 0.25*t;
subplot(211)
plot(t,x)
axis([0 25 -5 10])
title('Problem 8.12 (b): Plot of x(t)')
xlabel('Time (sec)')
ylabel('x(t)')
subplot(212)
b = [1 -2 1];
a = conv([1 0 0],[1 0 4]);
impulse(b,a)
print -depsc2 p8_12_c.eps

% (d)
figure(9)
t = 0:0.001:25;
```

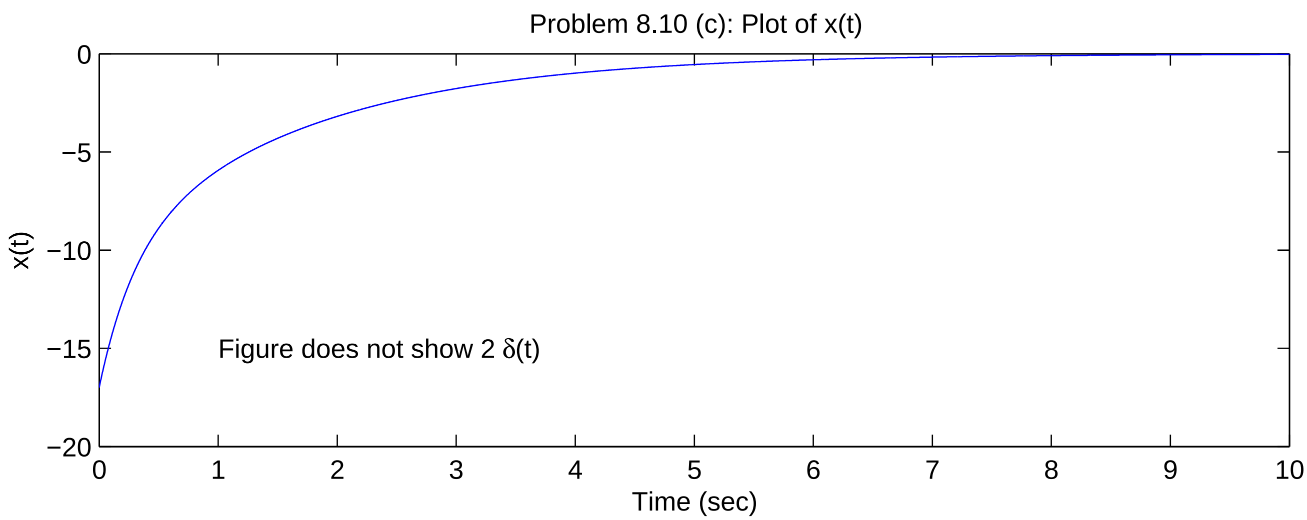
```
x = 0.126*cos(2*t−7*pi/180) + 0.1562*t.*cos(2*t−127*pi/180) − ...
    0.125 + 0.0625*t;
subplot(211)
plot(t,x)
axis([0 25 −5 5])
title('Problem 8.10 (d): Plot of x(t)')
xlabel('Time (sec)')
ylabel('x(t)')
subplot(212)
b = [1 −2 1];
a = conv([1 0 0],conv([1 0 4],[1 0 4]));
impulse(b,a)
print −depsc2 p8_12_d.eps


% (e)
figure(10)
t = 0:0.001:25;
x = 6.5*exp(−2*t).*cos(2*t−157*pi/180);
subplot(211)
plot(t,x)
axis([0 3 −6 2])
title('Problem 8.10 (e): Plot of x(t)')
xlabel('Time (sec)')
ylabel('x(t)')
text(0.5,−4,'Figure does not show \delta(t)')
subplot(212)
b = [1 −2 1];
a = conv([1 2],[1 2]) + [0 0 4];
impulse(b,a)
print −depsc2 p8_12_e.eps
```

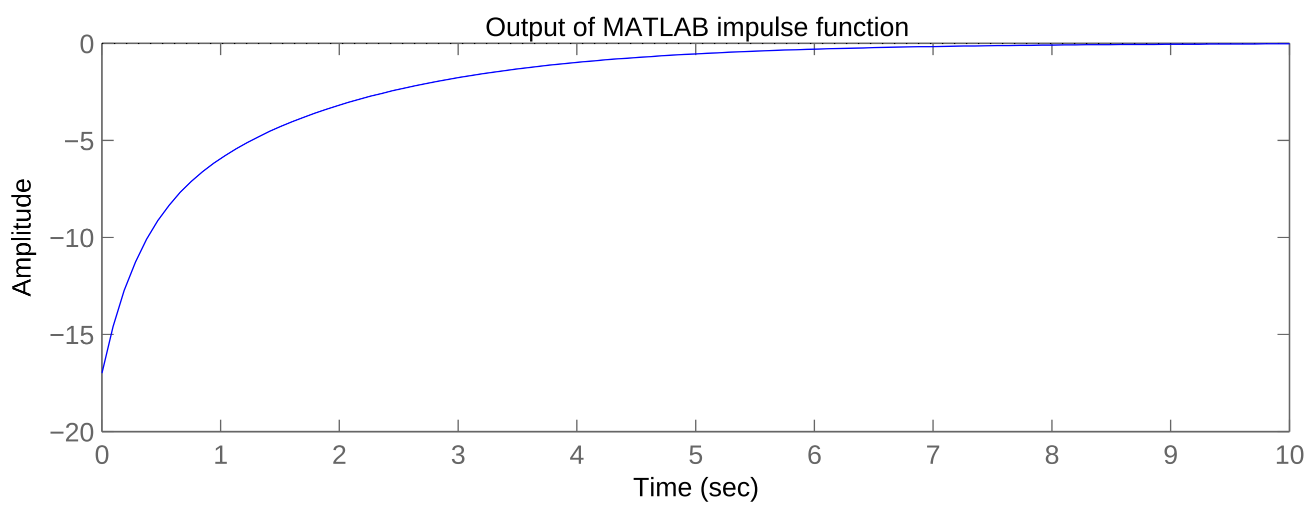

Problem 8.10 (a): Plot of x(t)
x(t)
1.2
1
0.8
0.6
0.4
0.2
0
−0.2
0
0.5
1
1.5
Time (sec)

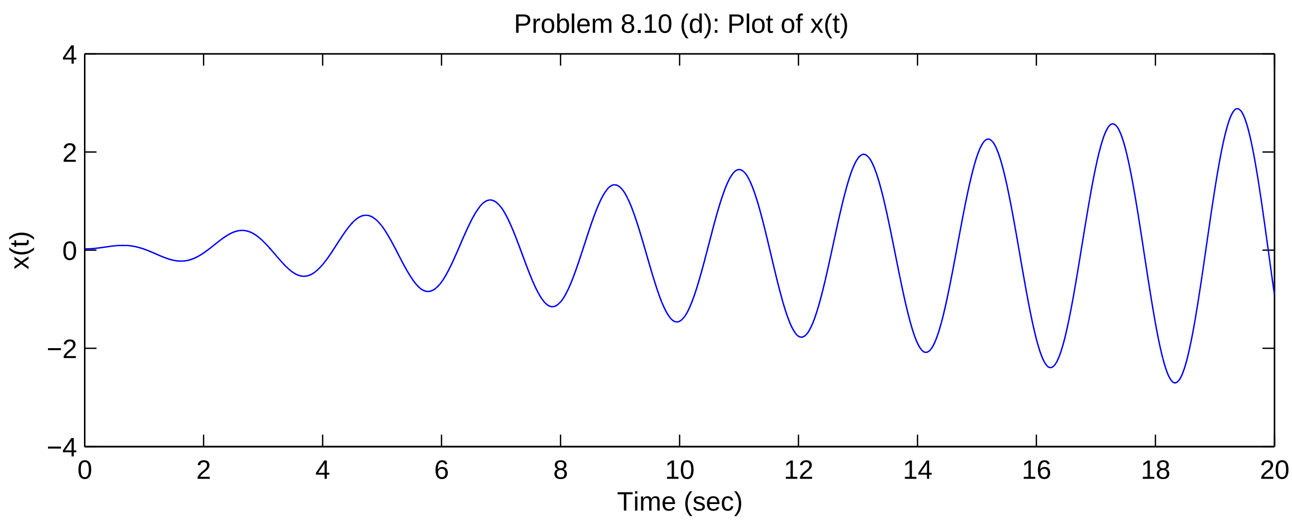

Impulse Response
Amplitude
1
0.5
0
−0.5
0
0.5
1
1.5
Time (sec)

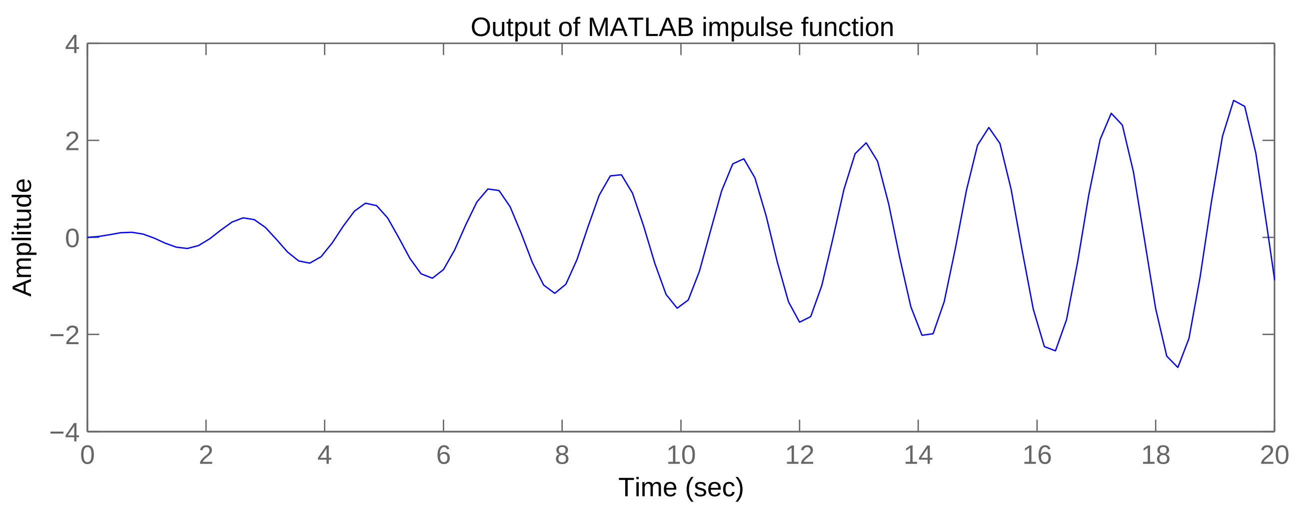

Problem 8.10 (b): Plot of x(t)
x(t)
Time (sec)
0
0.05
0.1
0.15
0.2
0.5
1
1.5
2
2.5
3
3.5
4

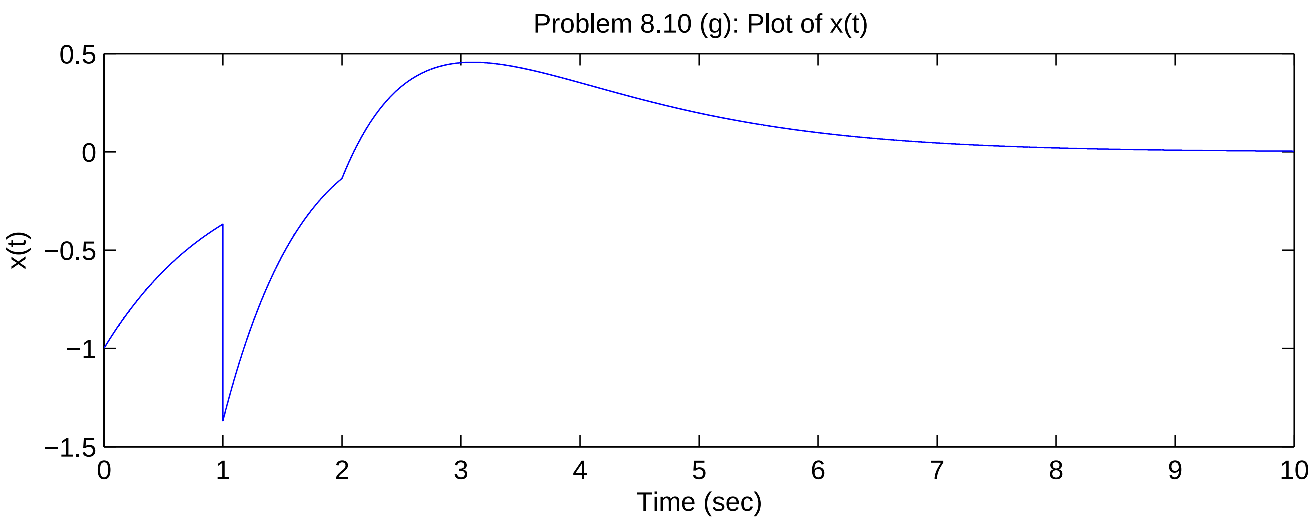

Impulse Response
Amplitude
Time (sec)
0
0.05
0.1
0.15
0.2
0.5
1
1.5
2
2.5
3
3.5
4

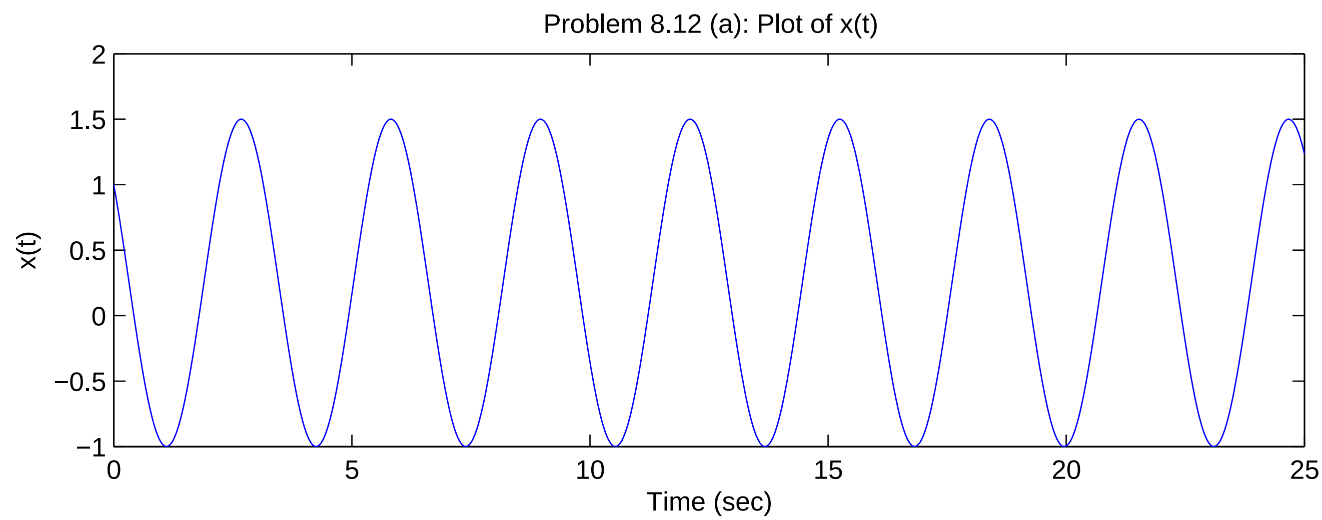

Problem 8.10 (c): Plot of x(t)
x(t)
Figure does not show 2 δ(t)
Time (sec)
0
−5
−10
−15
−20
0
1
2
3
4
5
6
7
8
9
10

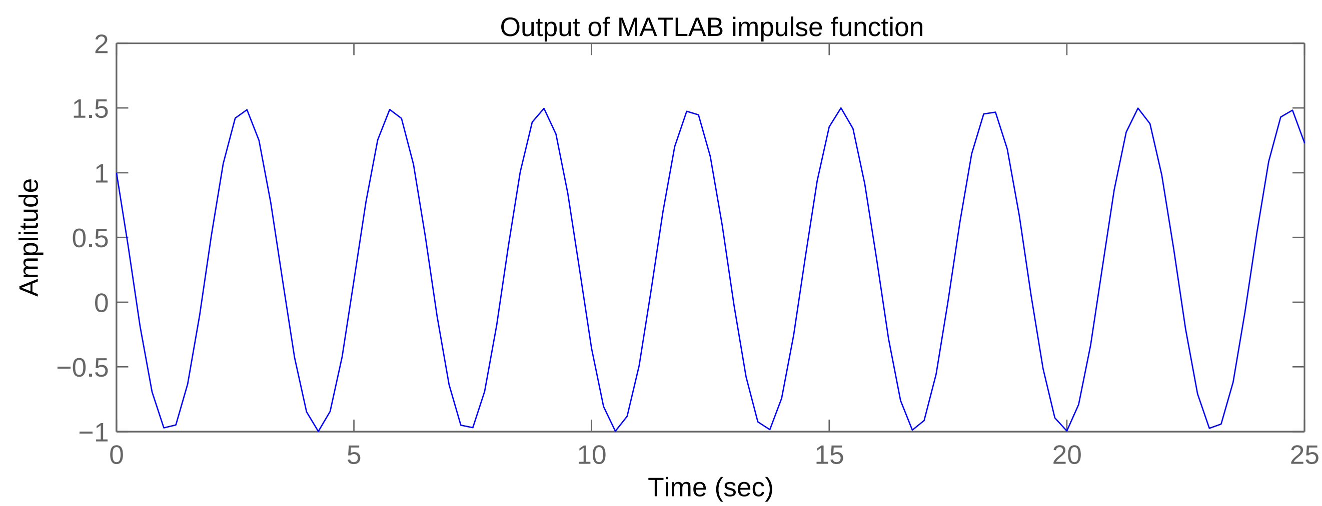

Output of MATLAB impulse function
Amplitude
Time (sec)
0
−5
−10
−15
−20
0
1
2
3
4
5
6
7
8
9
10

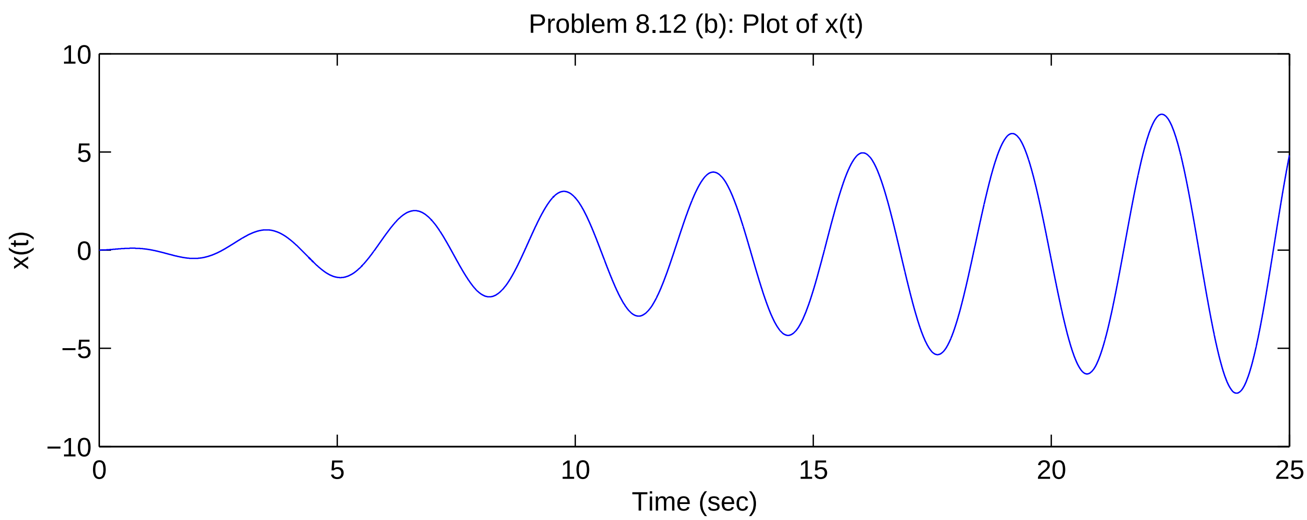

Problem 8.10 (d): Plot of x(t)
x(t)
Time (sec)
4
2
0
−2
−4
0
2
4
6
8
10
12
14
16
18
20

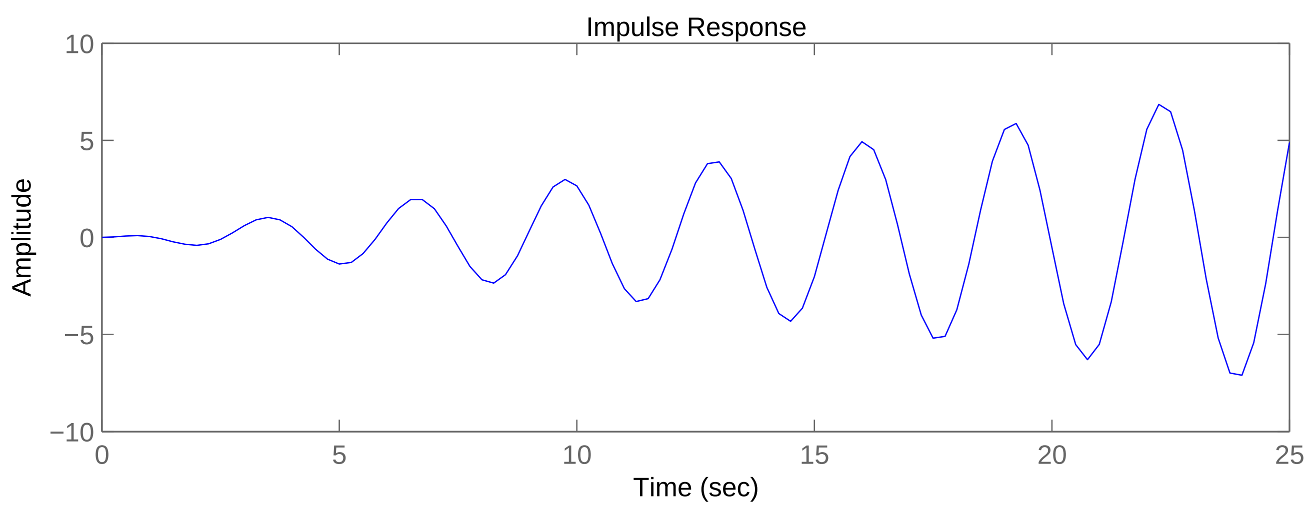

Output of MATLAB impulse function
Amplitude
Time (sec)
4
2
0
−2
−4
0
2
4
6
8
10
12
14
16
18
20

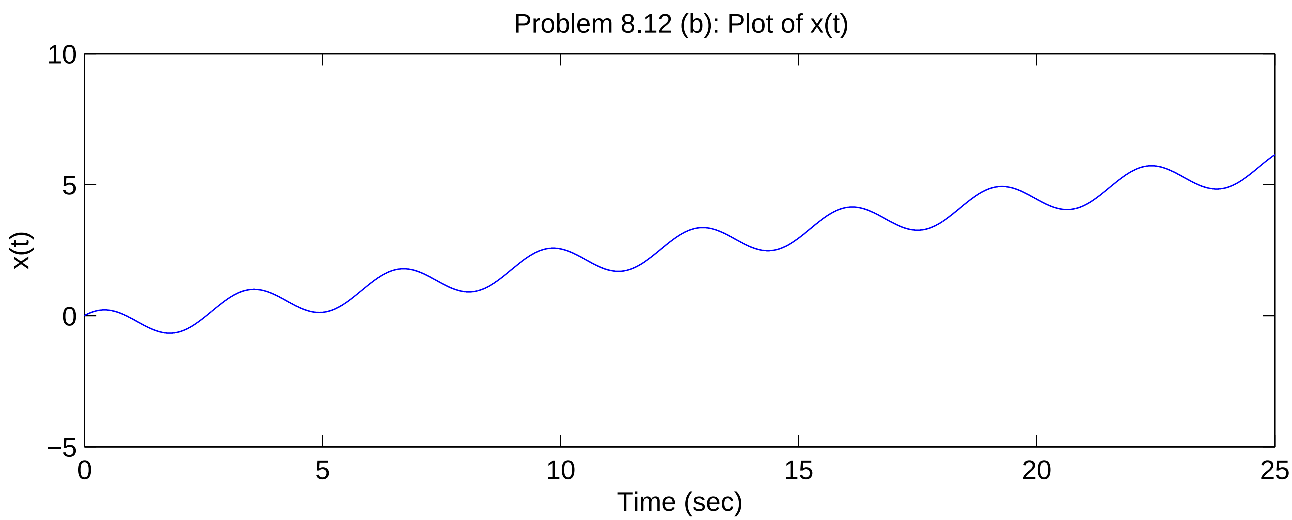

Problem 8.10 (g): Plot of x(t)
x(t)
Time (sec)
0.5
0
−0.5
−1
−1.5
0
1
2
3
4
5
6
7
8
9
10


Problem 8.12 (a): Plot of x(t)
x(t)
2
1.5
1
0.5
0
−0.5
−1
0
5
10
15
20
25
Time (sec)

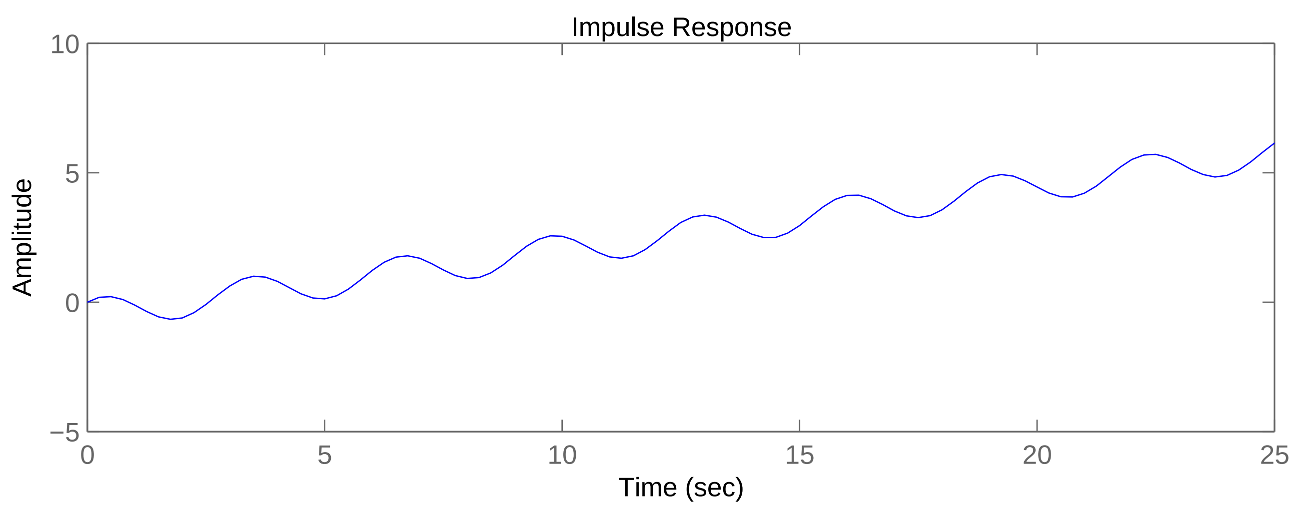

Output of MATLAB impulse function
Amplitude
2
1.5
1
0.5
0
−0.5
−1
0
5
10
15
20
25
Time (sec)


Problem 8.12 (b): Plot of x(t)
x(t)
Time (sec)
10
5
0
−5
−10
0
5
10
15
20
25

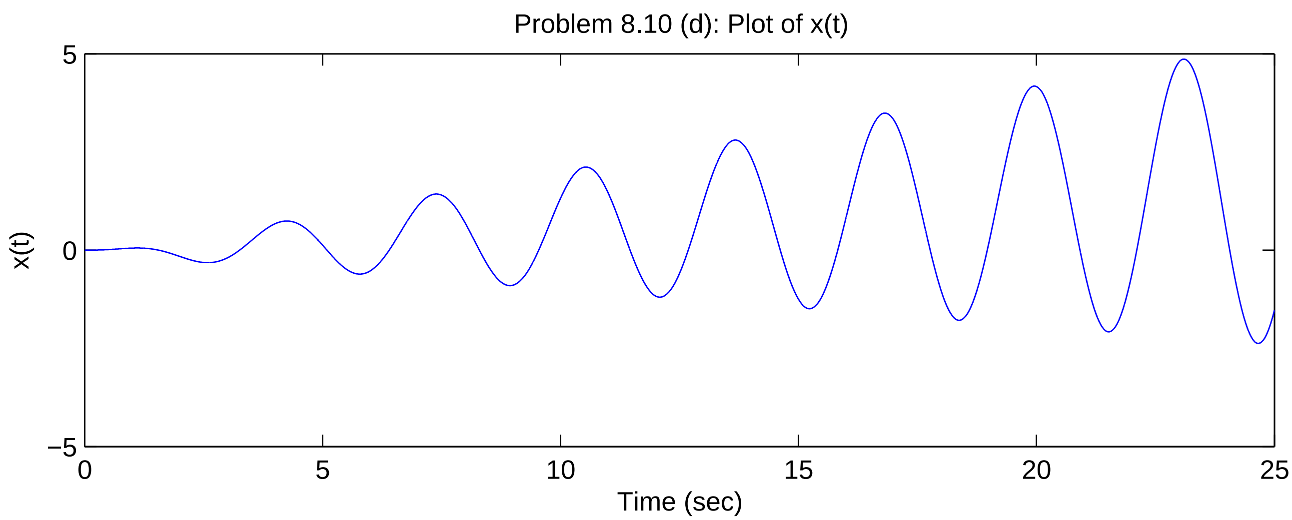

Impulse Response
Amplitude
Time (sec)
10
5
0
−5
−10
0
5
10
15
20
25

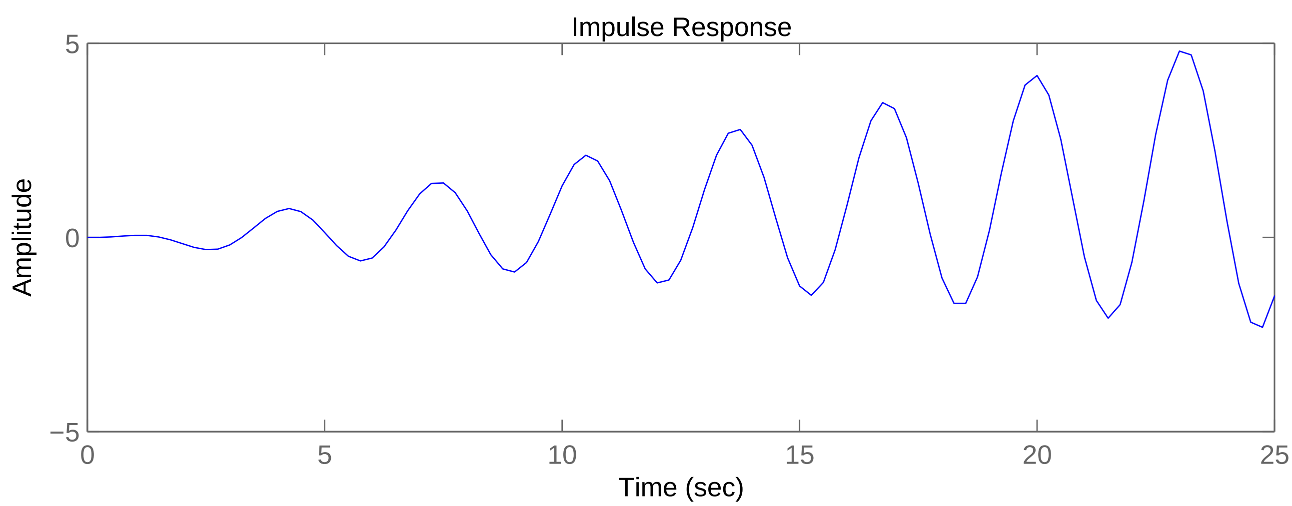


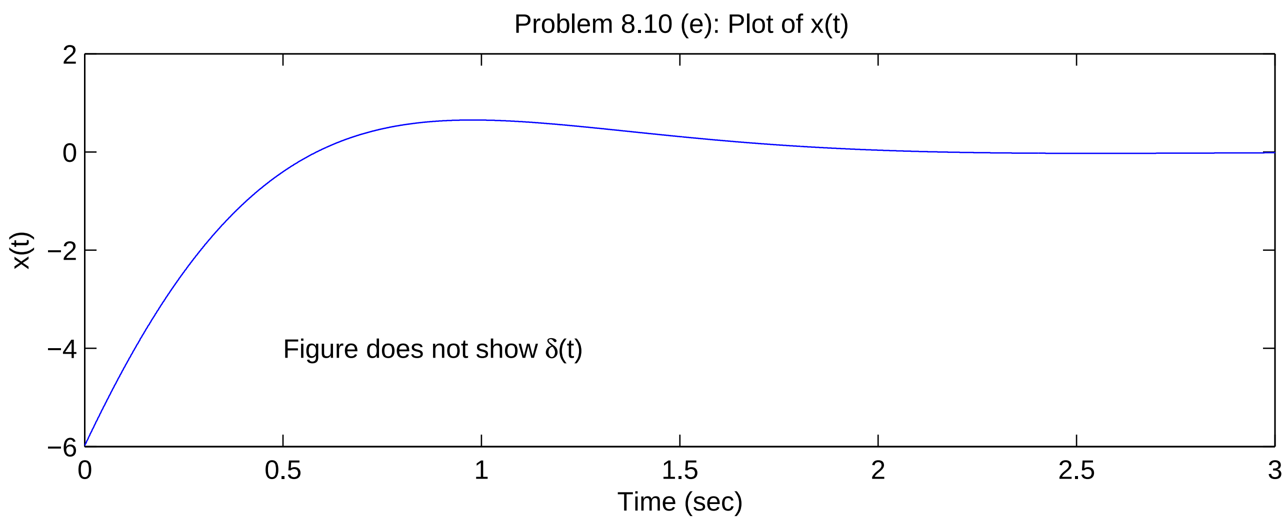
Problem 8.12 (b): Plot of x(t)
x(t)
10
5
0
−5
0
5
10
15
20
25
Time (sec)



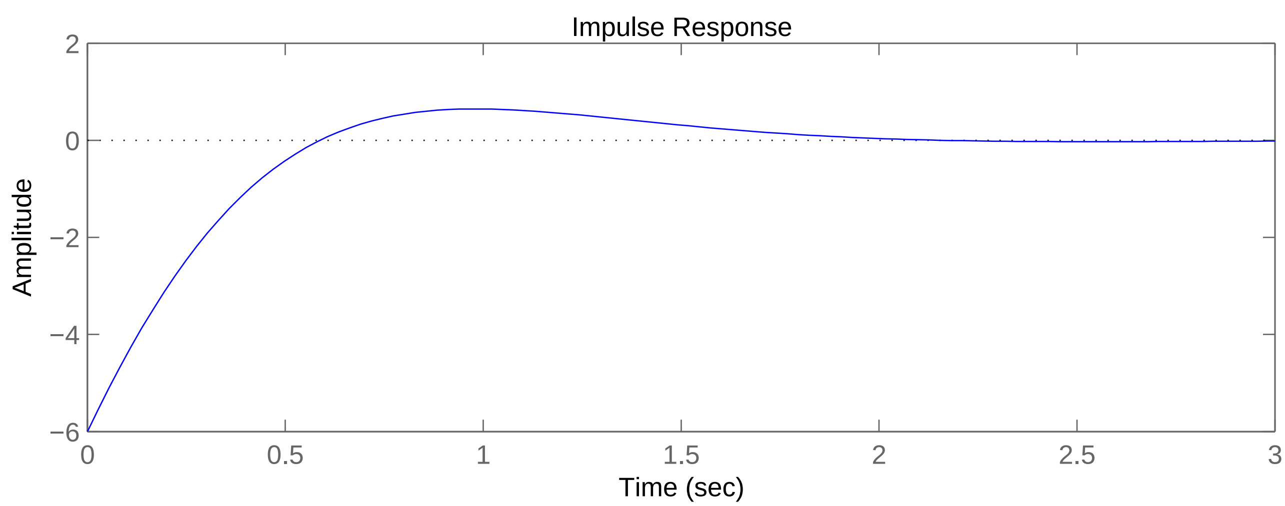
Impulse Response
Amplitude
10
5
0
−5
0
5
10
15
20
25
Time (sec)


Problem 8.10 (d): Plot of x(t)
5
0
−5
x(t)
0
5
10
15
20
25
Time (sec)


Impulse Response
5
0
−5
Amplitude
0
5
10
15
20
25
Time (sec)



Problem 8.10 (e): Plot of x(t)
x(t)
Time (sec)
Figure does not show δ(t)
2
0
−2
−4
−6
0
0.5
1
1.5
2
2.5
3



Impulse Response
Amplitude
Time (sec)
2
0
−2
−4
−6
0
0.5
1
1.5
2
2.5
3