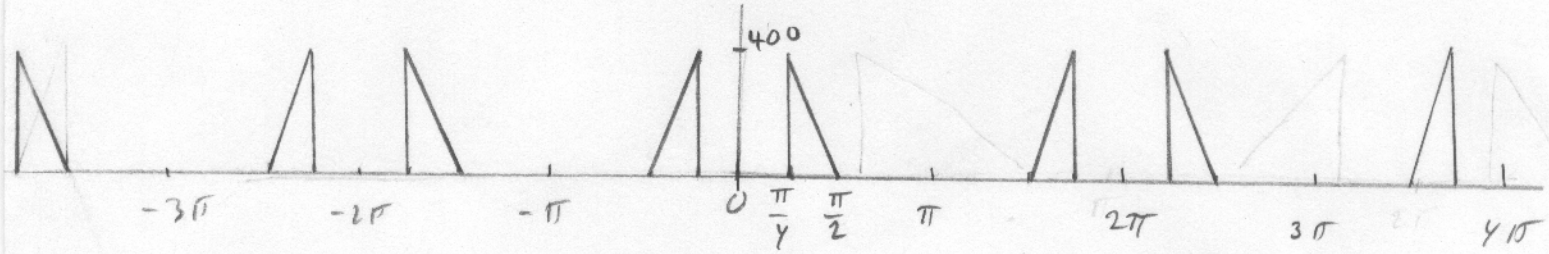
EE 342
HW #1 Soln

1. (a) $f_s = 400\text{Hz}$ $\quad f_N = f_s/2 = 200\text{ Hz}$ $\quad \Omega = 2\pi \frac{f}{f_s}$

100 Hz: $\Omega = 2\pi \frac{100\text{Hz}}{400\text{Hz}} = \frac{\pi}{2}$

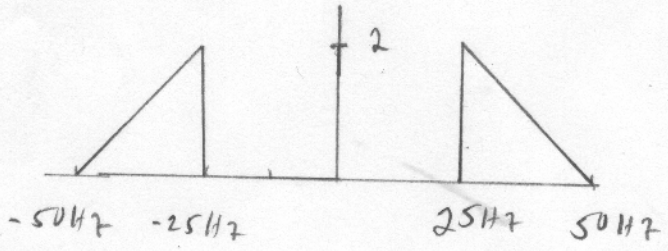
50 Hz: $\Omega = 2\pi \frac{50\text{Hz}}{400\text{Hz}} = \frac{\pi}{4}$



(b) On reconstruction, $f_s = 200\text{Hz}$; $f = f_s \frac{\Omega}{2\pi}$

$\frac{\pi}{2}$: $f = 200 \frac{\pi/2}{2\pi} = 50\text{Hz}$

$\frac{\pi}{4}$: $f = 200\text{Hz} \frac{\pi/4}{2\pi} = 25\text{Hz}$



2. $x(t) = 2\cos\left(\frac{5}{7}\pi t\right) - 2\cos\left(\frac{10}{3}\pi t\right)$

$\omega_1 = \frac{5}{7}\pi \quad T_1 = \frac{2\pi}{\omega_1} = \frac{14}{5}\text{ sec}$

$\omega_2 = \frac{10}{3}\pi \quad T_2 = \frac{2\pi}{\omega_2} = \frac{3}{5}\text{ sec}$

$\frac{T_1}{T_2} = \frac{14}{3} = \frac{p}{q} \quad T_0 = pT_2 = qT_1 = 14\left(\frac{3}{5}\right) = \frac{42}{5}\text{ sec}$

$\omega_0 = \frac{2\pi}{T_0} = \frac{5}{21}\pi \quad \omega_1 = \frac{5}{7}\pi = 3\omega_0 \quad \omega_2 = \frac{10}{3}\pi = 14\omega_0$

$$x(t) = 2\,\frac{e^{j\frac{5}{7}\pi t} + e^{-j\frac{5}{7}\pi t}}{2} - 2\,\frac{e^{j\frac{10}{3}\pi t} + e^{-j\frac{10}{3}\pi t}}{2}$$

$$= e^{j\frac{5}{7}\pi t} + e^{-j\frac{5}{7}\pi t} - e^{j\frac{10}{3}\pi t} - e^{-j\frac{10}{3}\pi t}$$

$$= e^{j3\omega_0 t} + e^{-j3\omega_0 t} - e^{j14\omega_0 t} - e^{-j14\omega_0 t}$$

$$C_3 = 1, \quad C_{-3} = 1, \quad C_{14} = -1, \quad C_{-14} = -1$$

3. Problem 8.1

3. (a) $\cos(\omega t)u(t) \leftrightarrow \frac{s}{s^2+\omega^2}$ $\quad \cos(3t)u(t) \leftrightarrow \frac{s}{s^2+9}$

(c) $e^{-6t}\cos\omega t\, u(t) \leftrightarrow \frac{s+6}{(s+6)^2+\omega^2}$ $\quad e^{-10t}\cos(3t)u(t) \leftrightarrow \frac{s+10}{(s+10)^2+9}$

(d) $e^{-10}\cos(3t-1)\,u(t) = e^{-10t}\cos(3t)\cos(1)\,u(t) + e^{-10t}\sin(3t)\sin(1)\,u(t)$

$$\leftrightarrow \frac{\cos(1)\,(s+10)}{(s+10)^2+9} + \frac{\sin(1)\,3}{(s+10)^2+9}$$

(f) $(t-1+e^{-10t}\cos(4t-\pi/3))\,u(t) = (t-1+e^{-10t}\cos(\tfrac{\pi}{3})\cos(4t) + e^{-10t}\sin(\pi/3)\sin(4t))u(t)$

$$\leftrightarrow \frac{1}{s^2} - \frac{1}{s} + \frac{1}{2}\,\frac{s+10}{(s+10)^2+4^2} + \frac{\sqrt{3}}{2}\,\frac{4}{(s+10)^2+4^2}$$

4. Problem 8.2

(d) $v(t) = \int_0^t x(\tau)\,d\tau \Rightarrow V(s) = \frac{1}{s}X(s) = \frac{s+1}{s(s^2+5s+1)}$

(f) $e^{-3t}x(t) \leftrightarrow X(s+3) = \frac{(s+3)+1}{(s+3)^2+5(s+3)+1} = \frac{s+4}{s^2+11s+31}$

(g) $x(t) * x(t) \leftrightarrow X(s)X(s) = \frac{(s+1)^2}{(s^2+5s+1)^2} = \frac{s^2+2s+1}{s^4+10s^3+27s^2+10s+1}$

5. Problem 8.3

(b) $x(t) = e^{-t}[u(t) - u(t-2)] + e^{t-4}[u(t-2) - u(t-4)]$

$= e^{-t}u(t) - e^{-2}e^{-(t-2)}u(t-2) + e^{-2}e^{t-2}u(t-2) - e^{t-4}u(t-4)$

$X(s) = \frac{1}{s+1} - e^{-2}\frac{e^{-2s}}{s+1} + e^{-2}\frac{e^{-2s}}{s-1} - \frac{e^{-4s}}{s-1}$

6. Problem 8.4

(a) $\cos^2 \omega t\, u(t) \longleftrightarrow \frac{s^2+2\omega^2}{s(s^2+4\omega^2)} = X(s)$

$e^{-bt}\cos^2 \omega t\, u(t) \longleftrightarrow X(s+b) = \frac{(s+b)^2+2\omega^2}{(s+b)[(s+b)^2+4\omega^2]}$

(c) $t\cos^2 \omega t\, u(t) \longleftrightarrow -\frac{d}{ds}X(s)$

$= (-1)\frac{2s\cdot s(s^2+4\omega^2) - (s^2+2\omega^2)(3s^2+4\omega^2)}{s^2(s^2+4\omega^2)^2}$

$= \frac{s^4+2\omega^2 s^2+8\omega^4}{s^2(s^2+4\omega^2)^2}$

(d) $t\sin^2 \omega t\, u(t) \longleftrightarrow -\frac{d}{ds}\frac{2\omega^2}{s(s^2+4\omega^2)} = \frac{6\omega^2 s^2+8\omega^4}{s^2(s^2+4\omega^2)^2}$

(e) $\cos^3 \omega t\, u(t) = \cos \omega t\,(\cos^2 \omega t\, u(t)) \longleftrightarrow \frac{1}{2}[X(s+j\omega) + X(s-j\omega)]$

$= \frac{1}{2}\left[\frac{(s+j\omega)^2+2\omega^2}{(s+j\omega)((s+j\omega)^2+4\omega^2)} + \frac{(s-j\omega)^2+2\omega^2}{(s-j\omega)((s-j\omega)^2+4\omega^2)}\right]$

$= \frac{s^5+8s^3\omega^2+7\omega^4 s}{s^6+11s^4\omega^2+19s^2\omega^4+9\omega^6} = \frac{s(s^2+7\omega^2)(\omega^2+s^2)}{(s^2+9\omega^2)(\omega^2+s^2)^2}$

$= \frac{s(s^2+7\omega^2)}{(s^2+9\omega^2)(\omega^2+s^2)}$