EE 342

HW # 3

1. $x(t) = \cos\left(\frac{3}{10}\pi t\right) - 2\sin\left(\frac{4}{25}\pi t\right)$

$\omega_1 = \frac{3}{10}\pi \quad T_1 = \frac{2\pi}{\omega_1} = \frac{20}{3} \qquad \omega_2 = \frac{4}{25}\pi \quad T_2 = \frac{2\pi}{\omega_2} = \frac{50}{4} = \frac{25}{2}$

$\frac{T_1}{T_2} = \frac{20/3}{25/2} = \frac{40}{75} = \frac{8}{15} = \frac{p}{q} \qquad T_0 = pT_2 = qT_1 = \frac{25}{2} \cdot 8 = 100 \text{ sec}$

$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{100} = \frac{\pi}{50}$

$x(t) = \cos\left(\frac{15}{50}\pi t\right) - 2\sin\left(\frac{8}{50}\pi t\right) = \cos(15\omega_0 t) - \sin(8\omega_0 t)$

$= \frac{e^{j15\omega_0 t} + e^{-j15\omega_0 t}}{2} - 2\,\frac{e^{j8\omega_0 t} - e^{-j8\omega_0 t}}{2j}$

$= \frac{1}{2}e^{j15\omega_0 t} + \frac{1}{2}e^{-j15\omega_0 t} + je^{j8\omega_0 t} - je^{-j8\omega_0 t}$

$\omega_0 = \frac{\pi}{50} \qquad c_{15} = \frac{1}{2} \quad c_{-15} = \frac{1}{2} \quad c_8 = j \quad c_{-8} = -j$

2. Problem 8.14

(a) $\frac{dy}{dt} + 2y = u(t) \qquad sY(s) - \cancel{y(0^-)}^{\,0} + 2Y(s) = \frac{1}{s}$

$Y(s) = \frac{1}{s(s+2)} = \frac{1/2}{s} + \frac{-1/2}{s+2} \qquad y(t) = \frac{1}{2}u(t) - \frac{1}{2}e^{-2t}u(t)$

(b) $\frac{dy}{dt} - 2y = u(t) \qquad sY(s) - \cancel{y(0^-)}^{\,1} - 2Y(s) = \frac{1}{s}$

$Y(s) = \frac{1}{s-2} + \frac{1}{s(s-2)} = \frac{1}{s-2} + \frac{-1/2}{s} + \frac{1/2}{s-2} = \frac{3/2}{s-2} - \frac{1/2}{s}$

$y(t) = \frac{3}{2}e^{2t}u(t) - \frac{1}{2}u(t)$

(c) $\frac{dy}{dt} + 10y = 4\sin(2t)\,u(t)$ $\quad sY(s) - \overset{1}{\cancel{y(0^-)}} + 10Y(s) = 4\frac{2}{s^2+2^2}$

$Y(s) = \frac{1}{s+10} + \frac{8}{(s+10)(s^2+4)} = \frac{\frac{14}{13}}{s+10} + \frac{\frac{-1-j5}{26}}{s-j2} + \frac{\frac{-1+j5}{26}}{s+j2}$

$y(t) = \frac{14}{13}e^{-10t}u(t) + \frac{\sqrt{26}}{13}\cos(2t-101^\circ)\,u(t)$

(d) $\frac{dy}{dt} + 10y = 8e^{-10t}u(t)$ $\quad sY(s) - \overset{0}{\cancel{y(0^-)}} + 10Y(s) = \frac{8}{s+10}$

$Y(s) = \frac{8}{(s+10)^2}$ $\quad y(t) = 8te^{-10t}u(t)$

(e) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = u(t)$

$s^2Y(s) - s\overset{0}{\cancel{y(0^-)}} - \overset{1}{\cancel{\dot{y}(0^-)}} + 6[sY(s) - \overset{0}{\cancel{y(0^-)}}] + 8Y(s) = \frac{1}{s}$

$Y(s) = \frac{1}{s^2+6s+8} + \frac{1}{s(s^2+6s+8)} = \frac{1}{(s+2)(s+4)} + \frac{1}{s(s+2)(s+4)}$

$= \frac{1/2}{s+2} + \frac{-1/2}{s+4} + \frac{1/8}{s} + \frac{-1/4}{s+2} + \frac{1/8}{s+4}$

$= \frac{1/4}{s+2} + \frac{-3/8}{s+4} + \frac{1/8}{s}$

$y(t) = \frac{1}{8}e^{-2t}u(t) - \frac{3}{8}e^{-4t}u(t) + \frac{1}{8}u(t)$

(f) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = \sin(2t)\,u(t)$

$s^2Y(s) - s\overset{0}{\cancel{y(0^-)}} - \overset{0}{\cancel{\dot{y}(0^-)}} + 6(sY(s) - \overset{0}{\cancel{y(0^-)}}) + 9Y(s) = \frac{2}{s^2+2^2}$

$Y(s) = \frac{2}{(s^2+6s+9)(s^2+4)} = \frac{12/169}{(s+3)} + \frac{2/13}{(s+3)^2} + \frac{\frac{-12-j5}{338}}{s-j2} + \frac{\frac{-12+j5}{338}}{s+j2}$

$y(t) = \frac{12}{169}e^{-3t}u(t) + \frac{2}{13}te^{-3t}u(t) + \frac{1}{13}\cos(2t-157^\circ)$

(g) $\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 13y = u(t)$

$s^2Y(s) - s\cancel{y(0^-)} - \cancel{\dot{y}(0^-)} + 6(sY(s) - \cancel{y(0^-)}) + 13Y(s) = \frac{1}{s}$

$Y(s) = \frac{s+7}{s^2+6s+13} + \frac{1}{s(s^2+6s+13)} = \frac{1/13}{s} + \frac{\frac{24-j49}{52}}{s+3-j2} + \frac{\frac{24+j49}{52}}{s+3+j2}$

$y(t) = \frac{1}{13}u(t) + 2|r_1|e^{-3t}\cos(2t - \measuredangle r_1)$

$= \frac{1}{13}u(t) + \frac{\sqrt{2977}}{26}e^{-3t}\cos(2t - 64°)u(t)$

3. See MATLAB

4. $s^2Y(s) - sy(0^-) - \dot{y}(0^-) + 4(sY(s) - y(0^-)) + 3Y(s) = 2s^2X(s) - 4sX(s) - X(s)$

$Y(s) = \frac{sy(0^-) + \dot{y}(0^-) + 4y(0^-)}{s^2+4s+3} + \frac{2s^2-4s-1}{s^2+4s+3}X(s)$

(a) $y(0^-) = -2$, $\dot{y}(0^-) = 1$, $x(t) = 0$

$Y(s) = \frac{-2s-7}{(s+1)(s+3)} = \frac{-5/2}{s+1} + \frac{1/2}{s+3}$ $\quad y(t) = -\frac{5}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t)$

(b) $y(0^-) = 0$ $\quad \dot{y}(0^-) = 0$ $\quad x(t) = \delta(t)$ $\quad X(s) = 1$

$Y(s) = \frac{2s^2-4s-1}{s^2+4s+3} = 2 + \frac{-12s-7}{(s+1)(s+3)} = 2 + \frac{5/2}{s+1} + \frac{-29/2}{s+3}$

$y(t) = 2\delta(t) + \frac{5}{2}e^{-t}u(t) - \frac{29}{2}e^{-3t}u(t)$

(c) $y(0^-) = 0$, $\dot{y}(0^-) = 0$, $x(t) = u(t)$ $\quad X(s) = \frac{1}{s}$

$Y(s) = \frac{2s^2-4s-1}{s(s+1)(s+3)} = \frac{-1/3}{s} + \frac{-5/2}{s+1} + \frac{25/6}{s+3}$

$y(t) = -\frac{1}{3}u(t) - \frac{5}{2}e^{-t}u(t) + \frac{25}{6}e^{-3t}u(t)$

(d) $y(0^-)=-2$, $\dot{y}(0^-)=1$, $x(t)=u(t)$

The initial conditions are the same as (a)

The input is the same as (c)

The output will be $y_a + y_c$

$$y(t) = -\frac{5}{2}e^{-t}u(t) + \frac{1}{2}e^{-3t}u(t) - \frac{1}{3}u(t) - \frac{5}{2}e^{-t}u(t) + \frac{25}{6}e^{-3t}u(t)$$

$$= -\frac{1}{3}u(t) - 5e^{-t}u(t) + \frac{14}{3}e^{-3t}u(t)$$

5. Problem 8.17

(a) $$I L_f s^3 Y(s) + (k_d L_f + R_f I) s^2 Y(s) + R_f k_d s Y(s) = k X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{k}{s^3 I L_f + s^2 (k_d L_f + R_f I) + s R_f k_d}$$

$$= \frac{k}{s(Is + k_d)(L_f s + R_f)}$$

$$= \frac{k / I L_f}{s\left(s + \frac{k_d}{I}\right)\left(s + \frac{R_f}{L_f}\right)}$$

(b) $$H(s) = \frac{r_1}{s} + \frac{r_2}{s + \frac{k_2}{I}} + \frac{r_3}{s + \frac{R_f}{L_f}}$$

$$r_1 = \frac{k}{k_d R_f}$$

$$r_2 = \frac{k I}{k_d (k_d L_f - R_f I)}$$

$$r_3 = \frac{k L_f}{R_f (R_f I - k_d L_f)}$$

6. Problem 8.25

(a) $\frac{dy}{dt} + e^{-t} y = x$

$\uparrow$

Time-varying coefficient $\Rightarrow$ non-linear $\Rightarrow$ no transfer fnc

(b) $\frac{dy}{dt} + v * y = x \qquad sY(s) + V(s)Y(s) = X(s) \qquad Y(s) = \frac{X(s)}{s+V(s)}$

$V(s) = \frac{1}{s^2+1} \qquad H(s) = \frac{Y(s)}{H(s)} = \frac{1}{s+V(s)} = \frac{1}{s+\frac{1}{s^2+1}} = \frac{s^2+1}{s^3+s^2+1}$

(c) $\frac{d^2y}{dt^2} + \int_0^t y(\lambda)\,d\lambda = \frac{dx}{dt} - x$

$s^2 Y(s) + \frac{1}{s} Y(s) = s X(s) - X(s)$

$s^3 Y(s) + Y(s) = s^2 X(s) - s X(s)$

$H(s) = \frac{Y(s)}{X(s)} = \frac{s^2 - s}{s^3+1}$

(d) $\frac{dy}{dt} = y * x \qquad s Y(s) = Y(s) X(s) \qquad s = X(s)$ No trasfer fnc

(e) $\frac{dy}{dt} - 2y = t\,x$

$\uparrow$

Time-varying coeff $\Rightarrow$ non-linear $\Rightarrow$ no transer fnc

7. Problem 8.26

$H(s) = \frac{s+7}{s^2+4} \Rightarrow b_1 = 1,\ b_0 = 7,\ a_1 = 0,\ a_0 = 4$

From Eqn 8.93 $\quad Y(s) = \frac{y(0^-)s + \dot{y}(0^-) + a_1 y(0^-)}{s^2 + a_1 s + a_0} + \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} X(s)$

$= \frac{y(0^-)s + \dot{y}(0^-)}{s^2+4} + \frac{s+7}{s^2+4} X(s)$

$y(t) =$ Inverse Laplace Transform[$Y(s)$]

8. problem 8.28

$y_1(t) = 3tu(t) + 2u(t) - e^{-t}u(t)$ for $x_1(t) = e^{-t}u(t)$

① $Y_1(s) = \frac{3}{s^2} + \frac{2}{s} - \frac{1}{s+1} = \frac{s^2+5s+3}{s^2(s+1)}$ for $X_1(s) = \frac{1}{s+1}$

② $y_2(t) = 2tu(t) + 2u(t) - e^{-2t}u(t)$ for $x_2(t) = e^{-2t}u(t)$

$Y_2(s) = \frac{2}{s^2} + \frac{2}{s} - \frac{1}{s+2} = \frac{s^2+6s+4}{s^2(s+2)}$

$Y_1(s)$ has poles at $0, 0, -1$. The pole at -1 is due to input $X_1(s)$
$Y_2(s)$ has poles at $0, 0, -2$ The pole at -2 is due to input $X_2(s)$

The system has two poles at $0, 0$, so is a 2^{nd} order system.

Denominator of 2^{nd} order system is $s^2 + a_1 s + a_0 = (s-p_1)(s-p_2)$
For this system, $p_1 = 0$ and $p_2 = 0$, so $s^2 + a_1 s + a_0 = s^2 \Rightarrow a_1 = 0,\ a_0 = 0$

For 2^{nd} order system (eqn 8.93):

$$Y(s) = \frac{y(0^-)s + \dot{y}(0^-) + a_1 y(0^-)}{s^2 + a_1 s + a_0} + \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} X(s)$$

Because $a_1 = 0$ and $a_0 = 0$, we have:

$$Y(s) = \frac{y(0^-)s + \dot{y}(0^-)}{s^2} + \frac{b_1 s + b_0}{s^2} X(s)$$

③ $$Y_1(s) = \frac{y(0^-)s + \dot{y}(0^-)}{s^2} + \frac{b_1 s + b_0}{s^2}\frac{1}{s+1} = \frac{y(0^-)s^2 + (\dot{y}(0^-) + y(0^-) + b_1)s + (\dot{y}(0^-) + b_0)}{s^2(s+1)}$$

Equating the numerator coefficients in ① and ③ gives

④ $y(0^-) = 1 \qquad \dot{y}(0^-) + y(0^-) + b_1 = 5 \qquad \dot{y}(0^-) + b_0 = 3$

(5) $$Y_2(s) = \frac{y(0^-)s + \dot y(0^-)}{s^2} + \frac{b_1 s + b_0}{s^2}\frac{1}{s+2} = \frac{y(0^-)s^2 + (\dot y(0^-) + 2y(0^-) + b_1)s + (2\dot y(0^-) + b_0)}{s^2(s+2)}$$

Equating the numerator coefficients in (2) and (5) gives

(6) $y(0^-) = 1 \quad \dot y(0^-) + 2y(0^-) + b_1 = 6 \quad 2\dot y(0^-) + b_0 = 4$

(4) and (6) give 6 eqns for 4 unknowns $y(0^-), \dot y(0^-), b_1, b_0$.

Solving gives: $y(0^-) = 1$

$\dot y(0^-) = 1$

$b_1 = 3$

$b_0 = 2$

$$H(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} = \frac{3s + 2}{s^2}$$

Another way: From Eqn 8.97) $Y(s) = \frac{C(s)}{A(s)} + H(s)X(s)$

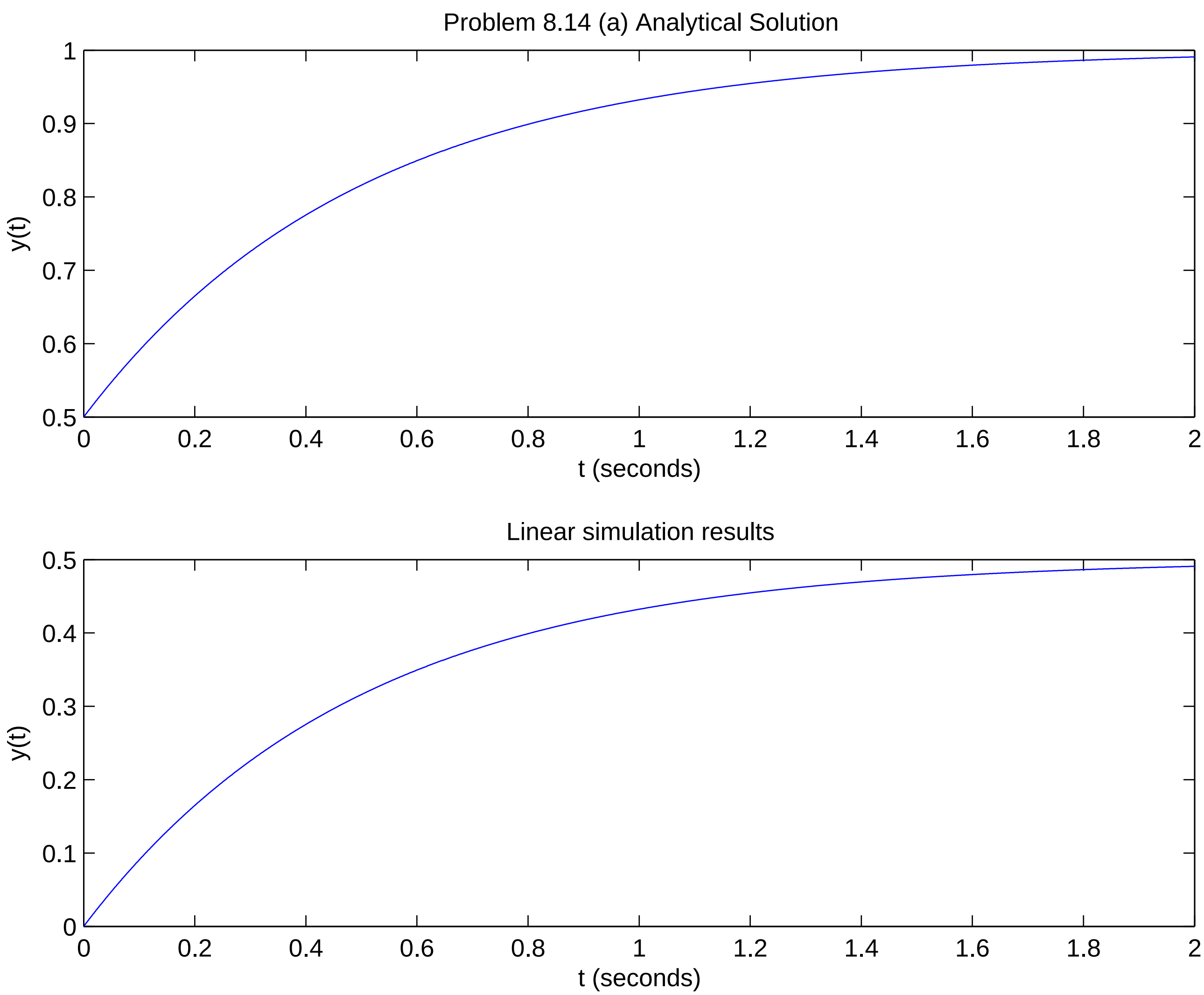
$Y_1(s) = \frac{C_1(s)}{A(s)} + H(s)X_1(s) \qquad Y_2(s) = \frac{C_2(s)}{A(s)} + H(s)X_2(s)$

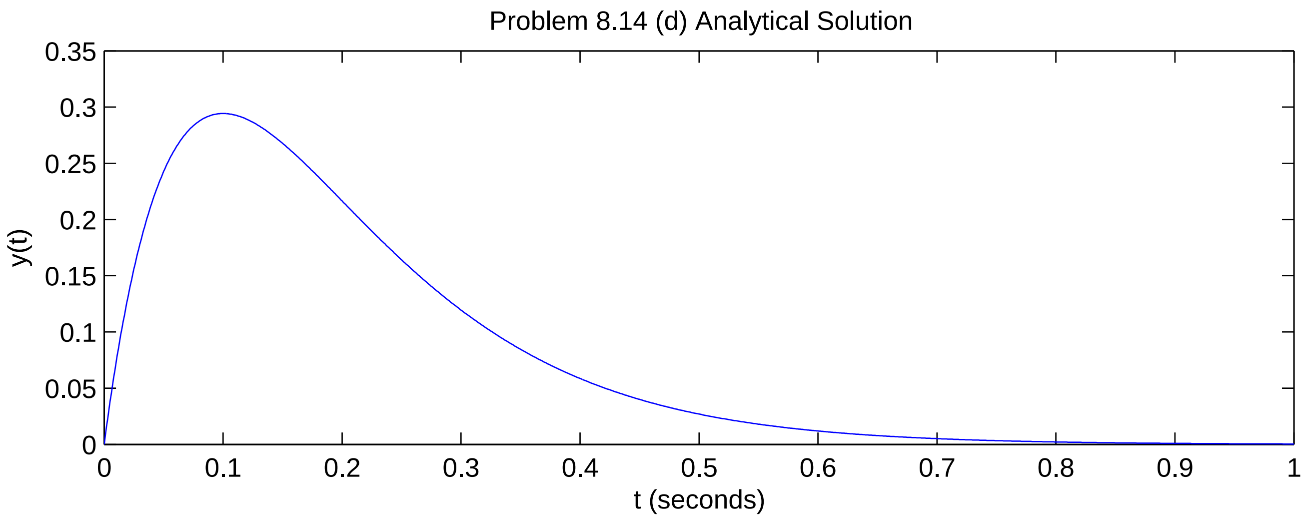
$C(s)$ depends on initial conditions; initial conditions are same, so $C_1(s) = C_2(s)$

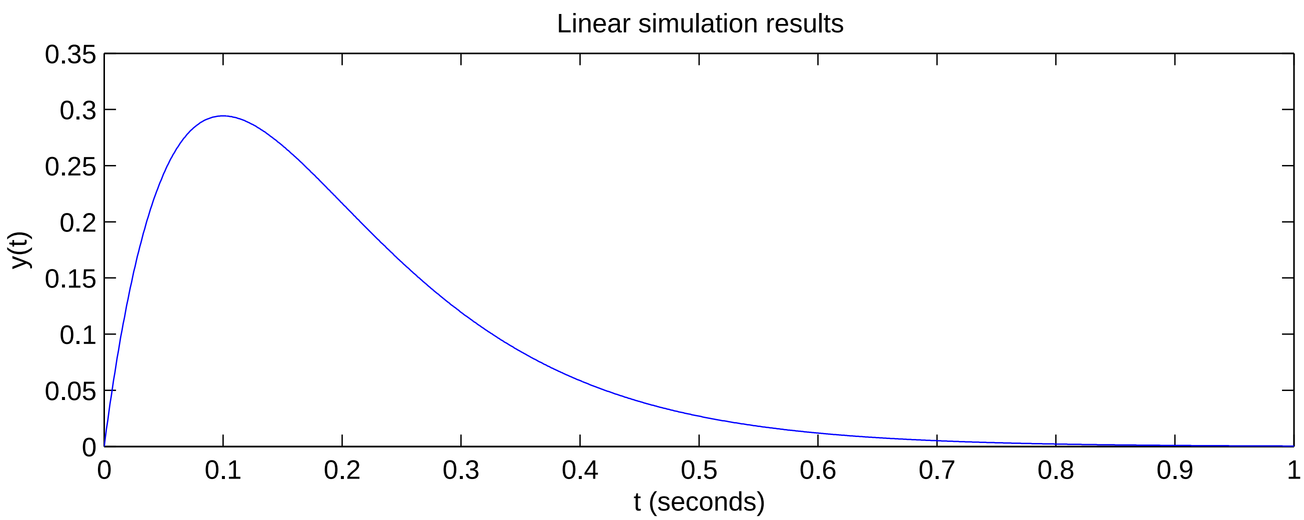
$Y_1(s) - Y_2(s) = H(s)[X_1(s) - X_2(s)] \qquad H(s) = \frac{Y_1(s) - Y_2(s)}{X_1(s) - X_2(s)}$

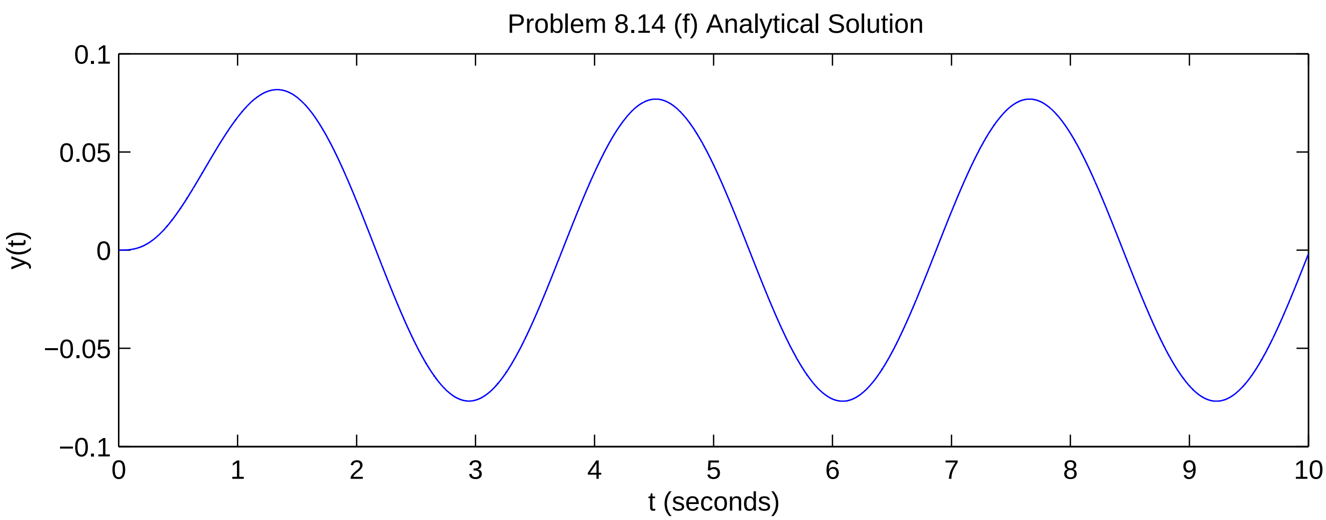
$$H(s) = \frac{\frac{(s^2 + 5s + 3)}{s^2(s+1)} - \frac{(s^2 + 6s + 4)}{s^2(s+2)}}{\frac{1}{(s+1)} - \frac{1}{(s+2)}} = \frac{3s + 2}{s^2}$$

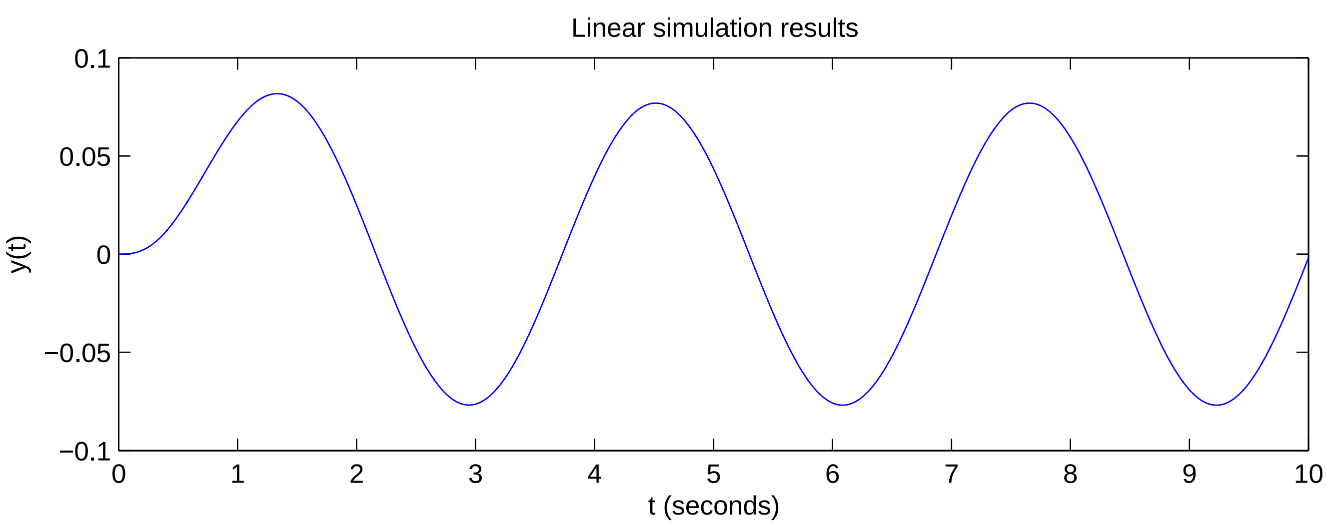
```
% EE 342 Homework      #3

% Problem 8.14

% (a)
figure(1)
clf
subplot(211)
t = 0:0.001:2;
y = 1 - 0.5*exp(-2*t);
plot(t,y)
xlabel('t (seconds)')
ylabel('y(t)')
title('Problem 8.14 (a) Analytical Solution')

subplot(212)
b = 1;
a = [1 2];
x = ones(size(t));
y = lsim(b,a,x,t);
plot(t,y)
xlabel('t (seconds)')
ylabel('y(t)')
title('Linear simulation results')
print -depsc2 p8_14a.eps

% (d)
figure(2)
clf
subplot(211)
t = 0:0.001:1;
y = 8*t.*exp(-10*t);
plot(t,y)
xlabel('t (seconds)')
ylabel('y(t)')
title('Problem 8.14 (d) Analytical Solution')

subplot(212)
b = 1;
a = [1 10];
x = 8*exp(-10*t);
y = lsim(b,a,x,t);
plot(t,y)
xlabel('t (seconds)')
ylabel('y(t)')
title('Linear simulation results')
print -depsc2 p8_14d.eps

% (f)
figure(3)
clf
subplot(211)
t = 0:0.001:10;
y = (12/169)*exp(-3*t)+(2/13)*t.*exp(-3*t)+(1/13)*cos(2*t+atan2(-5,-12));
plot(t,y)
xlabel('t (seconds)')
ylabel('y(t)')
title('Problem 8.14 (f) Analytical Solution')

subplot(212)
b = 1;
a = [1 6 9];
x = sin(2*t);
y = lsim(b,a,x,t);
plot(t,y)
xlabel('t (seconds)')
ylabel('y(t)')
title('Linear simulation results')
print -depsc2 p8_14f.eps
```


Problem 8.14 (a) Analytical Solution
y(t)
1
0.9
0.8
0.7
0.6
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
t (seconds)
Linear simulation results
y(t)
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
t (seconds)


Problem 8.14 (d) Analytical Solution
y(t)
t (seconds)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Linear simulation results
y(t)
t (seconds)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Problem 8.14 (f) Analytical Solution
y(t)
0.1
0.05
0
−0.05
−0.1
0 1 2 3 4 5 6 7 8 9 10
t (seconds)


Linear simulation results
y(t)
0.1
0.05
0
−0.05
−0.1
0 1 2 3 4 5 6 7 8 9 10
t (seconds)