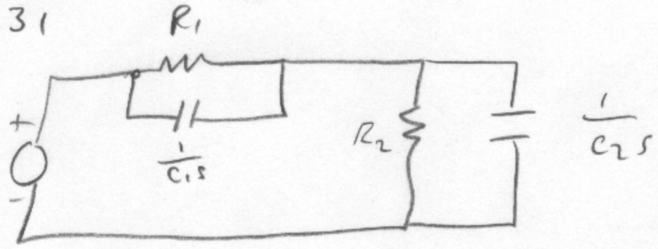
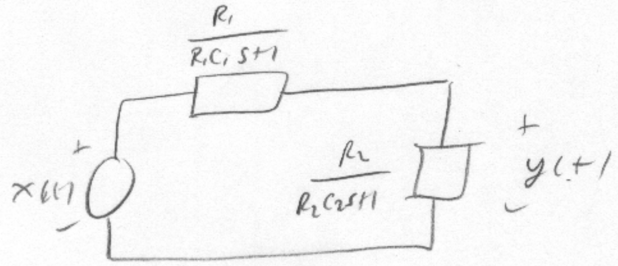
EE 342
HW #4

1. Problem 8.31

(a)

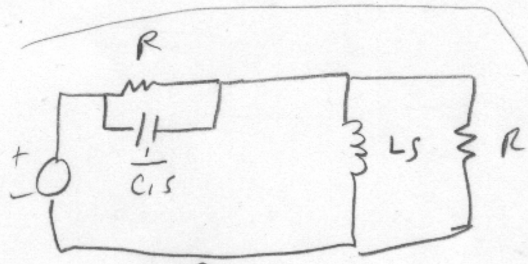


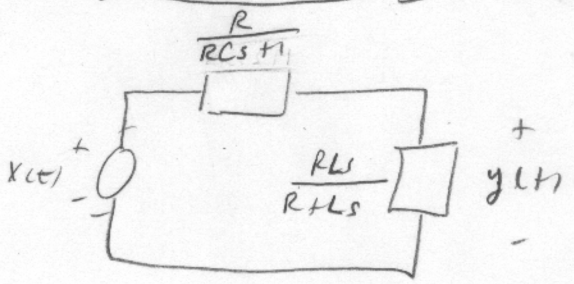


$$H(s) = \frac{\dfrac{R_2}{R_2C_2s+1}}{\dfrac{R_1}{R_1C_1s+1} + \dfrac{R_2}{R_2C_2s+1}} = \frac{R_1R_2C_1 s + R_2}{(C_1+C_2)R_1R_2 s + R_1 + R_2}$$

$$= \frac{\dfrac{C_1}{C_1+C_2} s + \dfrac{1}{(C_1+C_2)R_1}}{s + \dfrac{R_1+R_2}{(C_1+C_2)R_1R_2}}$$

(c)





$$H(s) = \frac{\dfrac{RLs}{R+Ls}}{\dfrac{RLs}{R+Ls} + \dfrac{R}{RCs+1}} = \frac{LRCs^2 + Ls}{LRCs^2 + 2Ls + R}$$

$$H(s) = \frac{s^2 + s/RC}{s^2 + \dfrac{2s}{RC} + \dfrac{1}{LC}}$$

2. Problem 2.34

(a) $h(t) = \cos 2t + 4 \sin 2t$

$$H(s) = \frac{s}{s^2+4} + 4\,\frac{2}{s^2+4} = \frac{s+8}{s^2+4}$$

(b) $x(t) = \frac{5}{7}e^{-t} - \frac{12}{7}e^{-8t}$ $\qquad X(s) =$

$$X(s) = \frac{5/7}{s+1} - \frac{12/7}{s+8} = \frac{-s+4}{(s+1)(s+8)}$$

$$Y(s) = H(s)\,X(s) = \frac{(s-4)(s+8)}{(s^2+4)(s+1)(s+8)} = \frac{-s+4}{(s^2+4)(s+1)}$$

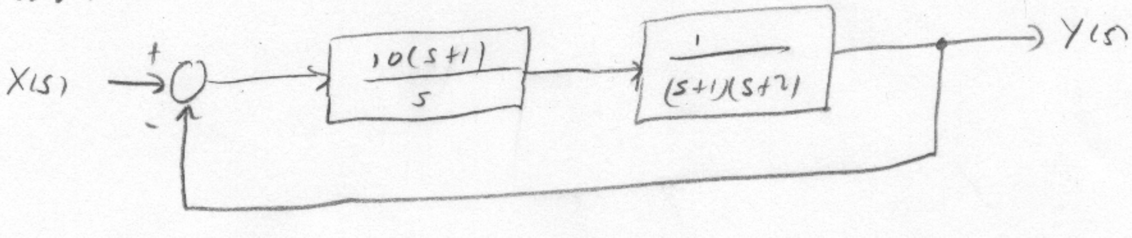
$$= \frac{1}{s+1} - \frac{s}{s^2+4}$$

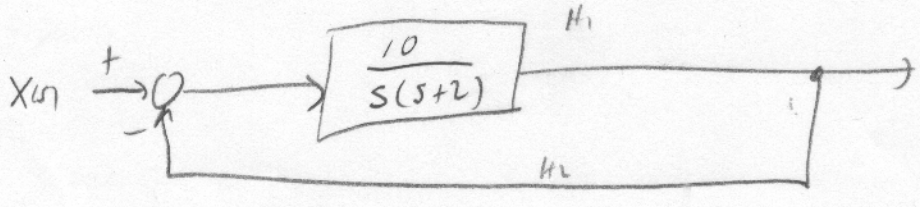
$$y(t) = e^{-t} - \cos(2t) \quad t \geq 0$$

(c) See MATLAB

3. Problem 8.38

(a)

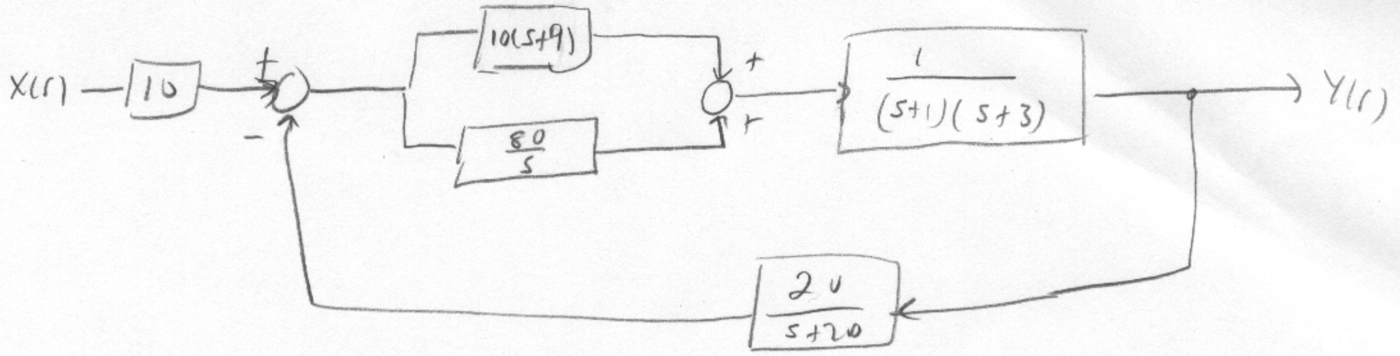


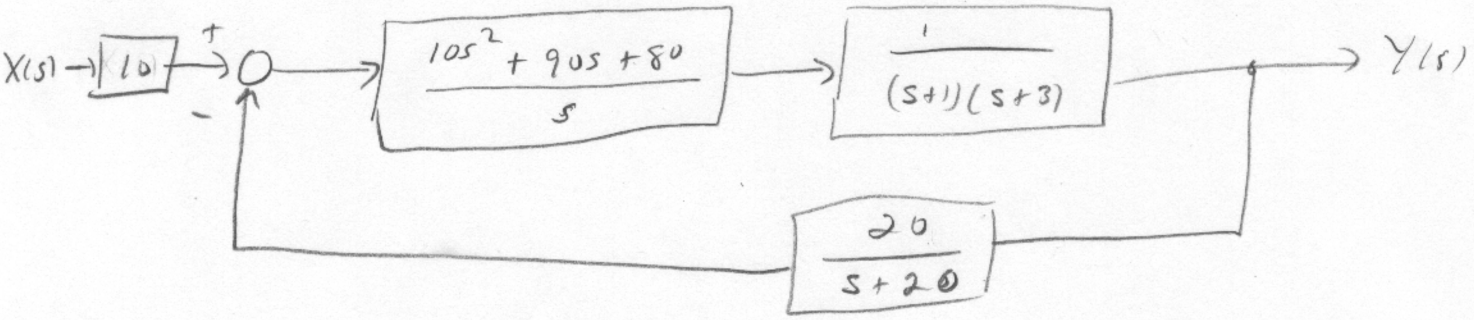


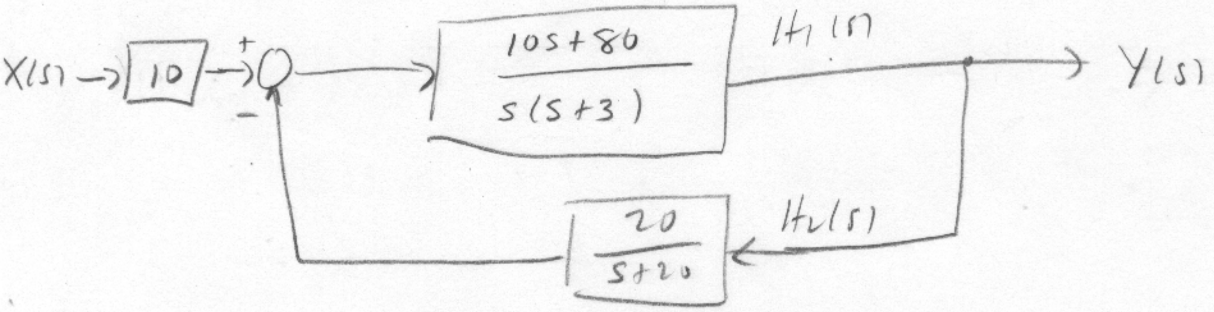
$$H(s) = \frac{H_1(s)}{1 + H_1(s)\,H_2(s)} = \frac{\frac{10}{s(s+2)}}{1 + \frac{10}{s(s+2)}}$$

$$= \frac{10}{s^2+2s+10}$$

(b)



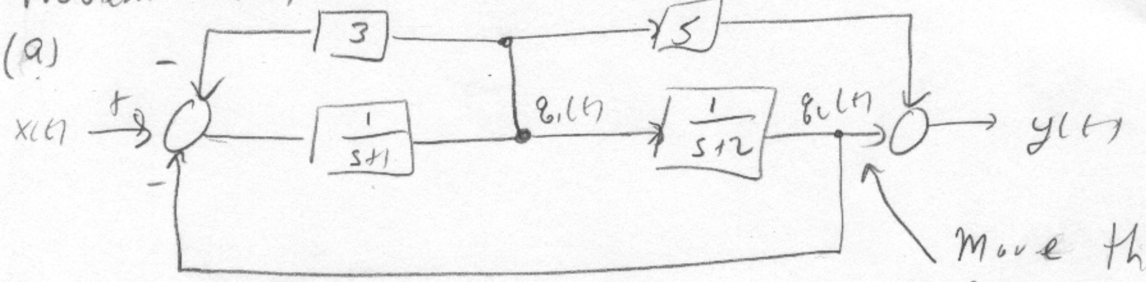




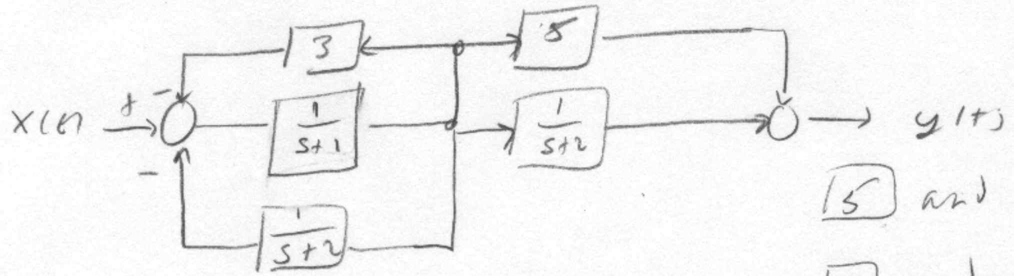
$$H(s) = \frac{10\, H_1(s)}{1 + H_1(s)\, H_2(s)} = \frac{100(s+8)(s+20)}{s^3 + 23s^2 + 260s + 1600}$$

4. Problem 8.39

(a)

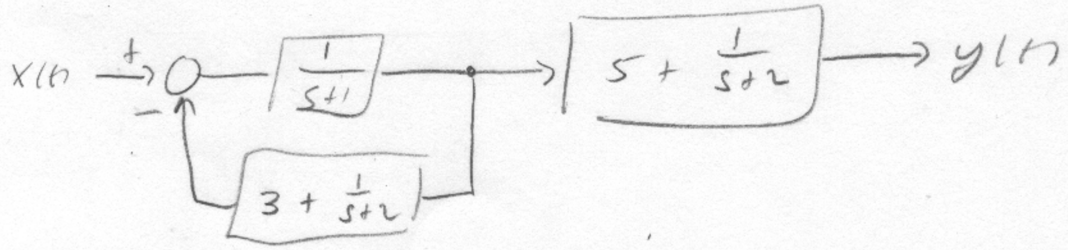


Move this pick-off point to the left of $\boxed{\frac{1}{s+2}}$



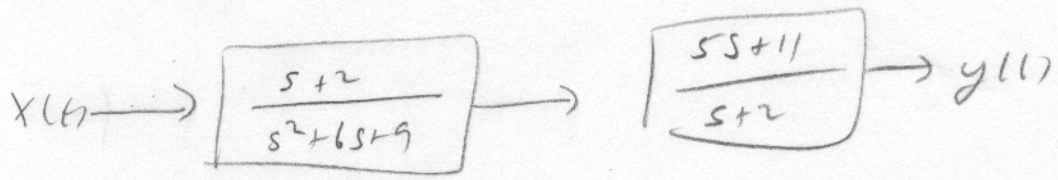
$\boxed{5}$ and $\boxed{\frac{1}{s+2}}$ in parallel

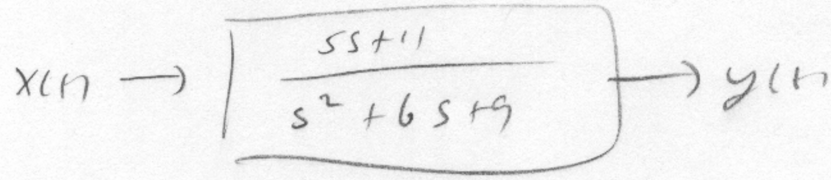
$\boxed{3}$ and $\boxed{\frac{1}{s+2}}$ in parallel



Feedback

$$\frac{H_1(s)}{1+H_1(s)H_2(s)}$$





(b) Need to find $y(0^-)$ and $\dot{y}(0^-)$

$q_1(0^-) = 1 \quad q_2(0^-) = -3$

$y(t) = 5q_1(t) + q_2(t) \qquad y(0^-) = 5q_1(0^-) + q_2(0^-) = 5\cdot 1 - 3 = 2$

$\dot{y}(0^-) = 5\dot{q}_1(0^-) + \dot{q}_2(0^-)$

Need to find $\dot{q}_1(0^-)$ and $\dot{q}_2(0^-)$

$Q_2(s) = \frac{1}{s+2} Q_1(s) \Rightarrow \qquad sQ_2(s) + 2Q_2(s) = Q_1(s)$

$\dot{q}_2(t) + 2q_2(t) = q_1(t)$

$\dot{q}_2(0^-) = q_1(0^-) - 2q_2(0^-) = 1 - 2(-3) = 7$

$Q_1(s) = \frac{1}{s+1}\left[X(s) - 3Q_1(s) - Q_2(s)\right] \Rightarrow sQ_1(s) + Q_1(s) = X(s) - 3Q_1(s) - Q_2(s)$

$\dot{q}_1(t) + 4q_1(t) = x(t) - q_2(t)$

$\dot{q}_1(0^-) = x(0^-) - q_2(0^-) - 4q_1(0^-) = 0 - (-3) - 4(1) = -1$

$\dot{y}(0^-) = 5\dot{q}_1(0^-) + \dot{q}_2(0^-) = 5(-1) + 7 = 2.$

$$Y(s) = \frac{y(0^-)s + \dot{y}(0^-) + a_1 y(0^-)}{s^2 + a_1 s + a_0} + \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} X(s)$$

$y(0^-) = 2, \quad \dot{y}(0^-) = 2, \quad a_1 = 6, \quad a_0 = 9, \quad b_1 = 5, \quad b_0 = 11, \quad X(s) = \frac{1}{s}$

$$Y(s) = \frac{2s + 2 + (6)(2)}{s^2 + 6s + 9} + \frac{5s + 11}{(s^2 + 6s + 9)} \frac{1}{s}$$

$$= \frac{2s^2 + 19s + 11}{s(s+3)^2}$$

$$= \frac{11/9}{s} + \frac{7/9}{s+3} + \frac{28/3}{(s+3)^2} \qquad \Bigg| \qquad y(t) = \frac{11}{9} + \frac{7}{9}e^{-3t} + \frac{28}{3}te^{-3t}, \quad t \geq 0$$

15. Problem 9.1

(a) $H(s) = \dfrac{s-4}{s(s+7)}$ Pole at 0, marginally stable

(b) $H(s) = \dfrac{s+3}{s^2+3} = \dfrac{s+3}{(s+j\sqrt{3})(s-j\sqrt{3})}$ Poles at $j\sqrt{3}$, $-j\sqrt{3}$, on Imag axis

Marginally stable

(c) $H(s) = \dfrac{2s^2+3s+1}{s^3+2s^2+4}$ Poles at -2.6, $0.3 \pm j1.2$ unstable

(d) $H(s) = \dfrac{3s^2-2s+6}{s^3+s^2+s+1}$ Poles at -1, $\pm j$ Marginally stable

6. Problem 9.2

$$H(s) = \frac{k}{L_f I s^3 + (L_f k_d + R_f I) s^2 + R_f k_d s}$$

$$= \frac{k}{(L_f s + R_f)(I s + k_d) s}$$

Poles at 0, $-\dfrac{R_f}{L_f}$, $-\dfrac{k_d}{I}$ Marginally stable

7. Problem 9.9

(a) $A(s) = s^3 + 30s^2 + (k+200)s + 40k$

	s^3	1	$k+200$
$\frac{1}{30}$	s^2	30	$40k$
$\frac{90}{600-k}$	s^1	$\frac{600-k}{3}$	0
	s^0	$40k$	

$$(k+200) - \frac{1}{30}\cdot 40k = \frac{600-k}{3}$$

Stable if $\frac{600-k}{3} > 0 \Rightarrow k < 600$

and $40k > 0 \Rightarrow k > 0$

$0 < k < 600$

(b) $A(s) = s^4 + s^3 + ks^2 + 2s + 3$

	s^4	1	k	3
1	s^3	1	2	0
$\frac{1}{k-2}$	s^2	$k-2$	3	0
	s^1	$\frac{2k-7}{k-2}$	0	0
	s^0	3	0	0

$$2 - \left(\frac{1}{k-2}\right)(3) = \frac{2k-7}{k-2}$$

Stable if $k-2 > 0 \Rightarrow k > 2$

and $\frac{2k-7}{k-2} > 0 \Rightarrow k > \frac{7}{2}$

Stable if $k > \frac{7}{2}$

(c) $A(s) = s^3 + s^2 + (k+3)s + (3k-5)$

$$
\begin{array}{c|c|cc}
 & s^3 & 1 & k+3 \\
1 & s^2 & 1 & 3k-5 \\
 & s^1 & 8-2k & 0 \\
 & s^0 & 3k-5 &
\end{array}
$$

$k+3-(1)(3k-5) = 8-2k$

Stable if $8-2k > 0 \Rightarrow k < 4$

and $3k-5 > 0 \Rightarrow k > 5/3$

$\frac{5}{3} < k < 4$ to be stable

(d) $A(s) = s^5 + 10s^4 + (9+k)s^3 + (90+2k)s^2 + 12ks + 10k$

$$
\begin{array}{c|c|ccc}
 & s^5 & 1 & 9+k & 12k \\
\frac{1}{10} & s^4 & 10 & 90+2k & 10k \\
\frac{25}{2k} & s^3 & \frac{4}{5}k & 11k & 0 \\
\frac{8k}{20k-475} & s^2 & \frac{4k-95}{2} & 10k & 0 \\
 & s^1 & \frac{k(28k-1045)}{4k-95} & 0 & 0 \\
 & s^0 & 10k & 0 & 0
\end{array}
$$

Stable if $\frac{4}{5}k > 0 \Rightarrow k > 0$

$\frac{4k-95}{2} > 0 \Rightarrow k > \frac{95}{4} = 23.75$

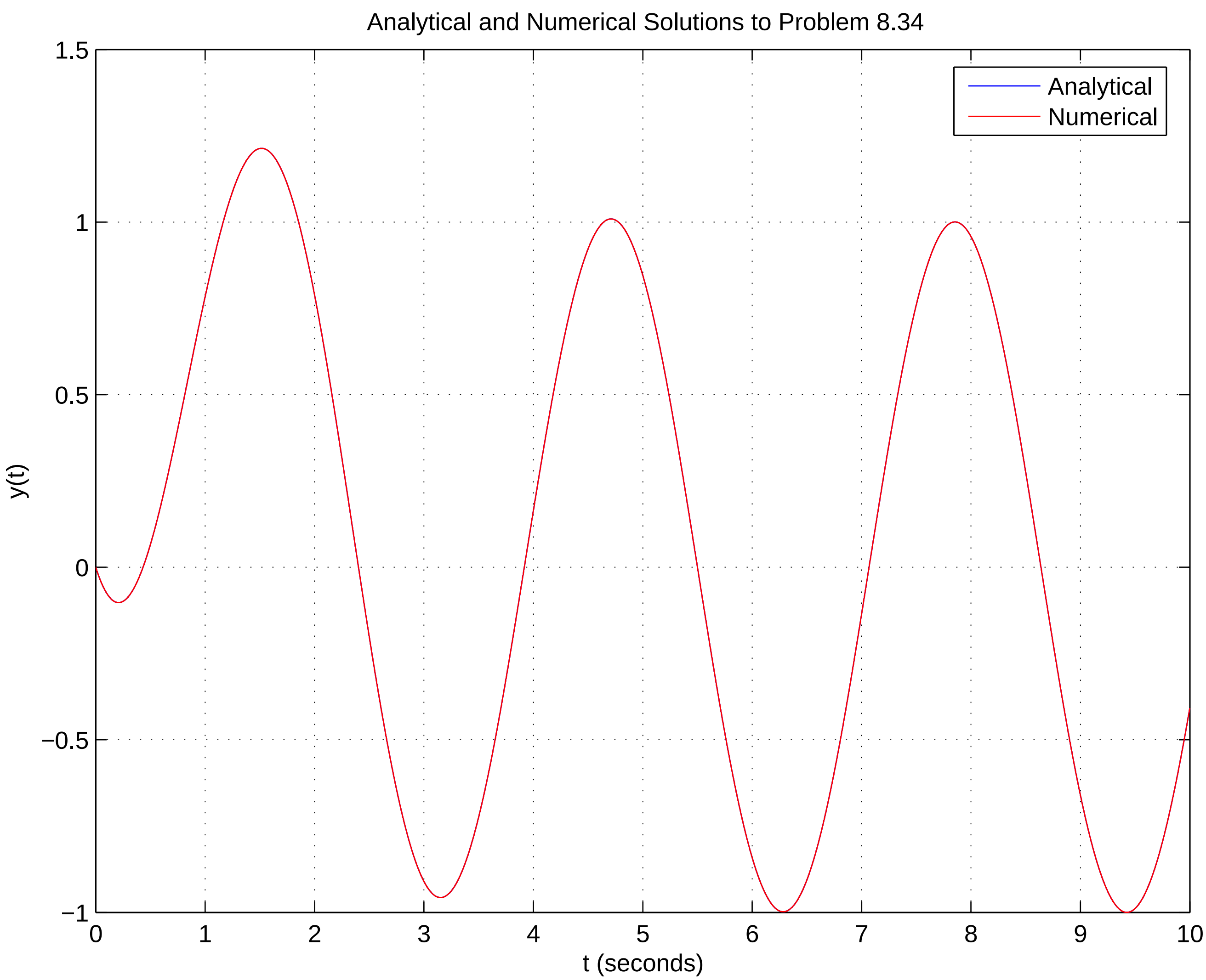
$\frac{k(28k-1045)}{4k-95} > 0 \Rightarrow k > \frac{1045}{28} = 37.32$

$10k > 0 \Rightarrow k > 0$

Stable if $k > 37.32$

```
% Problem 8.34 (c)

clf
t = 0:0.001:10;
y = exp(−t)−cos(2*t);
plot(t,y);
hold on
x = (5/7)*exp(−t) − (12/7)*exp(−8*t);
b = [1 8];
a = [1 0 4];
yn = lsim(b,a,x,t);
plot(t,yn,'r');
legend('Analytical','Numerical')
grid
xlabel('t (seconds)');
ylabel('y(t)')
title('Analytical and Numerical Solutions to Problem 8.34');
print −dpsc2 p8_34.ps
```


Analytical and Numerical Solutions to Problem 8.34
Analytical
Numerical
1.5
1
0.5
0
−0.5
−1
y(t)
0
1
2
3
4
5
6
7
8
9
10
t (seconds)