EE 342
HW #5

1. I forgot to include the value of k, so you could either leave k as a variable or let k = 1

In general, $|H(0)| = k \dfrac{\prod_{i=1}^{M} |z_i|}{\prod_{i=1}^{N} |p_i|}$

(a) $H(0) = 0$, because there is a zero at 0

Band pass filter - blocks 0, blocks ∞ $(N > M)$

(b) $H(0) = 0$, because there is a zero at 0

High pass filter - blocks 0, passes ∞ $(N = M)$

(c) $H(0) = \dfrac{k}{|-2+j2|\,|-2-j2|} = \dfrac{k}{(2\sqrt{2})(2\sqrt{2})} = \dfrac{k}{8}$

Low pass filter - passes 0, blocks ∞ $(N > M)$

(d) 6 poles at radius 20

$H(0) = \dfrac{k}{20^6} = 1.5625 \times 10^{-8}$ s

Low pass - passes 0, blocks ∞ $(N > M)$

2. Problem 9.16

(i) $$H(s) = \frac{242.5\,(s+8)}{(s+2)\,[(s+4)^2+81]\,(s+10)}$$

(a) $p_i = -2,\ -4 \pm j9,\ -10$ $\qquad z_i = -8$

(b) $$G(s) = H(s)\,\frac{1}{s} = \frac{242.5\,(s+8)}{(s+2)\,[(s+4)^2+81]\,(s+10)\,s}$$

$$g(t) = r_1 e^{-2t} + 2|r_2|\,e^{-4t}\cos(9t + \measuredangle r_2) + r_4 e^{-10t} + r_5$$

(c) $$g(\infty) = \lim_{s\to 0} s\,G(s) = \lim_{s\to 0} H(s) = \frac{242.5\,(8)}{(2)(4^2+81)(10)} = 1$$

(d) $p = -2$ is the dominant pole

(e) See MATLAB

(ii) $$H(s) = \frac{115.5\,(s+8)(s+2.1)}{(s+2)\,[(s+4)^2+81]\,(s+10)}$$

(a) $p_i = -2,\ -4 \pm j9,\ -10$ $\qquad z_i = -8,\ -2.1$

(b) Same as (i)

(c) $$g(\infty) = \lim_{s\to 0} H(s) = \frac{115.5\,(8)(2.1)}{(2)(4^2+81)(10)} = 1$$

(d) Same as (i)

(e) See MATLAB

3. Problem 9.21

$$H(s) = \frac{s^2+16}{s^2+7s+12} \qquad x(t) = 2\cos(4t)\,u(t) \qquad X(s) = \frac{2s}{s^2+16}$$

$$Y(s) = H(s)X(s) = \frac{2s}{s^2+7s+12} = \frac{2s}{(s+4)(s+3)} = \frac{8}{s+4} - \frac{6}{s+3}$$

$$y(t) = 8e^{-4t} - 6e^{-3t}, \quad t \geq 0$$

$y_{ss}(t) = 0$ (zeros $\pm j4$ block input $\cos(4t)$)

$$y_{tr}(t) = 8e^{-4t} - 6e^{-3t}$$

4. Problem 9.22

$$H(s) = \frac{s^2+1}{(s+1)(s^2+2s+17)}$$

(a) $x(t) = u(t)$ $\quad X(s) = \frac{1}{s}$

$$Y(s) = \frac{s^2+1}{(s+1)(s^2+2s+17)s} = \frac{1/17}{s} - \frac{1/8}{s+1} + \frac{\frac{9-j32}{272}}{s-4j+1} + \frac{\frac{9+j32}{272}}{s+4j+1}$$

$$y(t) = \frac{1}{17} + \frac{1}{8}e^{-t} + \frac{\sqrt{1105}}{136} e^{-t}\cos\left(4t + \tan^{-1}\left(\frac{-32}{9}\right)\right)$$

$$y(t) = 0.059 + 0.125e^{-t} + 0.244\,e^{-t}\cos(4t - 74^\circ)$$

$$y_{ss}(t) = 0.059$$

$$y_{tr}(t) = 0.125e^{-t} + 0.244\,e^{-t}\cos(4t - 74^\circ)$$

(b) $x(t) = \cos(4t)\,u(t) \qquad X(s) = \dfrac{s}{s^2+1}$

$$Y(s) = \frac{s}{(s+1)(s^2+2s+17)} = \frac{-1/16}{s+1} + \frac{\frac{1-j4}{32}}{s-4j+1} + \frac{\frac{1+j4}{32}}{s+4j+1}$$

$$y(t) = -\tfrac{1}{16} e^{-t} + \tfrac{1}{16}\sqrt{17}\, e^{-t} \cos\left(4t + \tan^{-1}\left(\tfrac{-4}{1}\right)\right)$$

$$y(t) = -0.0625 e^{-t} - 0.258\, e^{-t} \cos(4t - 76°)$$

$$y_{tr}(t) = -0.0625 e^{-t} - 0.258\, e^{-t} \cos(4t - 76°)$$

$$y_{ss}(t) = 0$$

(c) $x(t) = \cos(4t) \qquad X(s) = \dfrac{s}{s^2+16}$

$$Y(s) = \frac{(s^2+1)s}{(s+1)(s^2+2s+17)(s^2+16)}$$

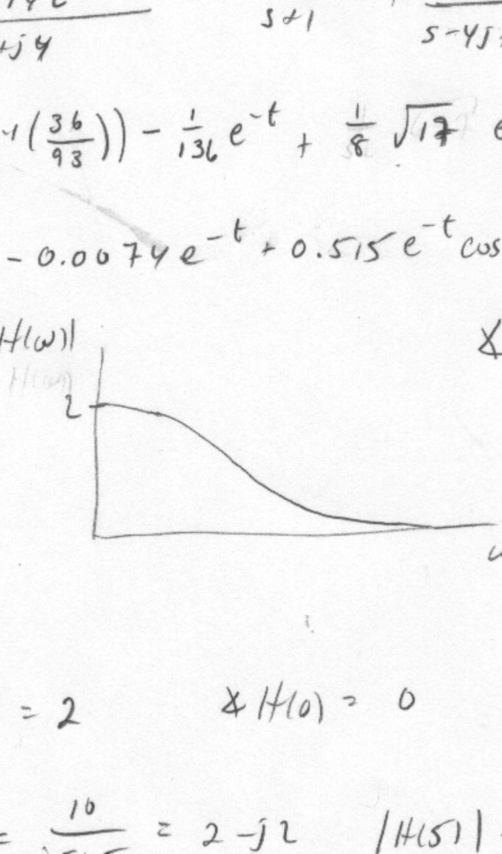
$$= \frac{\frac{93+j36}{442}}{s-j4} + \frac{\frac{93-j36}{442}}{s+j4} - \frac{1/136}{s+1} + \frac{\frac{-43-j32}{208}}{s-4j+1} + \frac{\frac{-43+j32}{208}}{s+4j+1}$$

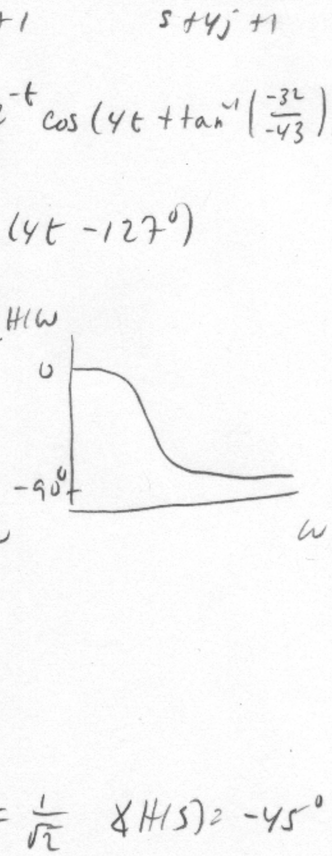
$$y(t) = \tfrac{3}{221}\sqrt{1105}\cos\left(4t + \tan^{-1}\left(\tfrac{36}{93}\right)\right) - \tfrac{1}{136} e^{-t} + \tfrac{1}{8}\sqrt{17}\, e^{-t}\cos\left(4t + \tan^{-1}\left(\tfrac{-32}{-43}\right)\right)$$

$$= 0.451\cos(4t + 21°) - 0.0074 e^{-t} + 0.515 e^{-t}\cos(4t - 127°)$$

5. Problem 9.37

(a) $H(s) = \dfrac{10}{s+5}$





$H(\omega) = \dfrac{10}{j\omega+5}$

$H(0) = \dfrac{10}{5} = 2 \qquad |H(0)| = 2 \qquad \measuredangle H(0) = 0$

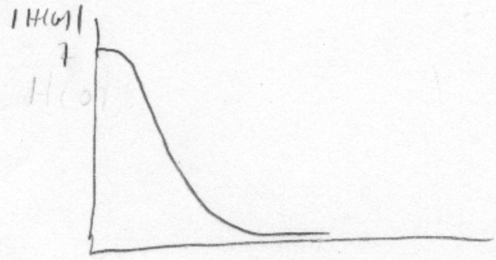
$\omega_{3dB} = 5$ rad/s $\qquad H(5) = \dfrac{10}{j5+5} = 2 - j2 \qquad |H(5)| = \dfrac{1}{\sqrt{2}} \qquad \measuredangle H(5) = -45°$

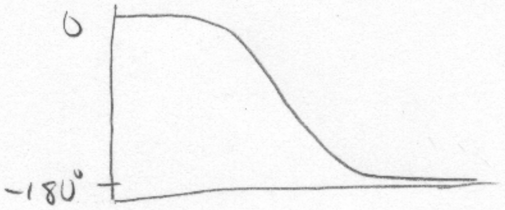
$H(\infty) = 0 \qquad |H(\infty)| = 0 \qquad \measuredangle H(\infty) = -90°$ (1 pole)

$H(\omega)$ is maximum at $\omega = 0$, so $\omega_p = 0$

$|H(0)| = 1$, $\quad \angle H(0) = 0$

(d) $H(s) = \dfrac{4}{(s+2)^2} \qquad H(\omega) = \dfrac{4}{(j\omega+2)^2} = \dfrac{4}{(4-\omega^2) + j4\omega}$





$H(0) = \dfrac{4}{4} = 1 \qquad |H(0)| = 1 \qquad \angle H(0) = 0$

ω_{3dB}? $\quad H(\omega_{3dB}) = \dfrac{1}{\sqrt{2}} = \left| \dfrac{4}{(4-\omega^2) + j4\omega} \right|$

$\dfrac{1}{2} = \dfrac{4}{(4-\omega^2) + j4\omega} \quad \dfrac{4}{(4-\omega^2) - j4\omega} = \dfrac{16}{\omega^4 + 8\omega^2 + 16}$

$\omega^4 + 8\omega^2 + 16 = 32 \Rightarrow \omega^4 + 8\omega^2 - 16 = 0 \Rightarrow \omega_{3dB} = 1.287$ rad/s

$H(\omega_{3dB}) = 0.293 - j0.644 = \dfrac{1}{\sqrt{2}} e^{-j1.144}$

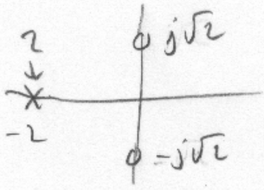
$|H(\omega_{3dB})| = \dfrac{1}{\sqrt{2}} \qquad \angle H(\omega_{3dB}) = -1.144 \text{ rad} = -65.5°$

$H(\infty) = \dfrac{4}{\infty} = 0 \qquad |H(\infty)| = 0 \qquad \angle H(\infty) = -180°$ (2 poles)

$H(\omega)$ is maximum at $\omega = 0$, so $\omega_p = 0$

$H(\omega_p) = 1 \qquad |H(\omega_p)| = 1 \qquad \angle H(\omega_p) = 0$

(f) $H(s) = \dfrac{s^2+2}{(s+2)^2}$

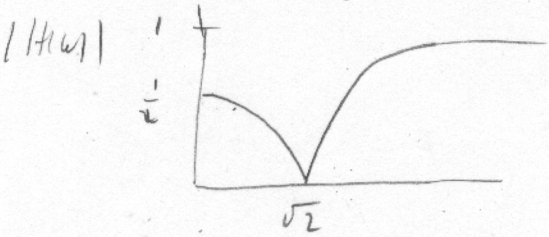


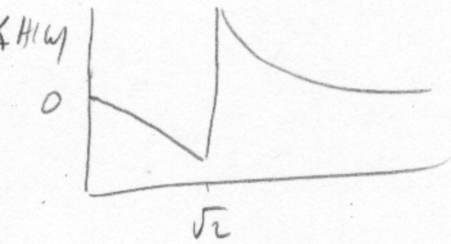
$$H(\omega) = \frac{(j\omega)^2+2}{(j\omega+2)^2} = \frac{2-\omega^2}{(4-\omega^2)+j4\omega}$$

$$H(0) = \frac{2}{4} = \frac{1}{2} \qquad |H(0)| = \frac{1}{2} \qquad \measuredangle H(0) = 0$$

$$H(\infty) = \frac{-\infty^2}{-\infty^2} = 1 \qquad |H(\infty)| = 1 \qquad \measuredangle H(\infty) = 0$$

Zeros at $\pm j\sqrt{2} \Rightarrow H(\sqrt{2}) = 0$





$H(\omega)$ is maximum at $\omega = \infty$, so $\omega_p = \infty$

$$|H(\omega_p)| = 1, \qquad \measuredangle H(\omega_p) = 0$$

ω_{3dB}: $\quad |H(\omega)| = \dfrac{1}{\sqrt{2}} \qquad H(\omega)\,H^*(\omega) = \dfrac{1}{2}$

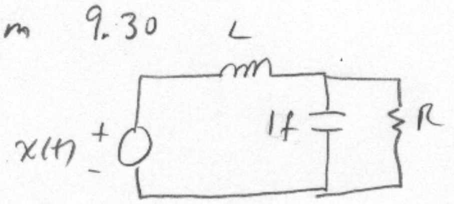
$$\left(\frac{2-\omega^2}{(4-\omega^2)+j4\omega}\right)\left(\frac{2-\omega^2}{(4-\omega^2)-j4\omega}\right) = \frac{\omega^4-4\omega^2+4}{\omega^4+8\omega^2+16} = \frac{1}{2}$$

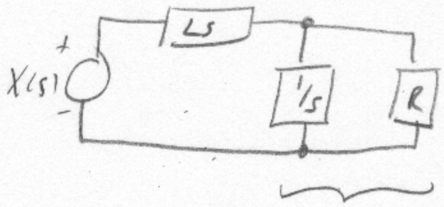
$$\omega^4 - 16\omega^2 - 8 = 0 \Rightarrow \omega_{3dB} = 4.06 \text{ rad/s}$$

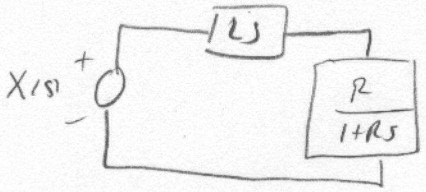
$$H(\omega_{3dB}) = 0.431 + j0.561 = \frac{1}{\sqrt{2}}e^{j0.915}$$

$$|H(\omega_{3dB})| = \frac{1}{\sqrt{2}} \qquad \measuredangle H(\omega_{3dB}) = 52^\circ$$

6. Problem 9.30







$$\frac{R/s}{R + 1/s} = \frac{R}{Rs+1}$$

$$H(s) = \frac{\frac{R}{Rs+1}}{\frac{R}{Rs+1} + Ls} = \frac{R}{LRs^2 + Ls + R} = \frac{1/L}{s^2 + \frac{s}{R} + \frac{1}{L}}$$

$$H(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (\text{eqn } 9.16) \Rightarrow \frac{1}{L} = \omega_n^2 \quad 2\zeta\omega_n = \frac{1}{R}$$

Want $\zeta = 1$, so $H(s) = \dfrac{K}{s^2 + 2\omega_n s + \omega_n^2} = \dfrac{K}{(s+\omega_n)^2}$

$$H(\omega) = \frac{K}{(\omega_n - j\omega)^2} \qquad |H(\omega)| = \frac{K}{\omega_n^2 + \omega^2}$$

$$|H(0)| = \frac{K}{\omega_n^2} \qquad \omega_{3dB} = 20 \Rightarrow |H(20)| = \frac{1}{\sqrt{2}}|H(0)| = \frac{K}{\sqrt{2}\,\omega_n^2}$$

$$\frac{K}{\sqrt{2}\,\omega_n^2} = \frac{K}{\omega_n^2 + 20^2} \Rightarrow \sqrt{2}\,\omega_n^2 = \omega_n^2 + 20^2$$

$$\Rightarrow (\sqrt{2} - 1)\,\omega_n^2 = 20^2 \Rightarrow \omega_n = 31.1 \text{ rad/s}$$

$$L = \frac{1}{\omega_n^2} = 1.036 \text{ mH}$$

$$R = \frac{1}{2\zeta\omega_n} = \frac{1}{2\omega_n} = 16.1 \text{ m}\Omega$$

7. Problem 9.35

(a) $H(s) = \dfrac{10(s+4)}{(s+1)(s+10)}$ $\quad H(0) = \dfrac{10(4)}{(1)(10)} = 4$ $\quad H(0)_{dB} = 20\log_{10}(4) = 12\text{ dB}$

Magnitude: Changes slope at 1 (pole), 4 (zero), 10 (pole)

<1	1-4	4-10	>10
Flat @ 12 dB	-20 dB/decade pole at 1	-20 dB/decade pole @ 1 +20 dB/decade zero at 4	-20 dB/decade pole @ 1 +20 dB/decade zero @ 4 -20 dB/decade pole @ 10
12 dB	-20 dB/decade Falls to 0dB	0 dB/decade Flat @ 0 dB	-20 dB/decade Falls from 0dB

Phase: Starts @ 0°, Falls @ -45°/decade from 0.1 to 10
Rises @ 45°/decade from 0.4 to 40
Falls @ -45°/decade from 1 to 100

<0.1	0.1-0.4	0.4-1	1-10	10-40	40-100	>100
Flat @ 0	Falls @ -45°/decade To -27°	Flat @ -27°	Falls @ -45°/d Falls to -72°	Flat @ -72°	Falls @ -45°/d Falls to -90°	Flat @ -90°

See MATLAB for plots

(d) $H(s) = \frac{10}{(s+1)(s^2+4s+16)}$ $H(0) = \frac{10}{(1)(16)} = \frac{10}{16}$ $H(0)_{dB} = -4\,dB$

Magnitude: Pole @ -1: flat to 1 rad/sec; falls at -20 dB/dec after

cc poles at $-2 \pm j3.46$ (radius = 4)

flat to 4 rad/sec, falls at -40 dB/dec after

<1	1 – 4	>4
Flat @ -4 dB	Falls @ -20 dB/dec From -4 dB to -16 dB	Falls @ -60 dB/decade

Phase: starts @ 0, Falls @ -45°/dec from 0.1 to 10

Falls @ -90°/dec from 0.4 to 40

Flat for ω > 40

<0.1	0.1 – 0.4	0.4 – 10	10 – 40	>40
Flat @ 0°	Falls @ -45°/dec To -27°	Falls @ 135°/dec To -215°	Falls @ -90°/dec To -270°	Flat @ -270°

See MATLAB for plots

(f) $H(s) = \frac{1000(s+1)}{(s+20)^2}$ $\quad H(0) = \frac{1000(1)}{(20)^2} = 2.5$ $\quad H(0)_{dB} = -8$ dB

Magnitude: Zero @ −1: flat to 1 rad/sec; rises at +20 dB/dec after

Poles @ −20: flat to 20 rad/sec; falls at −40 dB/dec after

<1	1−20	>20
Flat @ −8 dB	Rises @ +20 dB/dec Rises to +34 dB	Falls @ −20 dB/decade

Phase: Starts @ 0; Rises @ +45°/dec from 0.1 to 10

Falls @ −90°/dec from 2 to 200

<0.1	0.1 − 2	2 − 10	10 − 200	>200
Flat @ 0	Rises @ +45°/dec to +59°	Falls @ −45°/dec to +28°	Falls @ −90°/dec to −90°	Flat @ −90°

8) Problem 9.36

$$H(\omega) = \frac{-\omega^2 + j3\omega}{8 + j12\omega - 4\omega^2} = \frac{(j\omega)^2 + 3(j\omega)}{8 + 12(j\omega) + 4(j\omega)^2}$$

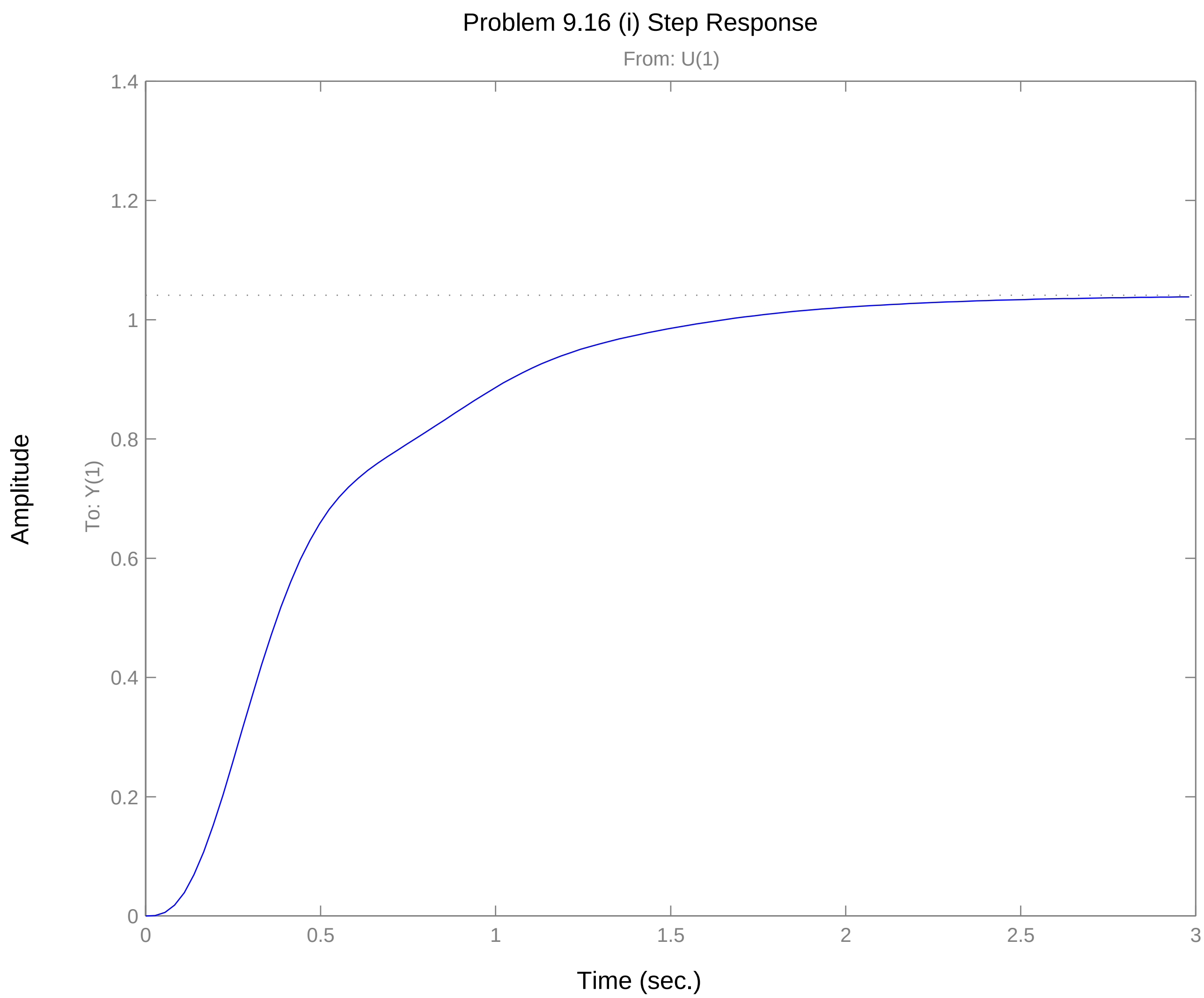
$H(\omega) = H(s)\big|_{s=j\omega}$ To find $H(s)$, replace $j\omega$ with s!

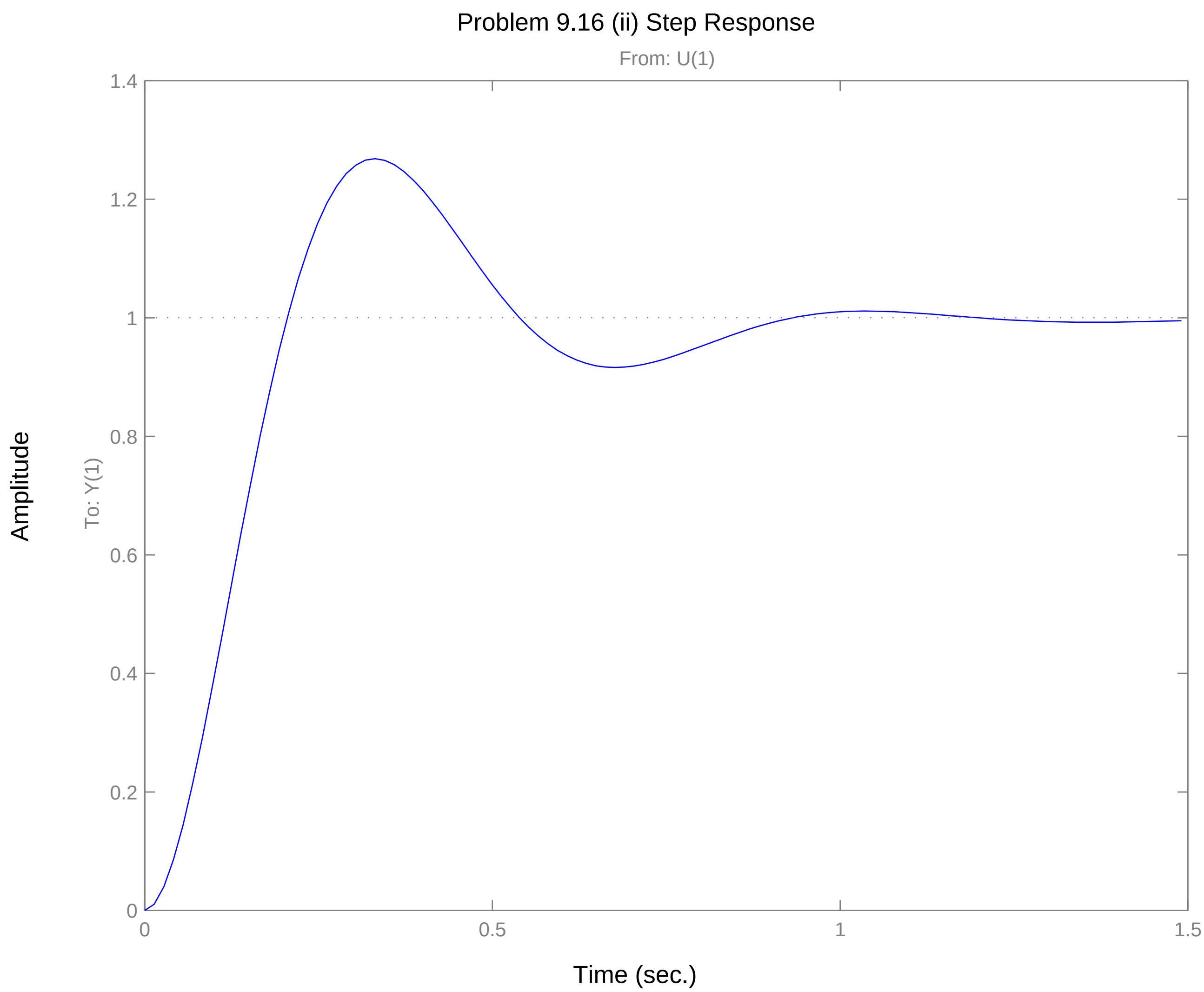
$$H(s) = \frac{s^2 + 3s}{8 + 12s + 4s^2} = \frac{\frac{1}{4}s(s+3)}{s^2 + 3s + 2}$$

```
% EE 342 HW    #5

% Problem 9.16

% (i)

figure(1)
clf
b = 252.5*[1 8];
a = conv([1 2],conv([1 8 16+81],[1 10]));
step(b,a)
title('Problem 9.16 (i) Step Response');
print -dpsc2 p9_16_i.ps


% (ii)

figure(2)
clf
b = 115.5*conv([1 8],[1 2.1]);
a = conv([1 2],conv([1 8 16+81],[1 10]));
step(b,a)
title('Problem 9.16 (ii) Step Response');
print -dpsc2 p9_16_ii.ps

% Problem 9.35

% (b)
figure(3)
clf

% Asymptotic:

wa = [0.01 1 4 10 1000];
Ha_mag = [12 12 0 0 -40];
subplot(211)
semilogx(wa,Ha_mag);
hold on

% Exact

b = 10*[1 4];
a = conv([1 1],[1 10]);
w = logspace(-2,3,1000);
Hex = freqs(b,a,w);
semilogx(w,20*log10(abs(Hex)),'r');
grid
legend('Asymptotic','Exact')

% Asymptotic:

wa = [0.01 0.1 0.4 1 10 40 100 1000];
Ha_phase = [0 0 -27 -27 -72 -72 -90 -90];
subplot(212)
semilogx(wa,Ha_phase);
hold on

% Exact

semilogx(w,unwrap(angle(Hex))*180/pi,'r');
grid
legend('Asymptotic','Exact')
print -dpsc2 p9_35_b.ps

% (d)
figure(4)
clf

% Asymptotic:
```

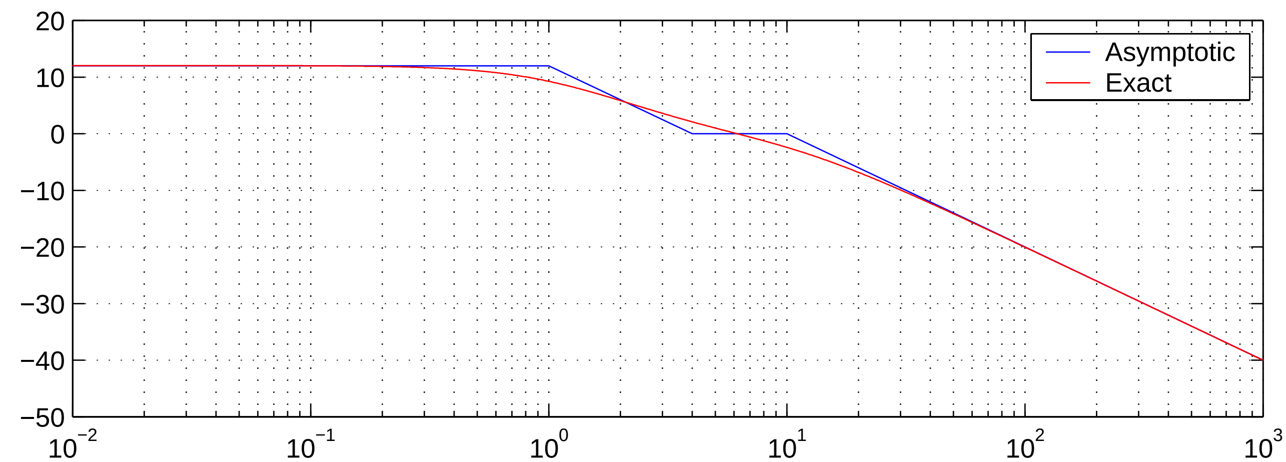


```
wa = [0.01 1 4 1000];
Ha_mag = [-4 -4 -16 -160];
subplot(211)
semilogx(wa,Ha_mag);
hold on

% Exact

b = 10;
a = conv([1 1],[1 4 16]);
w = logspace(-2,3,1000);
Hex = freqs(b,a,w);
semilogx(w,20*log10(abs(Hex)),'r');
grid
legend('Asymptotic','Exact')

% Asymptotic:

wa = [0.01 0.1 0.4 10 40 1000];
Ha_phase = [0 0 -27 -215 -270 -270];
subplot(212)
semilogx(wa,Ha_phase);
hold on

% Exact

semilogx(w,unwrap(angle(Hex))*180/pi,'r');
grid
legend('Asymptotic','Exact')
print -dpsc2 p9_35_d.ps

% (f)
figure(5)
clf

% Asymptotic:

wa = [0.01 1 20 1000];
Ha_mag = [8 8 34 0];
subplot(211)
semilogx(wa,Ha_mag);
hold on

% Exact

b = 1000*[1 1];
a = conv([1 20],[1 20]);
w = logspace(-2,3,1000);
Hex = freqs(b,a,w);
semilogx(w,20*log10(abs(Hex)),'r');
grid
legend('Asymptotic','Exact')

% Asymptotic:

wa = [0.01 0.1 2 10 200 1000];
Ha_phase = [0 0 59 28 -90 -90];
subplot(212)
semilogx(wa,Ha_phase);
hold on

% Exact

semilogx(w,unwrap(angle(Hex))*180/pi,'r');
grid
legend('Asymptotic','Exact')
print -dpsc2 p9_35_f.ps
```

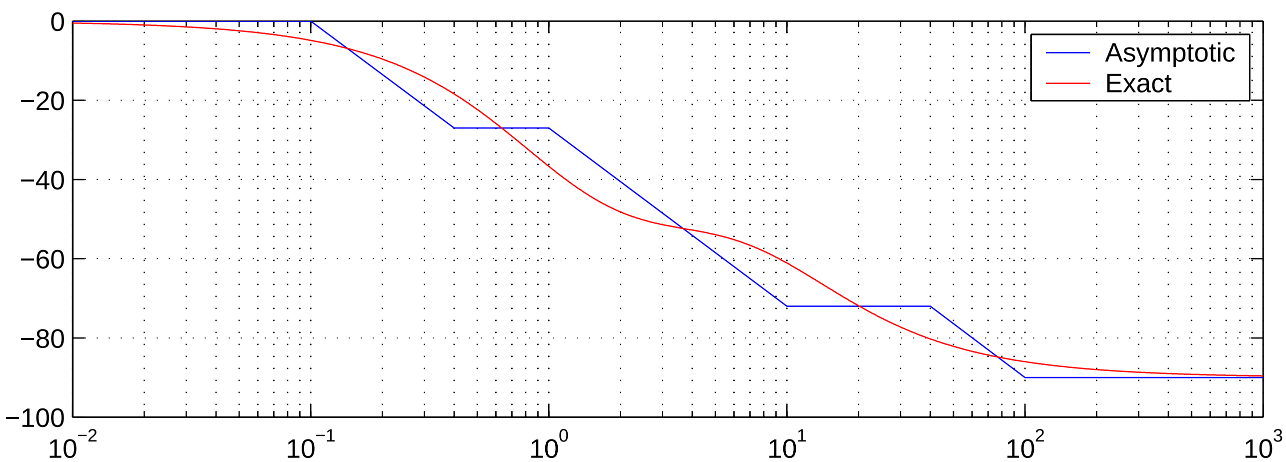

Problem 9.16 (i) Step Response
From: U(1)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
To: Y(1)
Amplitude
0
0.5
1
1.5
2
2.5
3
Time (sec.)



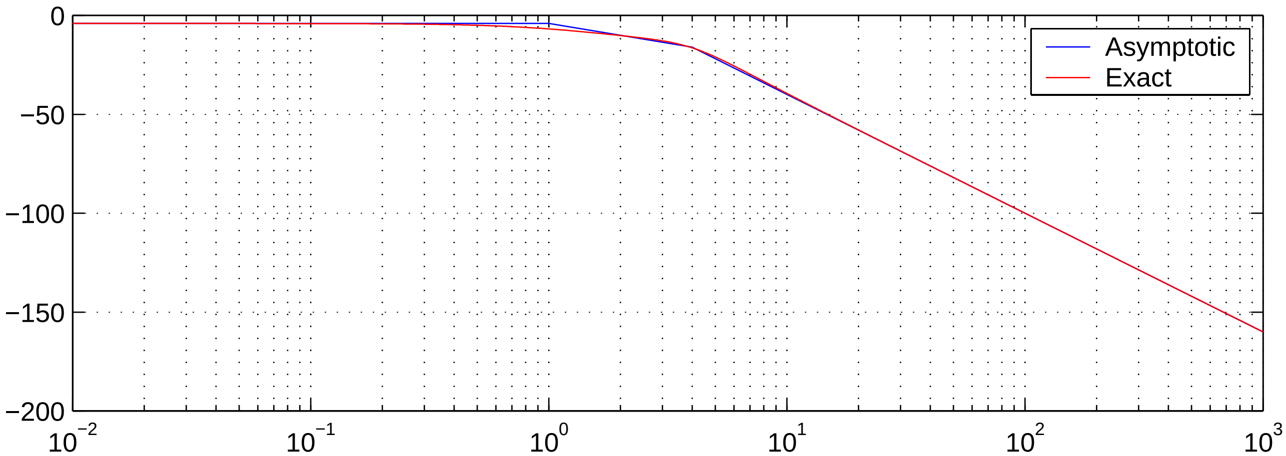
Problem 9.16 (ii) Step Response
From: U(1)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Amplitude
To: Y(1)
0
0.5
1
1.5
Time (sec.)



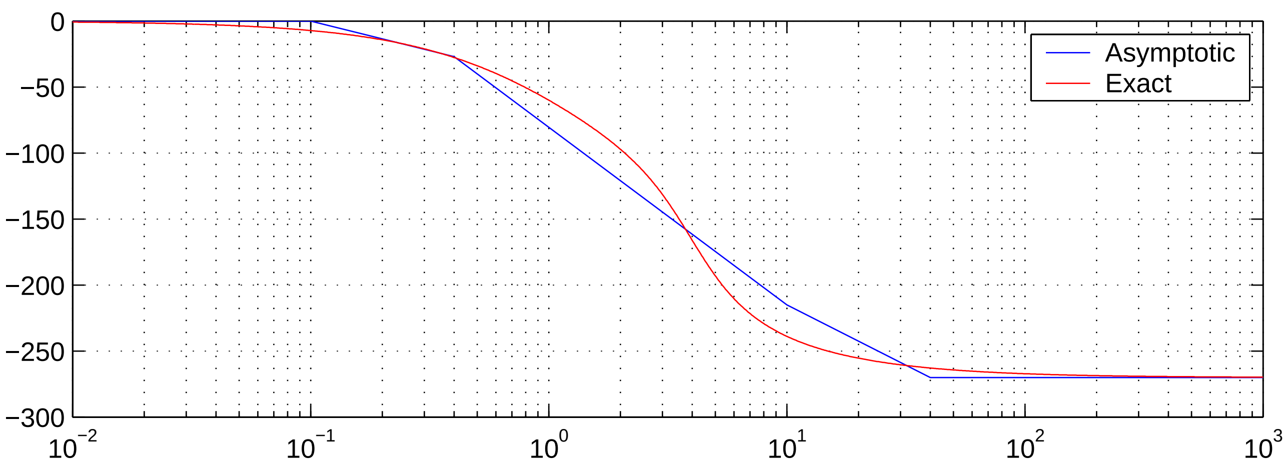
20
10
0
−10
−20
−30
−40
−50
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Asymptotic
Exact



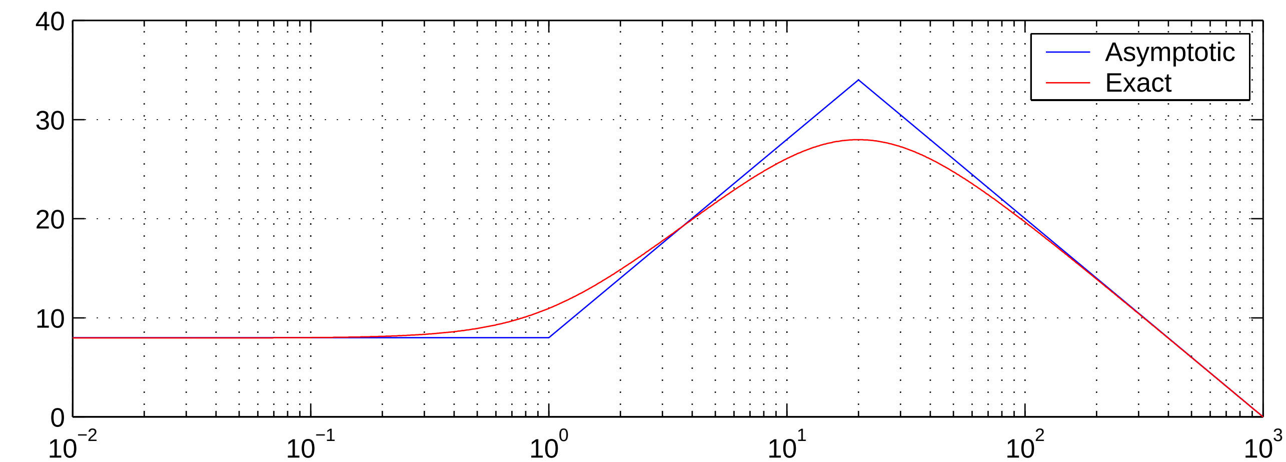
0
−20
−40
−60
−80
−100
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Asymptotic
Exact



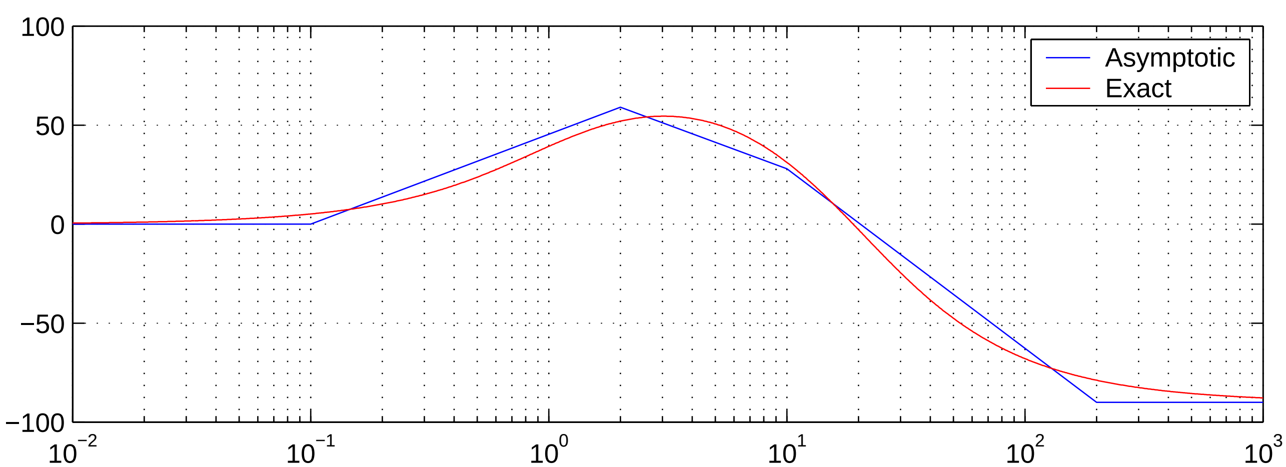
0
−50
−100
−150
−200
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Asymptotic
Exact



0
−50
−100
−150
−200
−250
−300
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Asymptotic
Exact



40
30
20
10
0
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Asymptotic
Exact



100
50
0
−50
−100
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Asymptotic
Exact