

EE 342 HW #6

1. Problem 9.37.

$$(a) \quad H(s) = \frac{\omega_c^3}{s^3 + 2\omega_c s^2 + 2\omega_c^2 s + \omega_c^3} = \frac{\omega_c^3}{(s + \omega_c)(s^2 + \omega_c s + \omega_c^2)}$$

$$= \frac{\omega_c}{s + \omega_c} + \frac{-\omega_c/2 - j\omega_c\sqrt{3}/6}{s + \frac{\omega_c}{2} - j\omega_c\frac{\sqrt{3}}{2}} + \frac{-\omega_c/2 + j\omega_c\sqrt{3}/6}{s + \frac{\omega_c}{2} + j\omega_c\frac{\sqrt{3}}{2}}$$

$$h(t) = \omega_c e^{-\omega_c t} + 2 \left| \frac{-\omega_c/2 + j\omega_c\sqrt{3}/6}{s + \frac{\omega_c}{2} + j\omega_c\frac{\sqrt{3}}{2}} \right| e^{-\omega_c t/2} \cos\left(\omega_c \frac{\sqrt{3}}{2} t + \angle(-\omega_c/2 + j\omega_c\sqrt{3}/6)\right)$$

$$h(t) = \omega_c e^{-\omega_c t} + \frac{2}{\sqrt{3}} \omega_c e^{-\frac{\omega_c}{2} t} \cos\left(\omega_c \frac{\sqrt{3}}{2} t - 150^\circ\right)$$

Plots see MATLAB

(b) $\tau \operatorname{sinc}\left(\frac{\tau t}{2\pi}\right) \leftrightarrow 2\pi P_{\tau}(w)$

$2 \operatorname{sinc}(t/\pi) \leftrightarrow 2\pi P_2(w)$

$h_i(t) = \frac{1}{\pi} \operatorname{sinc}(t/\pi)$ Ideal filter impulse response

See MATLAB for plots

Similarities - central peak with damped oscillations

Differences: $h_i(t)$ non-zero for $t < 0$

Peak of $h(t)$ does not occur at $t=0$ because filter is causal.

2. Problem 9.38

(a) $x(t) = u(t) - u(t-1)$

$y(t) = g(t) - g(t-1)$

$$G(s) = \frac{\omega_c^3}{(s + \omega_c)(s^2 + \omega_c s + \omega_c^2) s} = \frac{1}{s} - \frac{1}{s + \omega_c} + \frac{-j\sqrt{3}}{s + \frac{\omega_c}{2} - j\omega_c\frac{\sqrt{3}}{2}} + \frac{-j\sqrt{3}}{s + \frac{\omega_c}{2} + j\omega_c\frac{\sqrt{3}}{2}}$$

$$g(t) = \left[1 - e^{-\omega_c t} + \frac{2}{\sqrt{3}} e^{-\frac{\omega_c}{2} t} \cos\left(\frac{\sqrt{3}}{2} \omega_c t + 90^\circ\right) \right] u(t)$$

$$g(t-1) = \left[1 - e^{-\omega_c(t-1)} + \frac{2}{\sqrt{3}} e^{-\frac{\omega_c}{2}(t-1)} \cos\left(\frac{\sqrt{3}}{2} \omega_c (t-1) + 90^\circ\right) \right] u(t-1)$$

$y(t) = g(t) - g(t-1)$, where $\omega_c = 2\pi$

(b) $y(t) = g(t) - g(t-1)$ where $\omega_c = 4\pi$

(c) See MATLAB

(d) Higher cutoff frequency ($\omega_c = 4\pi$, $f_c = 2\text{Hz}$) passes the 1 second pulse better than the one with the lower cutoff ($\omega_c = 2\pi$, $f_c = 1\text{Hz}$)

3. Problem 9.45

(a) $H(s) = \frac{1}{(s+1)(s^2 + s + 1)}$

$H_{sb}(s) = HL \left(\frac{\omega_b s}{s^2 + \omega_b^2} \right)$

$\omega_b = \omega_2 - \omega_1 = 15 - 10 = 5 \text{ rad/sec}$

$\omega_0 = \sqrt{\omega_1 \omega_2} = \sqrt{150} = 5\sqrt{6}$

$H_{sb}(s) = HL \left(\frac{5s}{s^2 + 150} \right) = \frac{1}{\left(\frac{.5s}{s^2 + 150} + 1 \right) \left[\frac{25s^2}{(s^2 + 150)^2} + \frac{5s}{s^2 + 150s} + 1 \right]}$
 $= \frac{(s^2 + 150)^3}{(s^2 + 5s + 150)(s^4 + 5s^3 + 325s^2 + 750s + 22500)}$

See MATLAB for plots

(b) (i) At $\omega = 5$, $|H(\omega)| = 1$, & $\angle H(\omega) = -23^\circ$

$y_{ss}(t) = \sin(5t - 23^\circ)$

(ii) At $\omega = 12$, $|H(\omega)| = 0.01$, & $\angle H(\omega) = -259^\circ$

$y_{ss}(t) = 0.01 \sin(12t - 259^\circ)$

(iii) Sum of (i) and (ii)

$y_{ss}(t) = \sin(5t - 23^\circ) + 0.01 \sin(12t - 259^\circ)$

(c) See MATLAB

4. Problem 9.47

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^{2N}}}$$

Want $\omega_c = 10 \text{ rad/sec}$

Want $|H(\omega)| = 0.05$ for $\omega = 25 \text{ rad/sec}$

$$0.05 = \frac{1}{\sqrt{1 + (\frac{25}{10})^{2N}}} \Rightarrow N = \frac{1}{2} \frac{\log((\frac{1}{0.05})^2 - 1)}{\log(25/10)} = 3.26$$

Round up to nearest integer: $N = 4$

See MATLAB for filter design

(a) See MATLAB

(b) See MATLAB

(c) See MATLAB

(d) See MATLAB

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% Problem 9.37

% Impulse response of 3rd order Butterworth filter

figure(1)
t = -15:0.001:15;
wc = 1;
h = wc*exp(-wc*t)+(2/sqrt(3))*wc*exp(-wc*t/2).*cos(sqrt(3)*wc*t/2 - 150*pi/180);
h(t<0) = 0;
subplot(211)
plot(t,h)
grid
xlabel('t (seconds)');
ylabel('h(t)');
title('Problem 9.37');

% Impulse response of ideal filter
subplot(212)
hi = (1/pi)*sinc(t/pi);
plot(t,hi)
grid
xlabel('t (seconds)');
ylabel('h_i(t)');
print -dpasc2 p9_37.ps

% Problem 9.38

figure(2)
t = 0:0.001:4;
x = (t >= 0) - (t >= 1);
subplot(311)
plot(t,x)
axis([0 3 -1 2])
grid
xlabel('t (seconds)');
ylabel('x(t)');

%(a) wc = 2 pi
wc = 2*pi;
gt = (1 - exp(-wc*t) + (2/sqrt(3))*exp(-wc*t/2).*cos(sqrt(3)*wc*t/2+pi/2)).*(t >
= 0);
gtm1 = (1 - exp(-wc*(t-1)) + (2/sqrt(3))*exp(-wc*(t-1)/2).*cos(sqrt(3)*wc*(t-1)/
2+pi/2)).*(t >= 1);
y = gt - gtm1;
subplot(312)
plot(t,y);
grid
xlabel('t (seconds)');
ylabel('y_a(t)');
axis([0 3 -1 2])

%(b) wc = 4 pi
wc = 4*pi;
t = 0:0.001:4;
gt = (1 - exp(-wc*t) + (2/sqrt(3))*exp(-wc*t/2).*cos(sqrt(3)*wc*t/2+pi/2)).*(t >
= 0);
gtm1 = (1 - exp(-wc*(t-1)) + (2/sqrt(3))*exp(-wc*(t-1)/2).*cos(sqrt(3)*wc*(t-1)/
2+pi/2)).*(t >= 1);
y = gt - gtm1;
subplot(313)
plot(t,y);
axis([0 3 -1 2])

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grid
xlabel('t (seconds)');
ylabel('y_b(t)');
print -dpasc2 p9_38.ps

% Problem 9.45

% (a)
figure(3)
b = conv([1 0 150],conv([1 0 150],[1 0 150]));
a = conv([1 5 150],[1 5 325 750 22500]);
w = 0:0.01:25;
H = freqs(b,a,w);
subplot(211)
plot(w,abs(H))
axis([0 25 0 1.2])
grid
xlabel('\omega (rad/sec)');
ylabel('|H(\omega)|');
title('Problem 9.45 (a)')
subplot(212)
plot(w,unwrap(angle(H))*180/pi)
grid
xlabel('\omega (rad/sec)');
ylabel('\angle H(\omega)');
print -dpasc2 p9_45_a.ps

% (c)
figure(4)
clf
t=0:0.01:5;
x = sin(5*t) + sin(12*t);
y = lsim(b,a,x,t);
plot(t,y)
hold on
yss = sin(5*t - 23*pi/180) + 0.01*sin(12*t - 259*pi/180);
plot(t,yss,'r');
legend('y(t)', 'y_{ss}(t)')
grid
xlabel('t (seconds)');
ylabel('output')
title('Problem 9.45 (c)')
print -dpasc2 p9_45_c.ps

% Problem 9.47

N = 4;
[z,p,k] = buttap(N);
[b,a] = zp2tf(z,p,k);
[bl,al]=lp2lp(b,a,10);
w = 0:0.001:40;
H = freqs(bl,al,w);
figure(5)
subplot(211)
plot(w,abs(H));
grid
xlabel('\omega (rad/sec)');
ylabel('|H(\omega)|');
title('Problem 9.47')

subplot(212)
plot(w,unwrap(angle(H))*180/pi);

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grid
xlabel('\omega (rad/sec)');
ylabel('\angle H(\omega)');
print -dpsc2 p9_47_freq_resp.ps

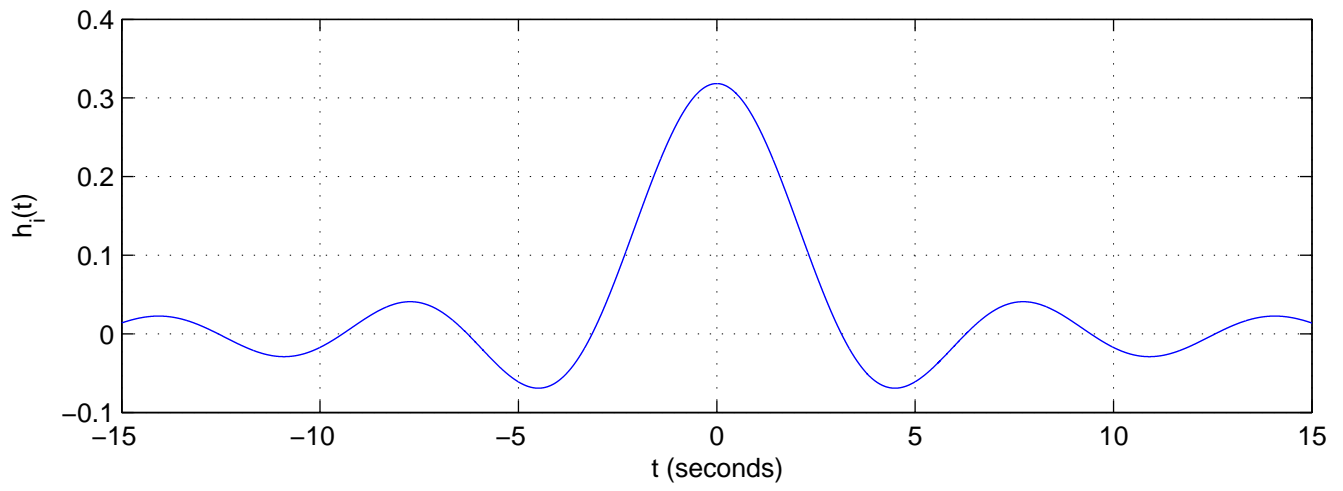
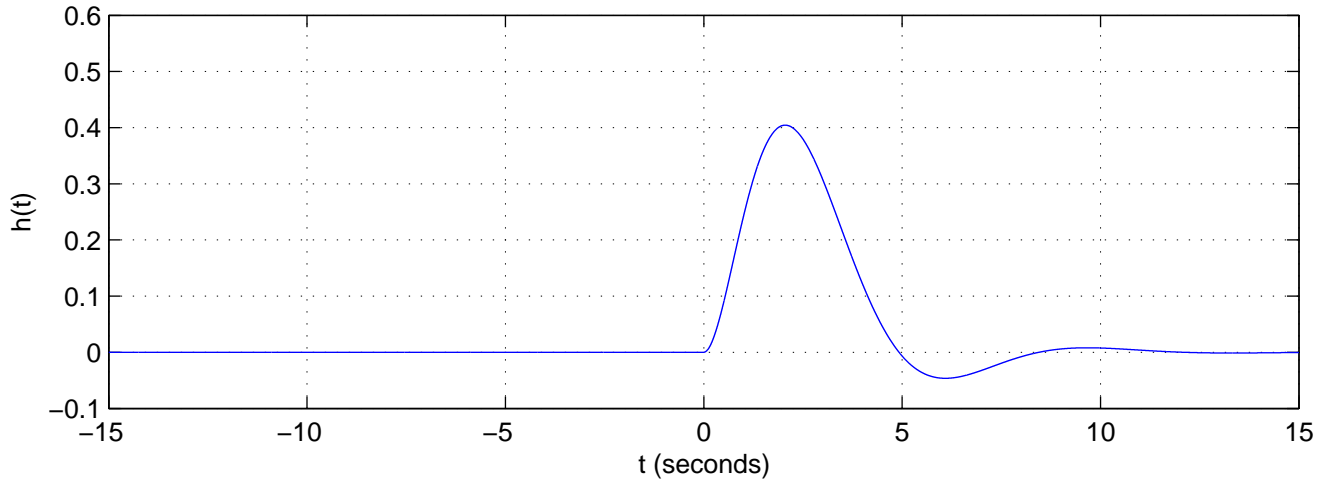
figure(6)
clf
% (a)
t = 0:0.01:5;
x = sin(5*t);
y = lsim(b1,a1,x,t);
subplot(411)
plot(t,x)
hold on
plot(t,y,'r')
grid
xlabel('t (seconds)');
ylabel('y_a(t)');
title('Problem 9.47');
legend('x(t)', 'y_a(t)')

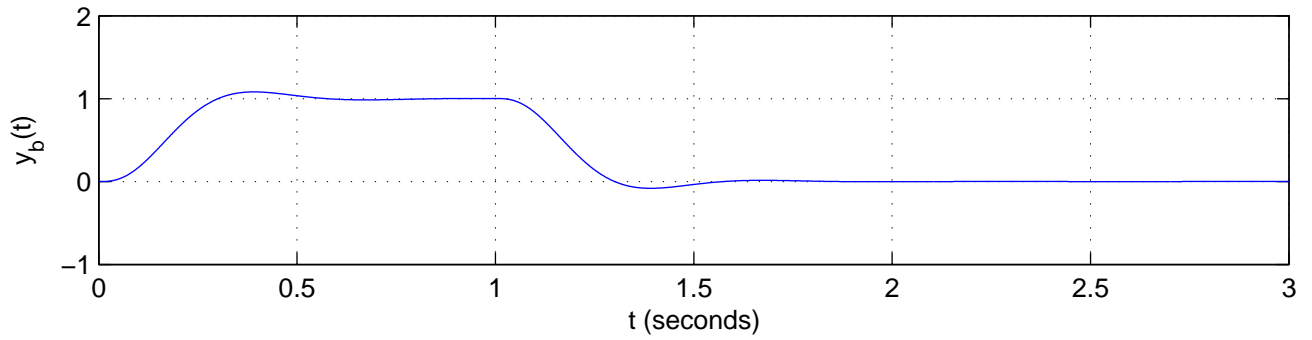
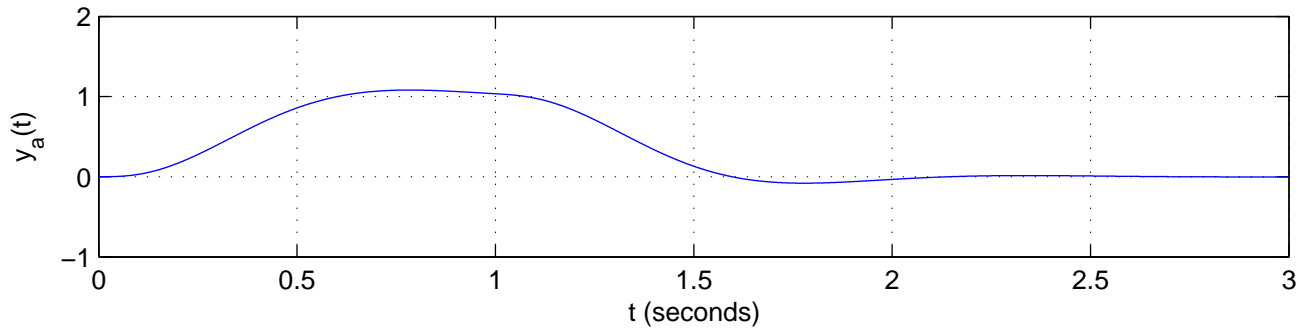
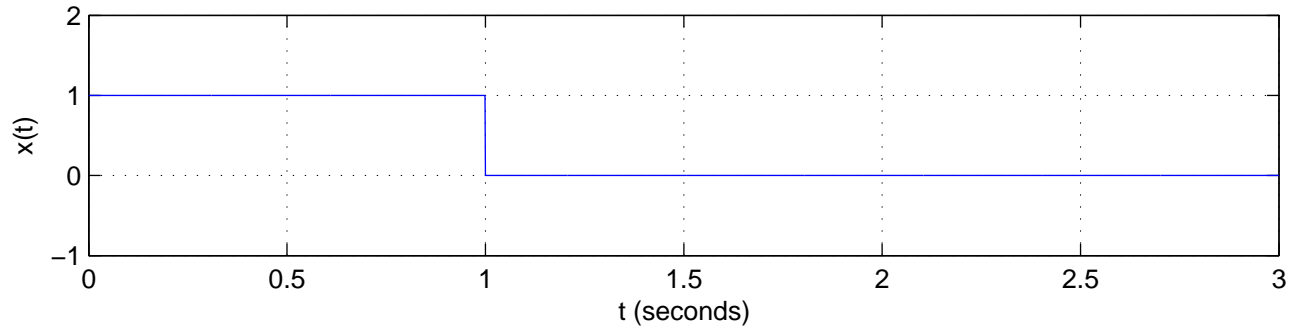
% (b)
x = sin(25*t);
y = lsim(b1,a1,x,t);
subplot(412)
plot(t,x)
hold on
plot(t,y,'r')
plot(t,y)
grid
xlabel('t (seconds)');
ylabel('y_b(t)');
legend('x(t)', 'y_b(t)')

% (c)
x = sin(5*t) + sin(25*t);
y = lsim(b1,a1,x,t);
subplot(413)
plot(t,x)
hold on
plot(t,y,'r')
grid
xlabel('t (seconds)');
ylabel('y_c(t)');
legend('x(t)', 'y_c(t)')

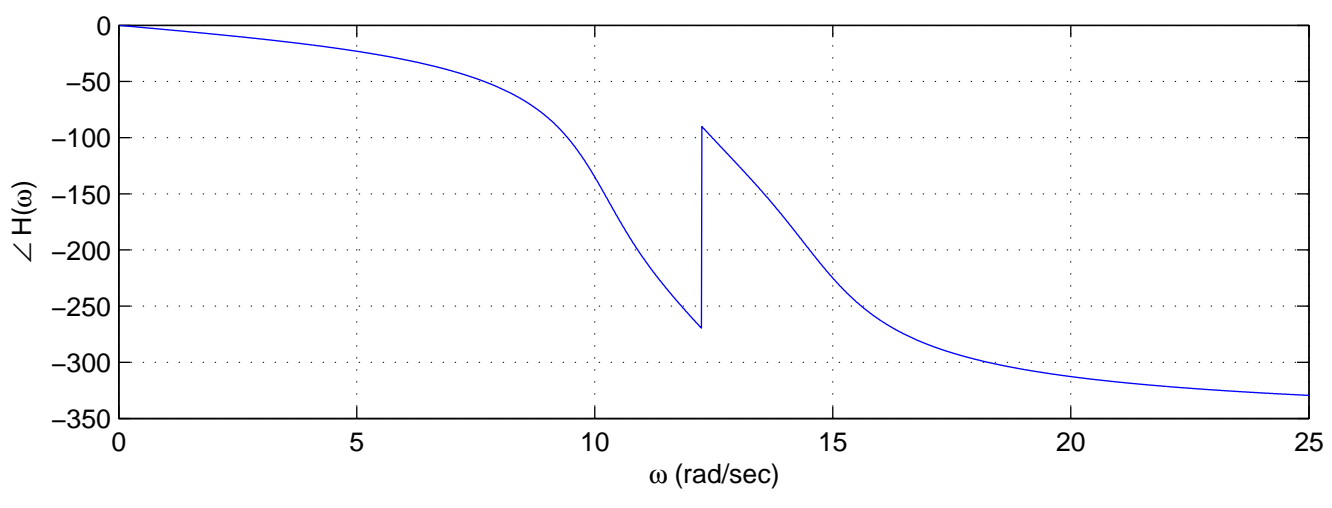
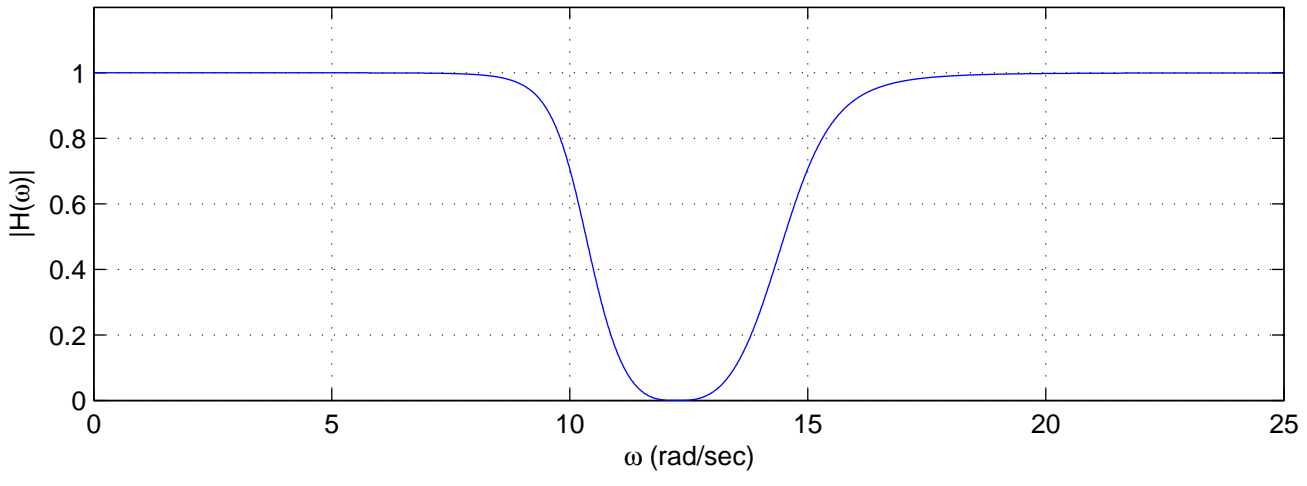
% (d)
t = 0:0.05:10;
x = rand(201,1);
y = lsim(b1,a1,x,t);
subplot(414)
plot(t,x)
hold on
plot(t,y,'r')
grid
xlabel('t (seconds)');
ylabel('y_d(t)');
legend('x(t)', 'y_d(t)')
print -dpsc2 p9_47_abcd.ps
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Problem 9.37

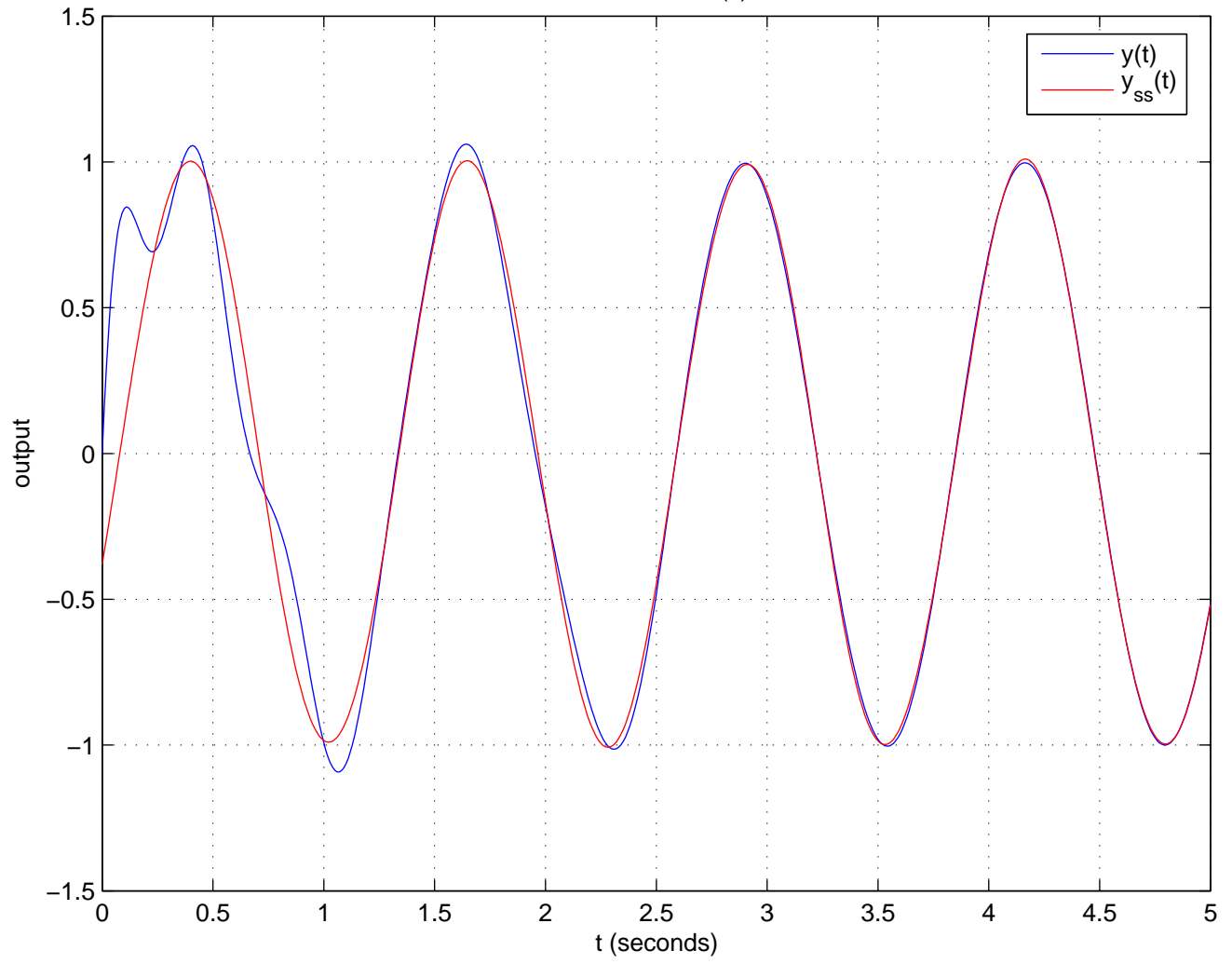




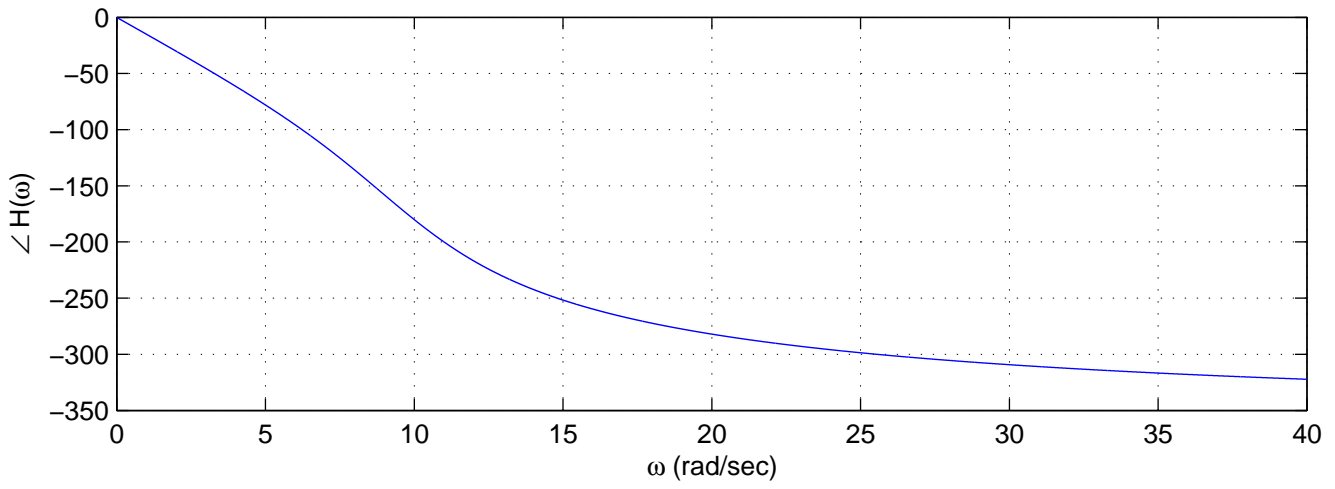
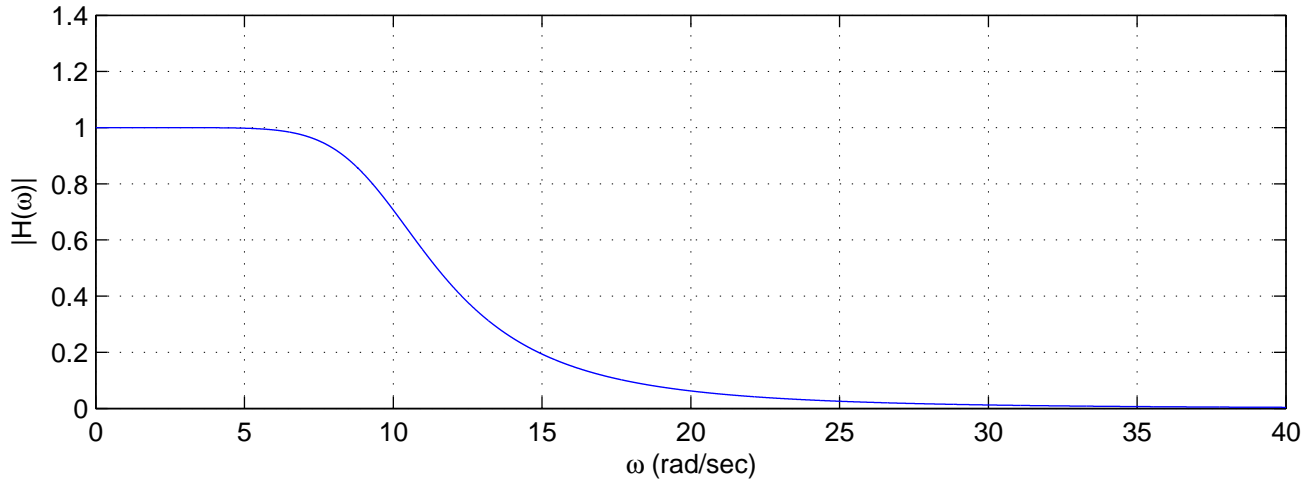
Problem 9.45 (a)



Problem 9.45 (c)



Problem 9.47



Problem 9.47

