EE 342 HW#7

1. (a), (b), (c), (d) See MATLAB

(e) Bessel filter has best time-domain response - no overshoot
Elliptic has worst - lots of ringing

2. Problem 10.1

(a) $G_p(s) = \dfrac{1}{s+0.1}$ Want $G_c(s) = \dfrac{K(s+0.1)}{(s+2)}$

$$Y(s) = G_c(s)G_p(s)R(s) = \frac{K(s+0.1)}{(s+2)} \frac{1}{(s+0.1)} \frac{r_0}{s} = \frac{r_0 K}{s(s+2)}$$

$$y(\infty) = sY(s)\Big|_{s=0} = \frac{r_0 K}{2}$$

To get $y(\infty) = r_0$, need $K=2$

$$G_c(s) = \frac{2(s+0.1)}{s+2}$$

(b) $G_p(s) = \dfrac{1}{s+0.2}$ $\quad Y(s) = \dfrac{2(s+0.1)}{(s+2)} \dfrac{1}{(s+0.2)} \dfrac{r_0}{s}$ $\quad y(\infty) = \dfrac{1}{2}$

(c) $G_{cl}(s) = \dfrac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} = \dfrac{2}{s+2}$ with $G_p(s) = \dfrac{1}{s+0.1}$

$$Y(s) = G_{cl}(s)R(s) = \frac{2}{s+2}\cdot\frac{r_0}{s} \qquad y(\infty) = r_0$$

(d) $G_{cl}(s) = \dfrac{G_c(s)G_p(s)}{1+G_c(s)G_p(s)} = \dfrac{2(s+0.1)}{s(s+0.2)+2(s+0.1)} = \dfrac{2(s+0.1)}{s^2+2.2s+0.2}$

$$Y(s) = G_{cl}(s)R(s) = \frac{2(s+0.1)}{s^2+2.2s+0.2}\frac{r_0}{s} \qquad y(\infty) = \frac{0.2}{0.2}r_0 = r_0$$

(e) (a) $Y(s) = G_c(s)\,G_p(s)\,R(s) = \dfrac{2(s+0.1)}{s+2}\,\dfrac{1}{s+0.1}\,R(s) = \dfrac{2}{s+2}\,R(s)$

See MATLAB for response

(b) $Y(s) = G_c(s)\,G_p(s)\,R(s) = \dfrac{2(s+0.1)}{s+2}\,\dfrac{1}{s+0.2}\,R(s) = \dfrac{2(s+0.1)}{(s+2)(s+0.2)}$

See MATLAB for response

(c) $Y(s) = \dfrac{2}{s+2}\,R(s)$

See MATLAB for step response

(d) $Y(s) = \dfrac{2(s+0.1)}{s^2+2.2s+0.2}\,R(s)$

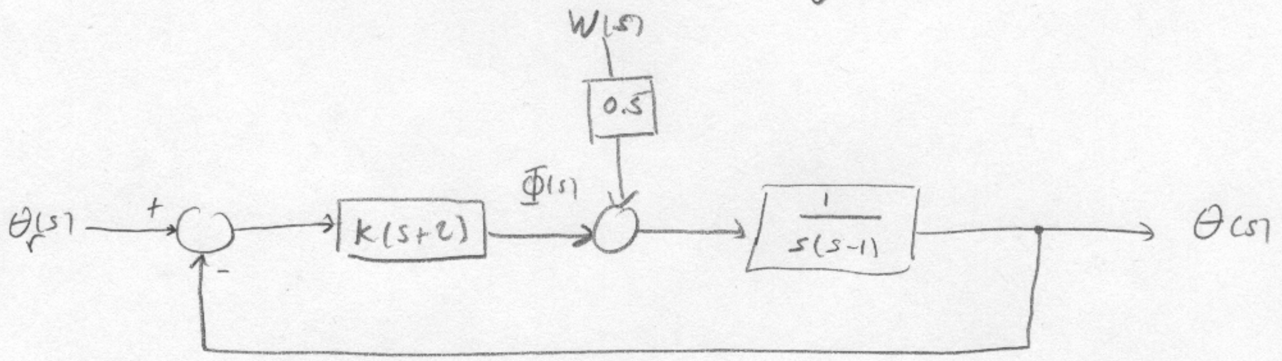
See MATLAB for step response

Closed-loop controller always reaches zero error, even if plant modeled slightly incorrectly

3. Problem 10.3

(a) No - Cannot compensate for wind gusts

(b)



$$\Theta(s) = \frac{1}{s(s-1)}\,\Phi(s) + \frac{0.5}{s(s-1)}\,W(s)$$

$$\Phi(s) = K(s+2)\,(\theta_r(s) - \theta(s))$$

$$\theta(s) = \frac{1}{s(s-1)}\,K(s+2)\left[\theta_r(s) - \theta(s)\right] + \frac{0.5}{s(s-1)}\,W(s)$$

$$\theta(s) = \frac{K(s+2)\,\theta_r(s) + 0.5\,W(s)}{s^2 + (K-1)s + 2K}$$

Part of response due to $W(s)$: $\dfrac{0.5\,W(s)}{s^2 + s(K-1) + 2K}$

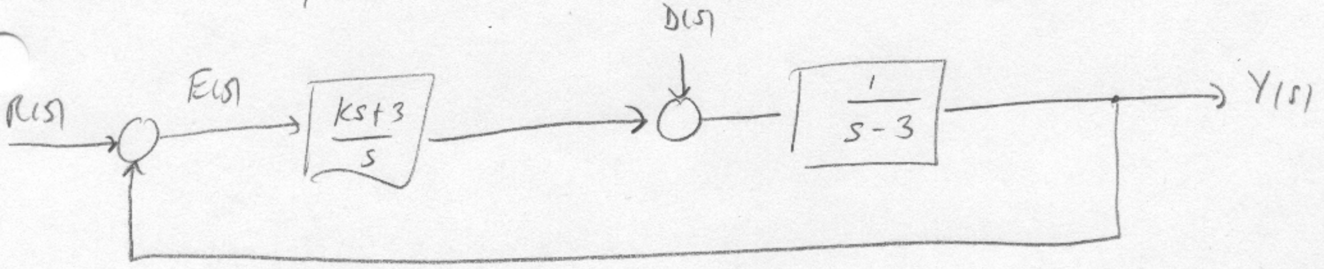
Let $s = j\omega$ to find magnitude:

$$\left|\frac{0.5\,W(j\omega)}{-\omega^2 + j\omega(K-1) + 2K}\right| = \frac{0.5\,|W(j\omega)|}{\sqrt{(2K-\omega^2)^2 + \omega^2(K-1)^2}}$$

As K gets bigger magnitude gets smaller

(c) See MATLAB

4. Problem 10.4



(a) $E(s) = R(s) - Y(s) = R(s) - \left[\frac{Ks+3}{s}\,\frac{1}{s-3}\,E(s) + \frac{1}{s-3}\,D(s)\right]$

$$E(s) = \frac{s(s-3)R(s) - s\,D(s)}{s^2+(K-3)s+3}$$

(b) For $R(s) = \frac{1}{s}$, $D(s) = 0$

$$E(s) = \frac{s-3}{s^2+(K-3)s+3} \qquad e(\infty) = sE(s)\Big|_{s\to 0} = \frac{s(s-3)}{s^2+(K-3)s+3}\Bigg|_{s=0} = 0$$

Need all poles in left hand plane for $e(\infty)$ to exist
For 2^{nd} order denominator need all coefficents > 0

$\therefore\ K-3 > 0 \qquad K > 3$

(c) For $R(s) = \frac{1}{s}$ $D(s) = \frac{1}{s}$

$$E(s) = \frac{(s-3) - 1}{s^2+(K-3)s+3} = \frac{s-4}{s^2+(K-3)s+3}$$

Same as (b): $K > 3$

(d) For $R(s) = \frac{1}{s}$ $D(s) = \frac{1}{s^2+1}$

$$E(s) = \frac{(s-3) - \frac{s}{s^2+1}}{s^2+(K-3)s+3} = \frac{(s^2+1)(s-3) - s}{(s^2+1)(s^2+(K-3)s+3)}$$

From Final Value therom, $e(\infty)=0$ if all poles in left hand plane.

Denominator: $\underbrace{(s^2+1)}_{\text{poles at } \pm j}(s^2+(3-k)s+3) =$

There is no k for which $e(t) \to 0$ as $t \to \infty$

(e) $G_c(s) = \dfrac{7s^3+K_1 s+K_2}{s(s^2+1)}$

$$E(s) = R(s) - Y(s) = R(s) - \left[\frac{7s^3+K_1 s+K_2}{s(s^2+1)} \frac{1}{s-3} E(s) + \frac{1}{s-3} D(s) \right]$$

$$E(s) = \frac{s(s^2+1)(s-3)R(s) - s(s^2+1)D(s)}{s(s^2+1)(s-3) + 7s^3 + K_1 s + K_2}$$

$$= \frac{s(s^2+1)(s-3)R(s) - s(s^2+1)D(s)}{s^4+4s^3+s^2+(K_1-3)s+K_2}$$

For $R(s) = \frac{1}{s}$ and $D(s) = \frac{1}{s^2+1}$

$$E(s) = \frac{(s^2+1)(s-3) - s}{s^4+4s^3+s^2+(K_1-3)s+K_2}$$

$e(t) \to 0$ as $t \to \infty$ if all poles in left hand plane

Use Routh-Hurwitz to check

$$\begin{array}{c|cccc}
 & s^4 & 1 & 1 & K_2 \\
\frac{1}{4} & s^3 & 4 & K_1-3 & 0 \\
 & s^2 & \frac{7-K_1}{4} & K_2 & 0 \\
\frac{16}{7-K_1} & s^1 & \frac{(7-K_1)(K_1-3)-16K_2}{7-K_1} & 0 & 0 \\
 & s^0 & K_2 & 0 & 0
\end{array}$$

$\frac{7-K_1}{4} > 0 \Rightarrow K_1 < 7$

$K_2 > 0$

$(7-K_1)(K_1-3) - 16K_2 > 0$

$\underbrace{(7-K_1)}_{>0}(K_1-3) > \underbrace{16K_2}_{>0}$

Must be >0, so $K_1 > 3$

$3 < K_1 < 7$

$0 < K_2 < \dfrac{(7-K_1)(K_1-3)}{16}$

5. Problem 10.5

(a) $h(t) = \sin(t)\,u(t) \Rightarrow G_p(s) = \frac{1}{s^2+1}$

For closed loop system want $h(t) = \sin(t)\, e^{-t} u(t)$

$$\Rightarrow G_{ce}(s) = \frac{1}{(s+1)^2+1} = \frac{1}{s^2+2s+2}$$

$$G_{ce}(s) = \frac{G_c(s)\, G_p(s)}{1 + G_c(s) G_p(s)}$$

$$\frac{1}{s^2+2s+2} = \frac{G_c(s)\, \frac{1}{s^2+1}}{1 + G_c(s)\, \frac{1}{s^2+1}} = \frac{G_c(s)}{s^2+1+G_c(s)}$$

$$s^2+1+G_c(s) = (s^2+2s+2)\, G_c(s)$$

$$s^2+1 = (s^2+2s+1)\, G_c(s) = (s+1)^2\, G_c(s)$$

$$G_c(s) = \frac{s^2+1}{(s+1)^2}$$

(b) For $R(s) = \frac{1}{s}$

$$Y(s) = G_{ce}(s)\, R(s) = \frac{1}{s\,(s^2+2s+2)} = \frac{1}{s\,(s+1-j)(s+1+j)}$$

$$= \frac{1/2}{s} + \frac{\frac{-1+j}{4}}{s+1-j} + \frac{\frac{-1-j}{4}}{s+1+j}$$

$$y(t) = \tfrac{1}{2}u(t) + \tfrac{\sqrt{2}}{2}\, e^{-t} \cos(t - 45^\circ)$$

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% EE 342 Homework 7

% Problem 1

% (a)
figure(1);
[z,p,k] = buttap(5);
[b,a]=zp2tf(z,p,k);
[b,a]=lp2lp(b,a,4*pi);
w = 0:0.001:20;
H = freqs(b,a,w);
subplot(311)
plot(w,abs(H));
grid
ylabel('|H(\omega)|')
title('5th Order Butterworth Filter')
subplot(312)
plot(w,unwrap(angle(H))*180/pi);
grid
ylabel('\angle H(\omega)')
xlabel('\omega')

% (b)
t = 0:0.01:4;
x=(t >= 0) - (t >= 1);
y = lsim(b,a,x,t);
subplot(313)
plot(t,y)
grid
axis([0 4 -0.5 1.5])
ylabel('Pulse Response')
print -dpsc2 hw7_p1a.ps

% (c)
figure(2);
[z,p,k] = besselap(5);
[b,a]=zp2tf(z,p,k);
[b,a]=lp2lp(b,a,4*pi);
w = 0:0.001:20;
H = freqs(b,a,w);
subplot(311)
plot(w,abs(H));
grid
ylabel('|H(\omega)|')
title ('5th Order Bessel Filter')
subplot(312)
plot(w,unwrap(angle(H))*180/pi);
grid
ylabel('\angle H(\omega)')
xlabel('\omega')

t = 0:0.01:4;
x=(t >= 0) - (t >= 1);
y = lsim(b,a,x,t);
subplot(313)
plot(t,y)
grid
axis([0 4 -0.5 1.5])
ylabel('Pulse Response')
print -dpsc2 hw7_p1c.ps

% (d)
figure(3);
[z,p,k] = ellipap(5,3,60);
[b,a]=zp2tf(z,p,k);
[b,a]=lp2lp(b,a,4*pi);
w = 0:0.001:20;
H = freqs(b,a,w);
subplot(311)
```



```
plot(w,abs(H));
grid
ylabel('|H(\omega)|')
title ('5th Order Elliptic Filter')
subplot(312)
plot(w,unwrap(angle(H))*180/pi);
grid
ylabel('\angle H(\omega)')
xlabel('\omega')

t = 0:0.01:4;
x=(t >= 0) - (t >= 1);
y = lsim(b,a,x,t);
subplot(313)
plot(t,y)
grid
axis([0 4 -0.5 1.5])
xlabel('t (sec)')
ylabel('Pulse Response')
print -dpsc2 hw7_p1d.ps

% Problem 10.1

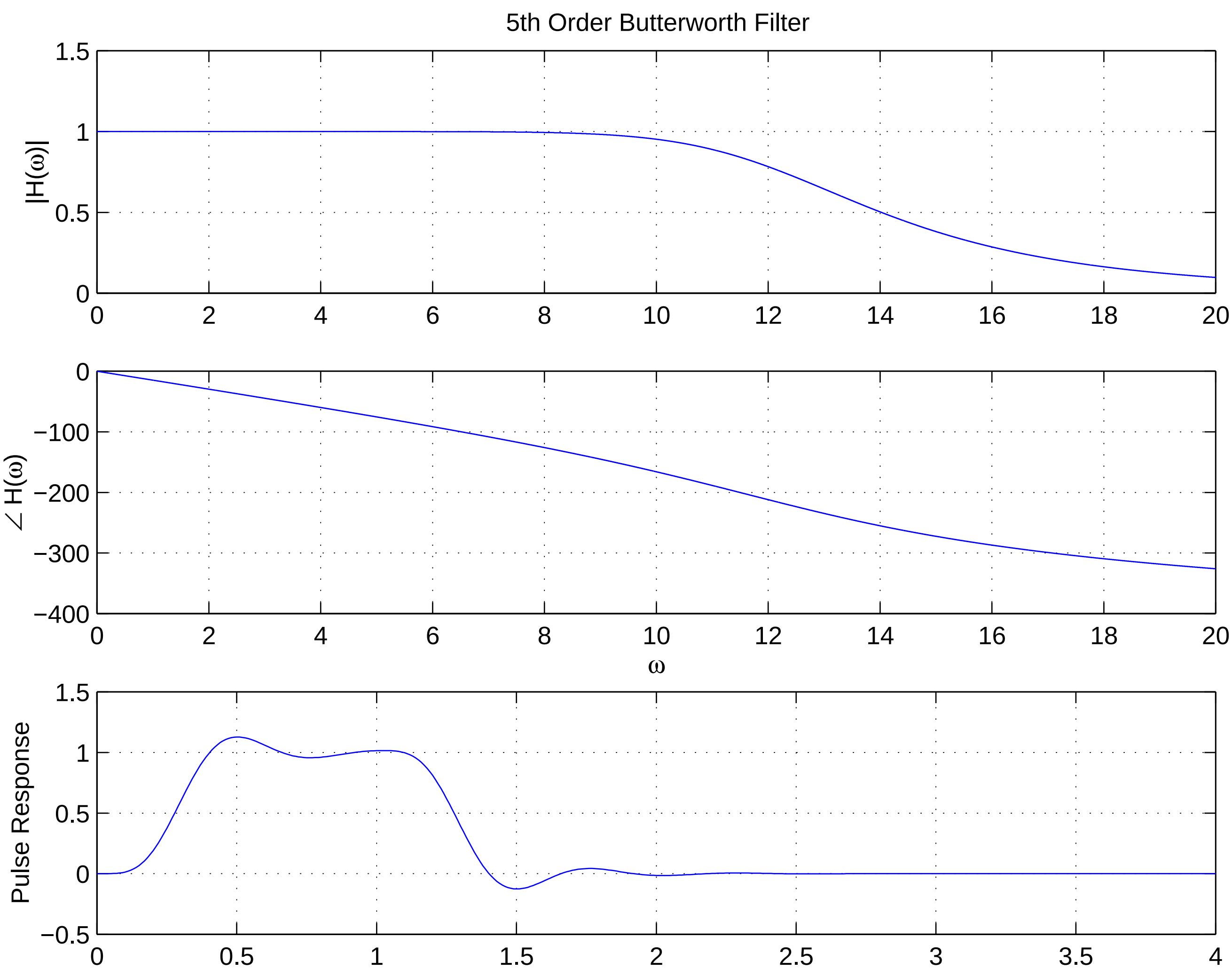
% (a)

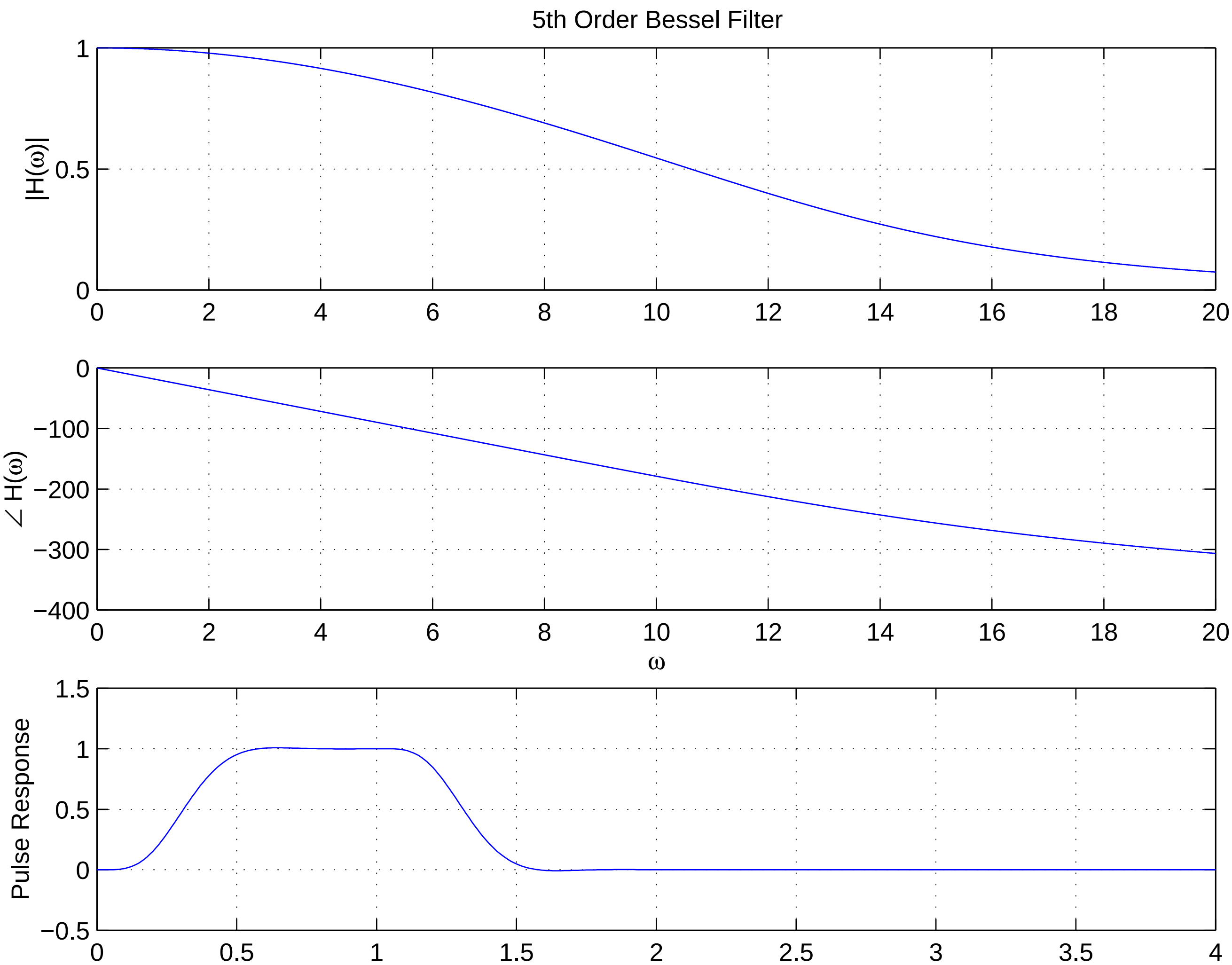
figure(4)
b = 2;
a=[1 2];
subplot(411)
step(b,a);
title('Problem 10.1');

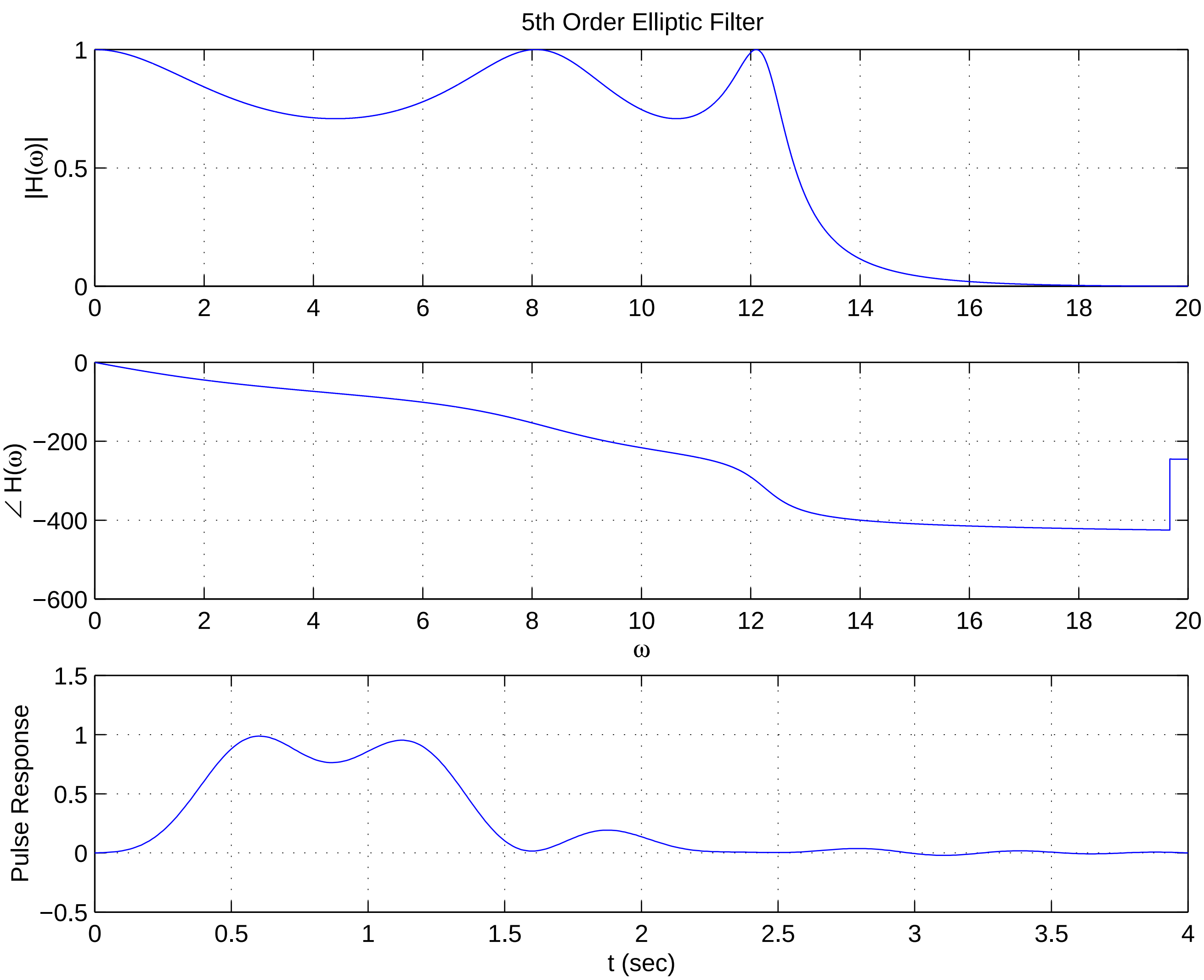
% (b)
b = 2*[1 0.1];
a=conv([1 2],[1 0.2]);
subplot(412)
step(b,a);

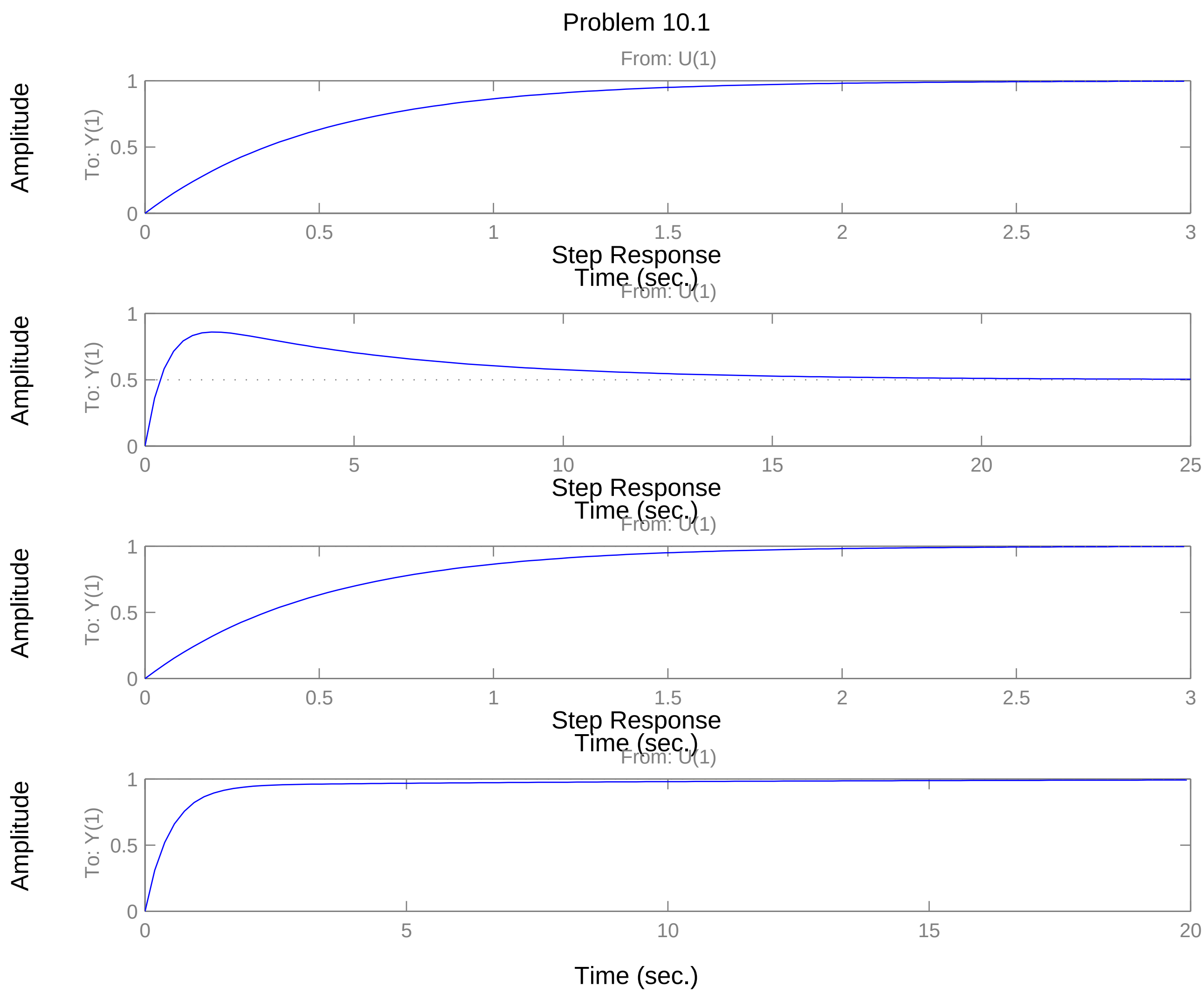
% (c)
b = 2;
a=[1 2];
subplot(413)
step(b,a);

% (d)
b = 2*[1 0.1];
a=[1 2.2 0.2];
subplot(414)
step(b,a);
print -dpsc2 hw7_p10_1.ps
```


5th Order Butterworth Filter
|H(ω)|
∠ H(ω)
ω
Pulse Response
0
2
4
6
8
10
12
14
16
18
20
1.5
1
0.5
−100
−200
−300
−400
−0.5
2.5
3
3.5


5th Order Bessel Filter
|H(ω)|
∠ H(ω)
ω
Pulse Response
1
0.5
0
2
4
6
8
10
12
14
16
18
20
−100
−200
−300
−400
1.5
−0.5
1
1.5
2.5
3
3.5
4


5th Order Elliptic Filter
|H(ω)|
∠ H(ω)
ω
Pulse Response
t (sec)
0
0.5
1
−200
−400
−600
2
4
6
8
10
12
14
16
18
20
1.5
−0.5
2.5
3
3.5


Problem 10.1
From: U(1)
Amplitude
To: Y(1)
1
0.5
0
0
0.5
1
1.5
2
2.5
3
Step Response
Time (sec.)
From: U(1)
Amplitude
To: Y(1)
1
0.5
0
0
5
10
15
20
25
Step Response
Time (sec.)
From: U(1)
Amplitude
To: Y(1)
1
0.5
0
0
0.5
1
1.5
2
2.5
3
Step Response
Time (sec.)
From: U(1)
Amplitude
To: Y(1)
1
0.5
0
0
5
10
15
20
Time (sec.)