EE 342
HW #8

1. Problem 10.7 See MATLAB

2. Problem 10.8

For 10.7 (a)

(a) Stable for all values of $K \geq 0$

(b) When critically damped, poles at -5.5,

$$G_c(s)\,G_p(s) = K\frac{1}{(s+1)(s+10)}$$

$$P(s) = \frac{1}{(s+1)(s+10)} \qquad K = \frac{1}{|P(p)|} = |(-5.5+1)(-5.5+10)| = 20.25$$

(c) Smallest time constant when $Re(p) = -5.5$

This happens for all $K > 20.25$ (from above)

For 10.7 (b)

$$G_c(s)\,G_p(s) = K\frac{1}{(s+1)(s+4)(s+10)} \qquad P(s) = \frac{1}{(s+1)(s+4)(s+10)}$$

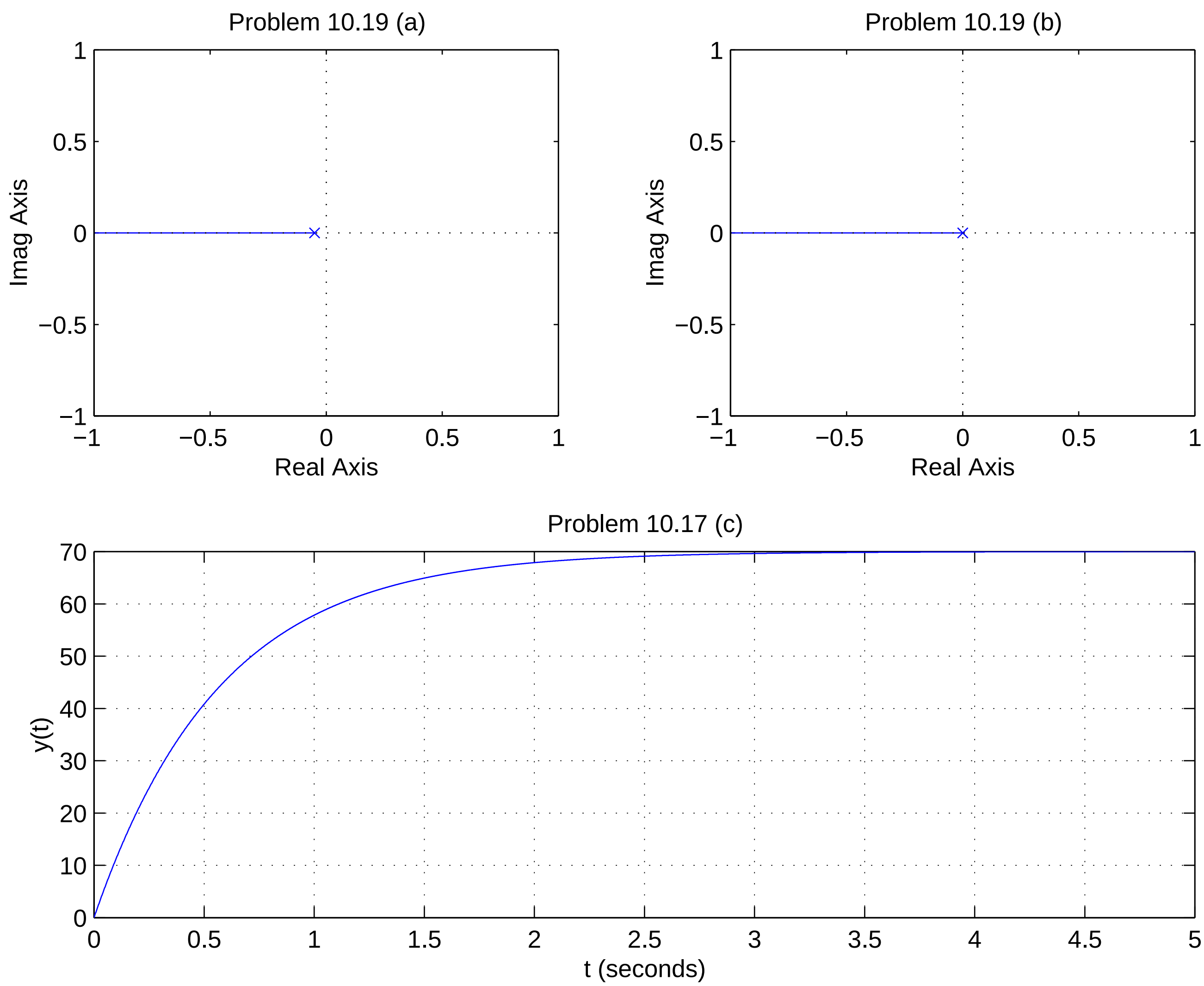
(a) System stable until poles at $\pm j7.33$

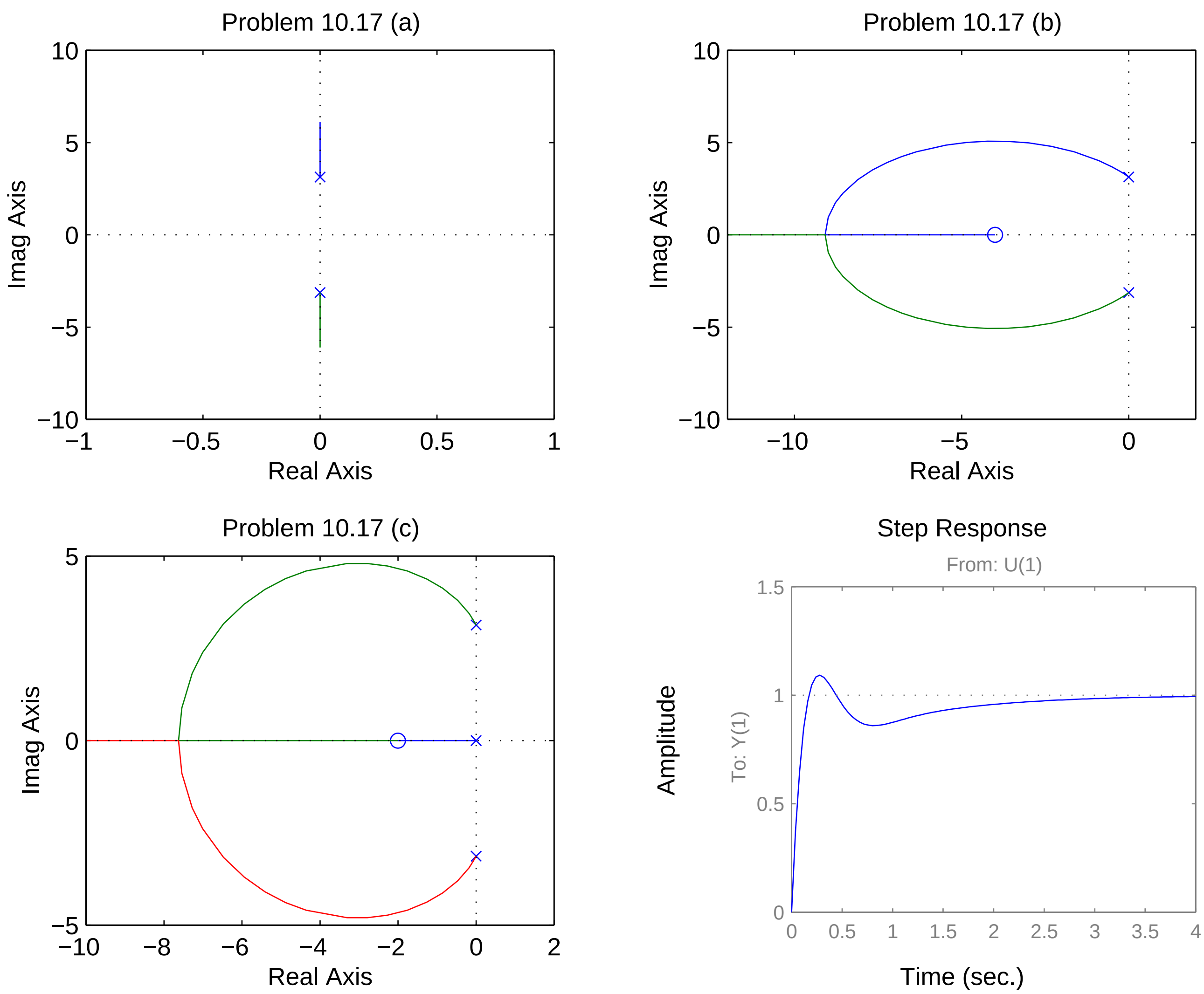
$$K = \frac{1}{|P(p)|} = |(j7.33+1)(j7.33+4)(j7.33+10)| \approx 766$$

Stable for $0 < K < 766$

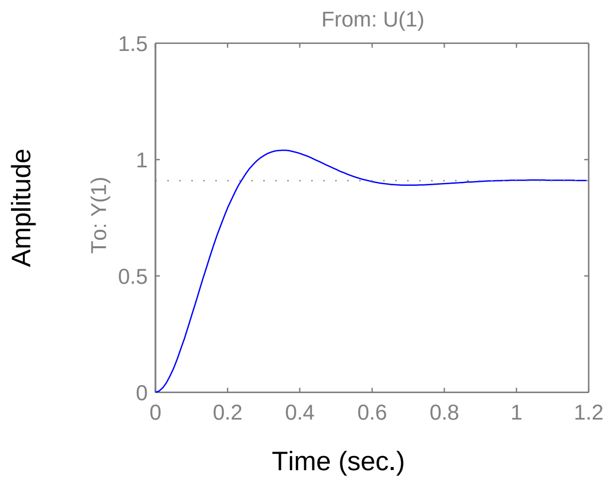
(b) When critically damped, poles at -2.3542

$$K = \frac{1}{|P(p)|} = |(-2.3542+1)(-2.3542+4)(-2.3542+10)| = 17$$

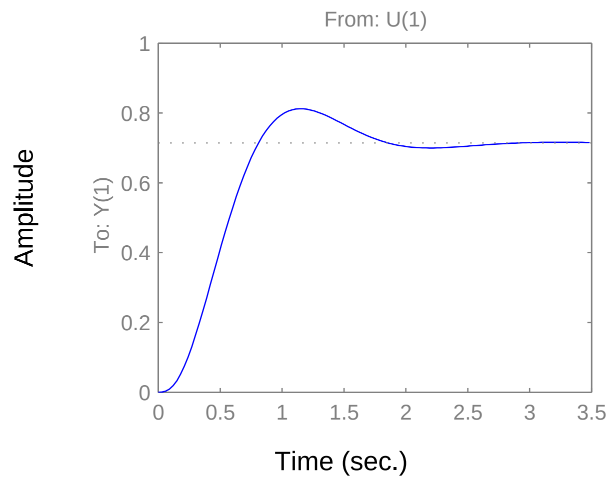

Problem 10.19 (a)
Imag Axis
Real Axis
1
0.5
0
−0.5
−1
−1
−0.5
0
0.5
1
Problem 10.19 (b)
Imag Axis
Real Axis
1
0.5
0
−0.5
−1
−1
−0.5
0
0.5
1
Problem 10.17 (c)
y(t)
t (seconds)
70
60
50
40
30
20
10
0
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Problem 10.17 (a)
Imag Axis
Real Axis
10
5
0
−5
−10
−1
−0.5
0
0.5
1
Problem 10.17 (b)
Imag Axis
Real Axis
10
5
0
−5
−10
−10
−5
0
Problem 10.17 (c)
Imag Axis
Real Axis
5
0
−5
−10
−8
−6
−4
−2
0
2
Step Response
From: U(1)
Amplitude
To: Y(1)
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4
Time (sec.)

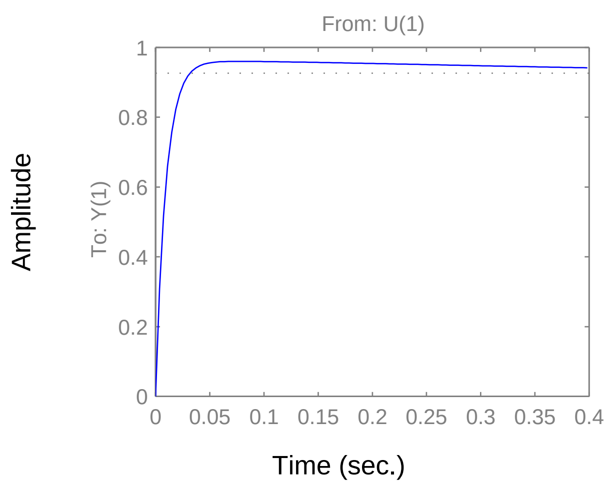
Step Response for 10.7(a)



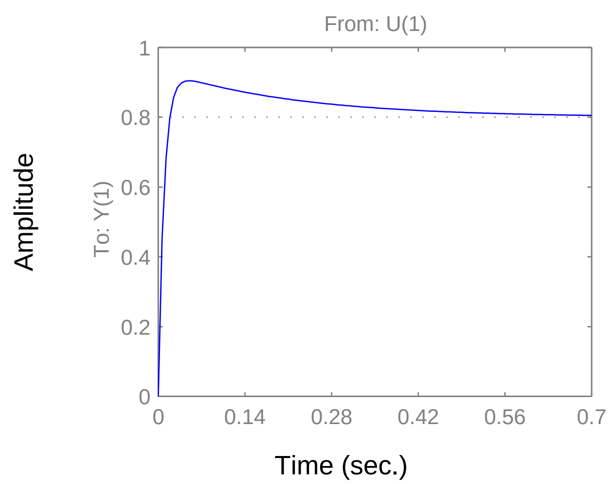
Step Response for 10.7(b)

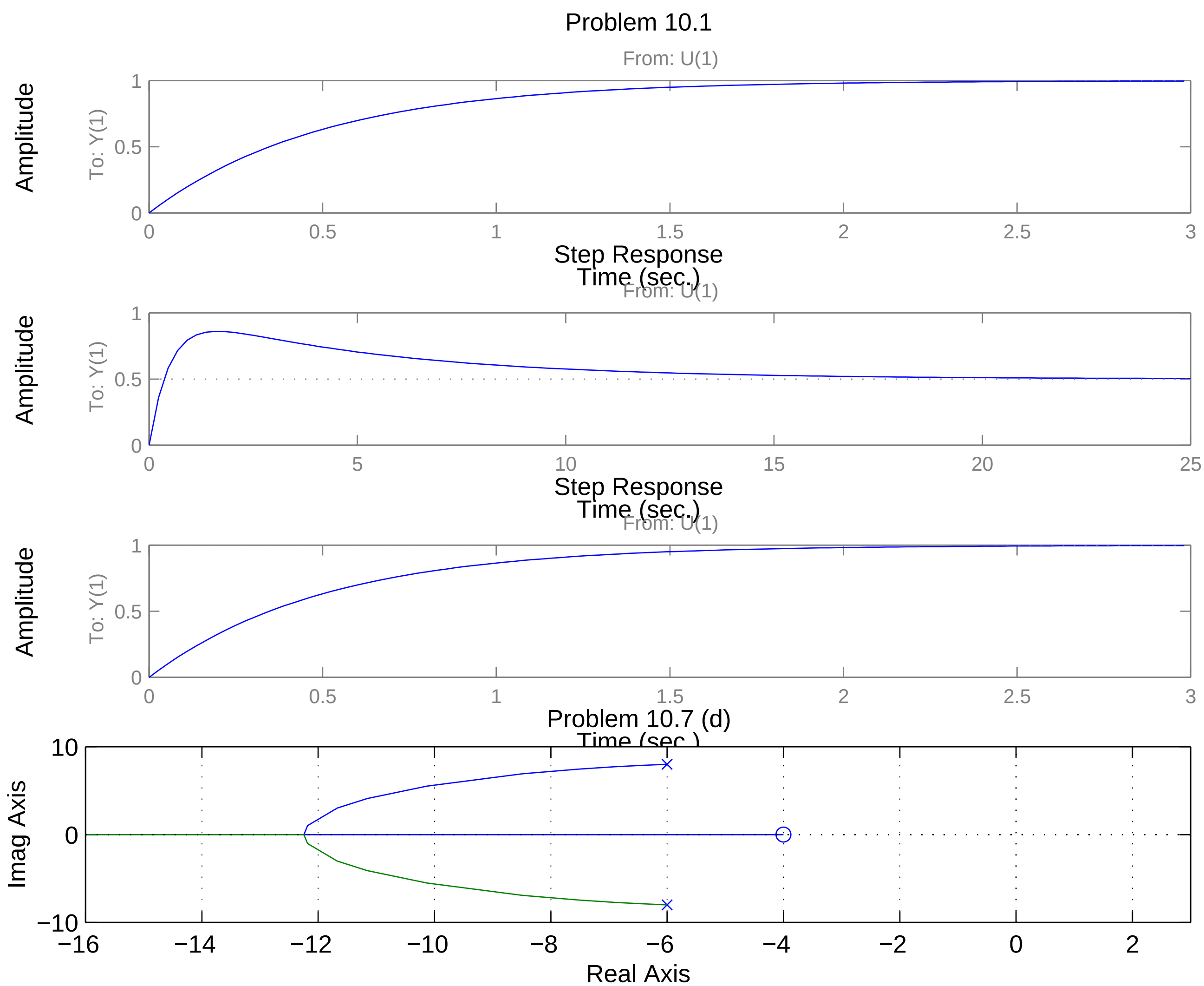


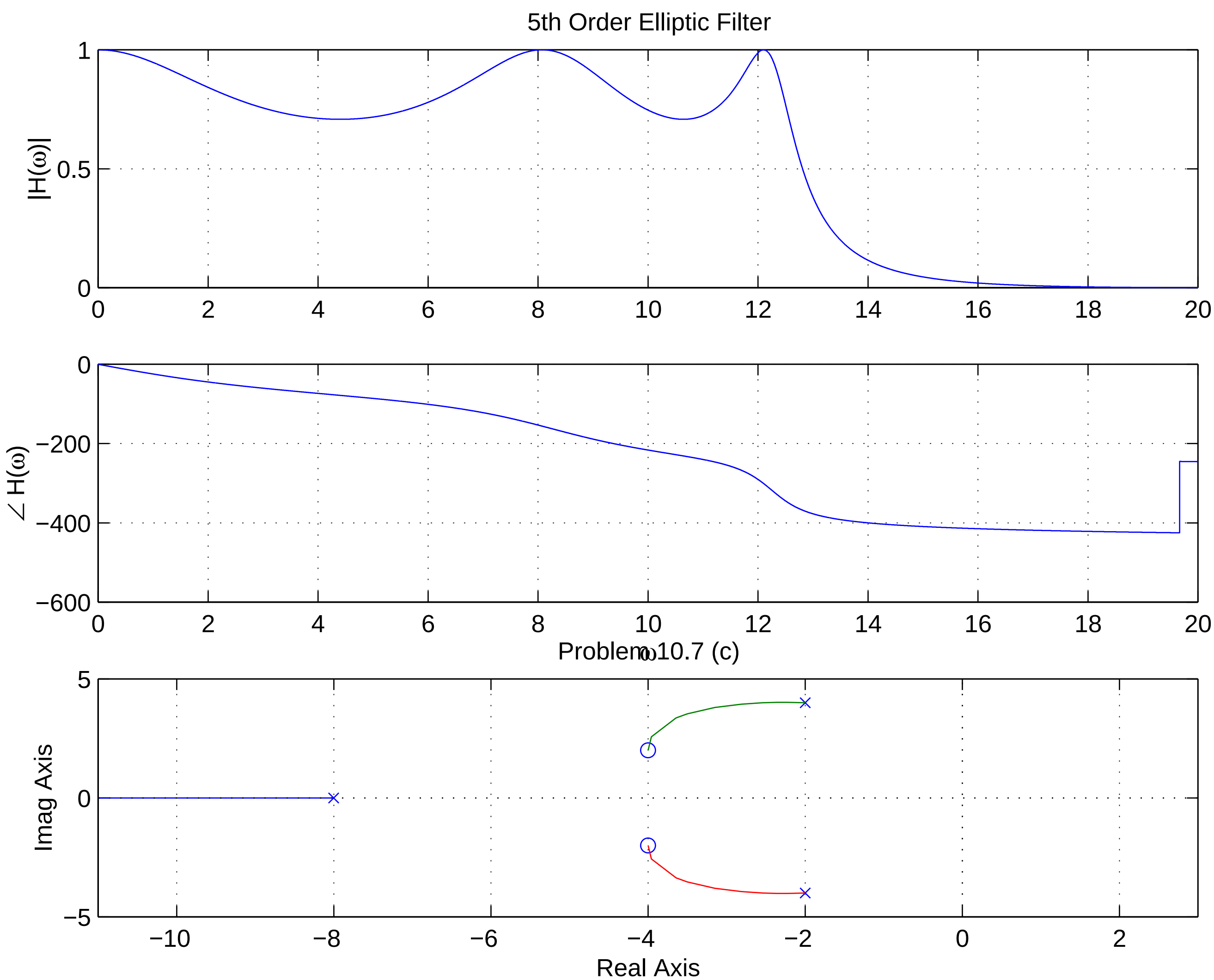
Step Response for 10.7(c)

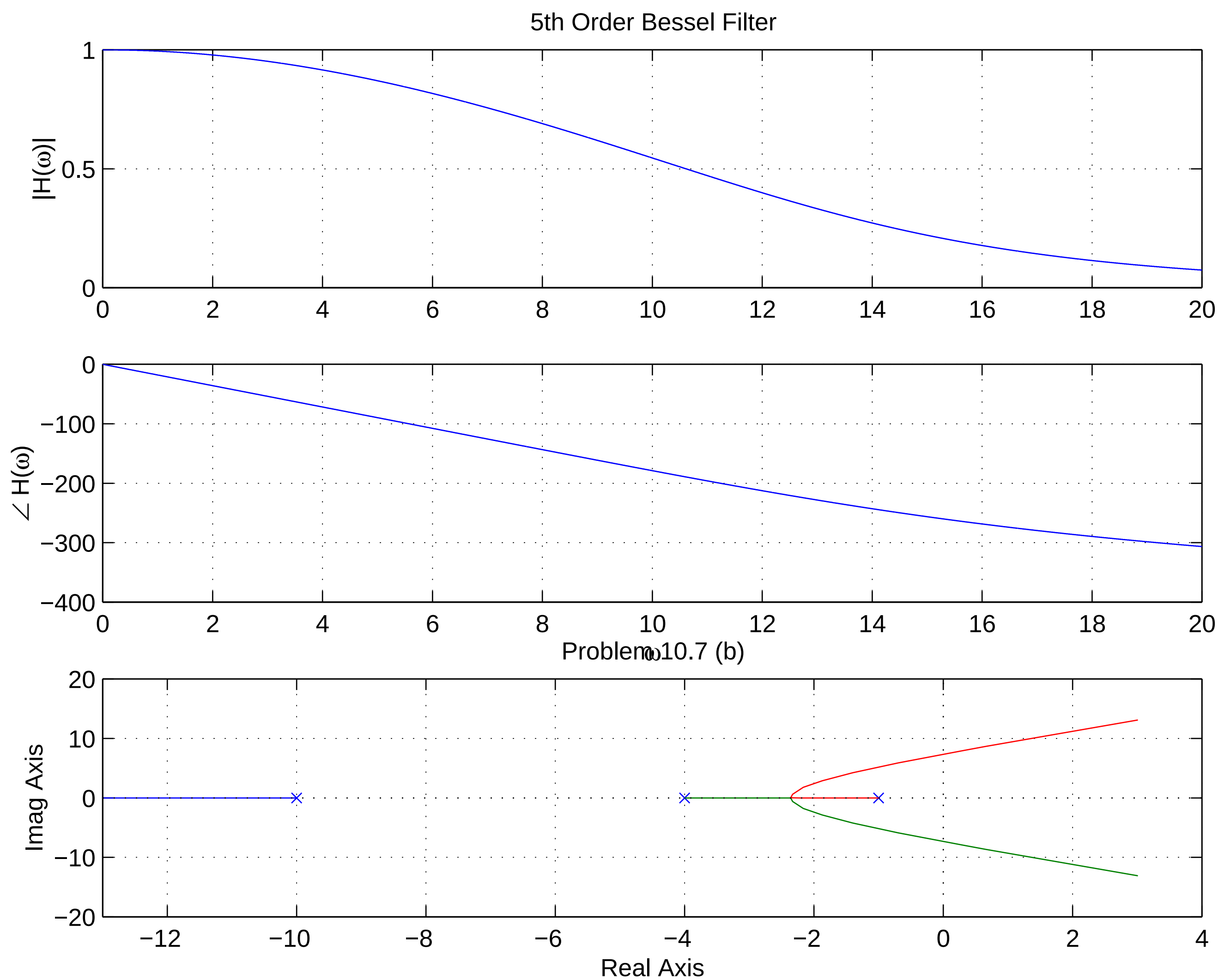


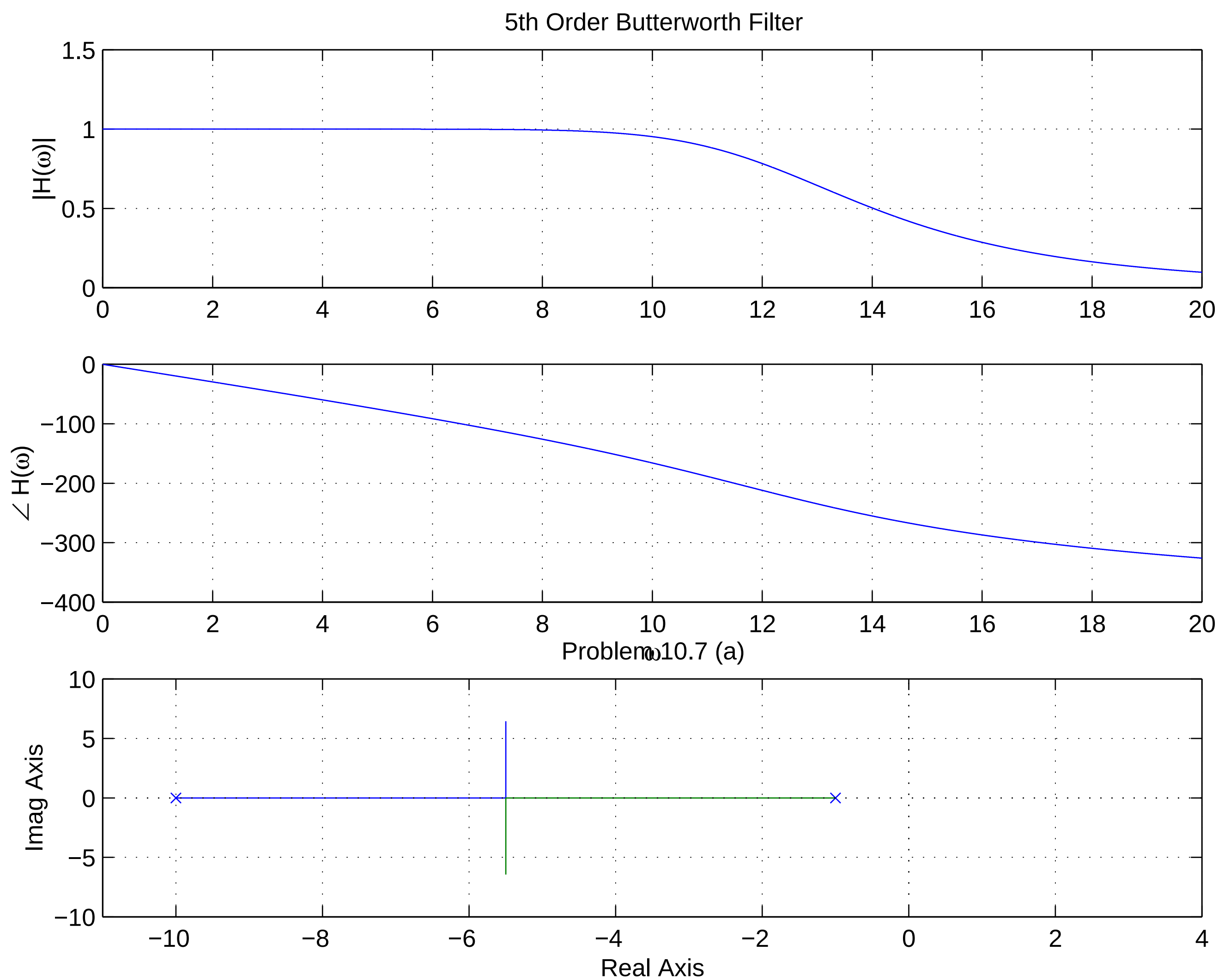
Step Response for 10.7(d)




Problem 10.1
From: U(1)
Amplitude
To: Y(1)
1
0.5
0
0
0.5
1
1.5
2
2.5
3
Step Response
Time (sec.)
From: U(1)
Amplitude
To: Y(1)
1
0.5
0
0
5
10
15
20
25
Step Response
Time (sec.)
From: U(1)
Amplitude
To: Y(1)
1
0.5
0
0
0.5
1
1.5
2
2.5
3
Problem 10.7 (d)
Time (sec.)
10
0
−10
Imag Axis
−16
−14
−12
−10
−8
−6
−4
−2
0
2
Real Axis


5th Order Elliptic Filter
|H(ω)|
1
0.5
0
0 2 4 6 8 10 12 14 16 18 20
∠ H(ω)
0
−200
−400
−600
0 2 4 6 8 10 12 14 16 18 20
ω
Problem 10.7 (c)
Imag Axis
5
0
−5
−10 −8 −6 −4 −2 0 2
Real Axis


5th Order Bessel Filter
|H(ω)|
1
0.5
0
0
2
4
6
8
10
12
14
16
18
20
∠ H(ω)
0
−100
−200
−300
−400
ω
Problem 10.7 (b)
Imag Axis
20
10
0
−10
−20
−12
−10
−8
−6
−4
−2
0
2
4
Real Axis



5th Order Butterworth Filter
|H(ω)|
∠ H(ω)
ω
Problem 10.7 (a)
Imag Axis
Real Axis

```
%Problem 10.7

% (a)
figure(1)
b=1;
a=conv([1 1],[1 10]);
rlocus(b,a)
title('Problem 10.7 (a)')
grid
print -dpsc2 p10_7a.ps

% (b)
figure(2)
b=1;
a=conv([1 1],conv([1 4],[1 10]));
rlocus(b,a)
title('Problem 10.7 (b)')
grid
print -dpsc2 p10_7b.ps

% (c)
figure(3)
b=conv([1 4],[1 4]) + [0 0 4];
a=conv(conv([1 2],[1 2])+[0 0 16],[1 8]);
rlocus(b,a)
title('Problem 10.7 (c)')
grid
print -dpsc2 p10_7c.ps

% (d)
figure(4)
b=[1 4];
a=conv([1 6],[1 6])+[0 0 64];
rlocus(b,a)
title('Problem 10.7 (d)')
grid
print -dpsc2 p10_7d.ps

% Problem 10.9
figure(5)

% 10.7 (a)
subplot(221)
K = 100;
b = K;
a = conv([1 1],[1 10]) + [0 0 K];
step(b,a)
title('Step Response       for  10.7(a)')

% 10.7 (b)
subplot(222)
K = 100;
b = K;
a = conv([1 1],conv([1 4],[1 10])) + [0 0 0 K];
step(b,a)
title('Step Response       for  10.7(b)')

% 10.7 (c)
subplot(223)
K = 100;
b = K*(conv([1 4],[1 4]) + [0 0 4]);
a = conv(conv([1 2],[1 2])+[0 0 16],[1 8]) + [0 K*(conv([1 4],[1 4]) + [0 0 4])]
;
step(b,a)
title('Step Response       for  10.7(c)')

% 10.7 (d)
subplot(224)
K = 100;
```



```
b = K*[1 4];
a = (conv([1 6],[1 6]) + [0 0 64]) + K*[0 1 4];
step(b,a)
title('Step Response       for  10.7(d)')
print -dpsc2 p10_9.ps

% Problem 10.17

%(a)
figure(6)
subplot(221)
b=1;
a=[1 0 9.8];
rlocus(b,a)
title('Problem 10.17 (a)')

%(b)
subplot(222)
b=[1 4];
a=[1 0 9.8];
rlocus(b,a)
title('Problem 10.17 (b)')

%(c)
subplot(223)
b=[1 4 4];
a=[1 0 9.8 0];
rlocus(b,a)
title('Problem 10.17 (c)')

%(d)
subplot(224)
KD = 10.8;
b=KD*[1 4 4];
a = [1 0 9.8 0] + KD*[0 1 4 4];
step(b,a)
print -dpsc2 p10_17.ps

% Problem 10.19

%(a)
figure(7)
subplot(221)
b=0.05;
a=[1 0.05];
rlocus(b,a)
title('Problem 10.19 (a)')

%(b)
subplot(222)
b = 1;
a=[1 0];
rlocus(b,a)
title('Problem 10.19 (b)')

%(c)
subplot(212)
t=0:0.001:5;
y = -70*exp(-1.75*t)+70;
plot(t,y)
title('Problem 10.17 (c)')
grid
xlabel('t (seconds)')
ylabel('y(t)')
print -dpsc2 p10_19.ps
```

(c) Smallest time constant when critically damped
$K = 17$

For 10.7 (c)

(a) Stable for all $K \geq 0$

(b) No critically damped response

(c) Smallest time constant for $K = \infty$

For 10.7 (d)

$$G_c(s)\,G_p(s) = K \frac{s+4}{(s+6)^2 + 64} \qquad P(s) = \frac{s+4}{(s+6)^2+64}$$

(a) Stable for all $K \geq 0$

(b) When critically damped, poles at -12.2462

$$K = \frac{1}{|P(s)|} = \left| \frac{(-12.2462 + 6)^2 + 64}{-12.2462 + 4} \right| = 12.49$$

(c) Smallest time constant when critically damped: $K = 12.49$

3. Problem 10.9

$$E(s) = \frac{1}{1 + G_p(s)\,G_c(s)} R(s) = \frac{1}{(1 + G_p(s)\,G_c(s))\,s} \quad \text{for } r(t) = u(t)$$

For 10.7 (a)

(a) $$E(s) = \frac{1}{\left(1 + K \frac{1}{(s+1)(s+10)}\right) s} = \frac{(s+1)(s+10)}{((s+1)(s+10) + K)\,s}$$

$$e_{ss} = \lim_{s \to 0} (s\,E(s)) = \frac{(1)(10)}{((1)(10) + K)} = \frac{10}{10+K} = \frac{10}{110} = 0.091$$

(b) Step response should go to $1 - e_{ss} = 1 - 0.091 = 0.909$

(b) $G_{cl}(s) = \dfrac{G_p(s)\,G_c(s)}{1+G_p(s)G_c(s)} = \dfrac{\dfrac{K}{(s+1)(s+10)}}{1+\dfrac{K}{(s+1)(s+10)}} = \dfrac{K}{(s+1)(s+10)\ K}$

$= \dfrac{100}{(s+1)(s+10)+100}$

See MATLAB for plot. Step response $\rightarrow$ 0.909

For 10.7 (b)

(a) $E(s) = \dfrac{1}{\left(1+\dfrac{K}{(s+1)(s+4)(s+10)}\right)s} = \dfrac{(s+1)(s+4)(s+10)}{\big((s+1)(s+4)(s+10)+K\big)s}$

$e_{ss} = \dfrac{(1)(4)(10)}{(1)(4)(10)+K} = \dfrac{40}{40+K} = \dfrac{40}{140} = 0.286$

Step response should go to $1-0.286 = 0.714$

(b) $G_{cl}(s) = \dfrac{\dfrac{K}{(s+1)(s+4)(s+10)}}{1+\dfrac{K}{(s+1)(s+4)(s+10)}} = \dfrac{K}{(s+1)(s+4)(s+10)+K}$

See MATLAB for plot. Step response $\rightarrow$ 0.714

For 10.7 (c)

(a) $E(s) = \dfrac{1}{\left(1+K\dfrac{(s+4)^2+4}{[(s+2)^2+16](s+8)}\right)s} = \dfrac{[(s+2)^2+16](s+8)}{\big([(s+2)^2+16](s+8)+K[(s+4)^2+4]\big)s}$

$e_{ss} = \dfrac{(2^2+16)(0+8)}{(2^2+16)(8)+K(4^2+4)} = \dfrac{160}{160+K\,20} = 0.0741$

Step response should go to $1-0.0741 = 0.926$

(b) $G_{cl}(s) = \dfrac{K[(s+4)^2+4]}{[(s+2)^2+16](s+8) + K[(s+4)^2+4]}$

See MATLAB for plot   Step response → 0.926

For 10.7(d)

(a) $E(s) = \dfrac{1}{\left(1 + K\dfrac{s+4}{[(s+6)^2+64]}\right)s} = \dfrac{(s+6)^2+64}{([(s+6)^2+64] + K(s+4))s}$

$e_{ss} = \dfrac{6^2+64}{6^2+64+4K} = \dfrac{100}{100+4K} = 0.2$

Step response should go to $1-0.2 = 0.8$

(b) $G_{cl}(s) = \dfrac{K(s+4)}{[(s+6)^2+64] + K(s+4)}$

See MATLAB for plot

4. Problem 10.17 $G_p(s) = \dfrac{1}{s^2+9.8}$

(a) $G_c(s) = K_P$ $G_p(s)G_c(s) = K\dfrac{1}{s^2+9.8}$

See MATLAB for root-locus plot

System is marginally stable for all K - will oscillate forever

(b) $G_c(s) = K_D s + K_P = K_D(s+4)$

$G_p(s)G_c(s) = K_D\dfrac{s+4}{s^2+9.8}$

From root-locus plot, system stable for all $K_D > 0$. As K_D increases, system goes from underdamped to critically damped to overdamped. When critically damped, time constant is smallest

(c) To have zero error, must have s in denominator of $G_p(s) G_c(s)$

Try PI controller: $G_c(s) = K_P + K_I/s = \frac{K_P(s+z)}{s}$

Root-locus plots show this will never be stable

Try PID controller: $G_c(s) = K_D s + K_P + K_I/s$

$$= K_D \left(\frac{s^2 + \frac{K_P}{K_D} s + \frac{K_I}{K_P}}{s} \right)$$

By trial-and-error, let $G_c(s) = K_D \frac{s^2+4s+4}{s}$

root-locus plot shows that this is stable for all $K_D > 0$

Larger K_D give smaller time constant

To get $\tau = 1$ sec (a pole at -1):

$$G_p(s) G_c(s) = K_D \frac{s^2+4s+4}{s} \frac{1}{s^2+9.8}$$

$$P(s) = \frac{s^2+4s+4}{s(s^2+9.8)} \qquad K_D = \frac{1}{|P(p)|} = \frac{(-1)[(-1)^2+9.8]}{(-1)^2+4(-1)+4} = -10.8$$

$$G_{cl}(s) = \frac{G_p(s) G_c(s)}{1+G_p(s) G_c(s)} = \frac{K_D(s^2+4s+4)}{s(s^2+9.8)+K_D(s^2+4s+4)}$$

$$= \frac{10.8(s^2+4s+4)}{s(s^2+9.8)+10.8(s^2+4s+4)}$$

See MATLAB for step response

5 Problem 10.19 $G_p(s) = \dfrac{0.05}{s+0.05}$

(a) $G_c(s) = K_p$ $G_p(s)\,G_c(s) = K_p \dfrac{0.05}{s+0.05}$

See MATLAB for root-locus plot

System stable for all K_p

Want error of 2°

$$E(s) = \frac{1}{1+G_p(s)G_c(s)} R(s) = \frac{1}{1+K_p \frac{0.05}{s+0.05}} R(s) = \frac{s+0.05}{s+0.05+K_p 0.05} \frac{70^\circ}{s}$$

$$e_{ss} = \frac{0.05}{0.05+K_p 0.05} 70^\circ = \frac{70^\circ}{K_p+1}$$

For $e_{ss} = 2^\circ$, need $K_p = 34$

Denominator is $s + 0.05 + K_p 0.05 = s + 1.75$

$$\tau = \frac{1}{1.75} = 0.57 \text{ sec}$$

(b) $G_c(s) = K_p + K_I/s = \dfrac{K_p(s + K_I/K_p)}{s}$

To have zero of controller at pole of system, let $K_I/K_p = 0.05$

$$G_c(s) = \frac{K_p(s+0.05)}{s} \qquad G_p(s)\,G_c(s) = K_p \frac{s+0.05}{s} \frac{0.05}{s+0.05} = 0.05 K_p \frac{1}{s}$$

$P(s) = \frac{1}{s}$ Want pole at $-1.75 \Rightarrow K = \dfrac{1}{|P(p)|} = 1.75$

$K = 0.05 K_p = 1.75 \Rightarrow K_p = 35$

(c) $G_c = 35 \frac{(s+0.05)}{s}$ $\qquad G_p(s) = \frac{0.05}{s+0.05}$

$$G_{cl}(s) = \frac{G_p(s)\,G_c(s)}{1 + G_p(s)G_c(s)} = \frac{\frac{1.75}{s}}{1 + \frac{1.75}{s}} = \frac{1.75}{s+1.75}$$

$$Y(s) = G_{cl}(s)\,R(s) = \frac{1.75}{s+1.75}\,\frac{70}{s} = \frac{122.5}{s(s+1.75)}$$

$$= \frac{-70}{s+1.75} + \frac{70}{s}$$

$$y(t) = \left(-70 e^{-1.75t} + 70\right)u(t)$$

See MATLAB for plot

Problem 11.3

(a) $x[n] = \delta[n] + 2\,\delta[n-1]$

$$X(z) = 1 + 2z^{-1} = \frac{z+2}{z}$$

(b) $x[n] = 2u[n] - \delta[n] - \delta[n-1]$

$$X(z) = \frac{2}{1-z^{-1}} - 1 - z^{-1} = \frac{z^2+1}{z(z-1)}$$

(c) $x[n] = e^{0.5n}\,u[n] + u[n-2]$

$$X(z) = \frac{z}{z-e^{0.5}} + z^{-2}\left(\frac{z}{z-1}\right) = \frac{z^3 - z^2 + z - e^{0.5}}{z(z-1)(z-e^{0.5})}$$

(d) $x[n] = e^{0.5}\delta(n) + e^{0.5}\delta(n-1) + u[n-2]$

$$X(z) = e^{0.5} + z^{-1}e^{0.5} + z^{-2}\frac{z}{z-1} = \frac{e^{0.5}(z^2-1)+1}{z(z-1)}$$

(e). $x(n) = \sin(\pi n/2)\, u[n-2] = \sin(\pi(n-2)/2 + \pi)\, u(n-2)$

$= -\sin[\pi(n-2)/2]\, u(n-2)$

$$X(z) = z^{-2} \frac{-\sin(\pi/2)\, z}{z^2 - 2\cos(\frac{\pi}{2}) z + 1} = \frac{-1}{z(z^2+1)}$$

(f) $x(n) = (0.5)^n\, n\, u(n)$

$$X(z) = \frac{0.5 z}{(z-0.5)^2}$$

(g) $x(n) = u(n) - n u(n-1) + (\frac{1}{3})^n u(n-2)$

$= u(n) - (n-1+1) u(n-1) + (\frac{1}{3})^{n-2} (\frac{1}{3})^2 u(n-2)$

$= u(n) - (n-1) u(n-1) - u(n-1) + \frac{1}{9} (\frac{1}{3})^{n-2} u[n-2]$

$$X(z) = \frac{z}{z-1} - z^{-1} \frac{z}{(z-1)^2} - z^{-1} \frac{z}{z-1} + \frac{1}{9} z^{-2} \frac{z}{z-1/3}$$

$$= \frac{z(z-1)^2(z-\frac{1}{3}) - z(z-\frac{1}{3}) + \frac{1}{9}(z-1)^2}{z(z-1)^2(z-\frac{1}{3})}$$

(h) $x(n) = 0\delta(n) + 1\delta(n-1) + 2\delta(n-2) - n u(n-3)$

$= \delta(n-1) + 2\delta(n-2) - (n-3) u(n-3) - 3u(n-3)$

$$X(z) = z^{-1} + 2z^{-2} - z^{-3} \frac{z}{(z-1)^2} - 3z^{-3} \frac{z}{z-1}$$

$$= \frac{z^3 - 6z + 4}{z^2(z-1)^2}$$

(i) $x(n) = (n-1) u(n) - n u(n-3)$

$= n u(n) - u(n) - (n-3) u(n-3) - 3u(n-3)$

$$X(z) = \frac{z}{(z-1)^2} - \frac{z}{z-1} - z^{-3} \frac{z}{(z-1)^2} - 3z^{-3} \frac{z}{z-1}$$

$$= \frac{-z^4 + 2z^3 - 3z + 2}{z^2(z-1)^2}$$

(j) $x(n) = (0.25)^{-n} u(n-2) = 4^{n} u(n-2) = 4^{n-2} 4^{2} u(n-2)$

$= 16 \; 4^{(n-2)} u(n-2)$

$X(z) = 16 \, z^{-2} \frac{z}{z-4} = \frac{16}{z(z-4)}$