EE 389 – HW 7

Due March 27, 2008

1. (a) Set up the second-order differential equation for a large-amplitude pendulum

$$Ml\frac{d^2\theta}{dt^2} = -Mg\sin(\theta)$$

   as a set up coupled first-order differential equations. Let $M = 1$ kg, and $L = 1$ m.

   (b) Write a program to numerically solve the coupled equations using Euler's method. Save the output (θ vs. t) to a file. Use $\theta(0) = 45°$ and $\dot{\theta}(0) = 0$ as initial conditions.

   (c) Write a MATLAB m file to numerically solve the coupled equations using `ode45`. Save the output (θ vs. t) to a file.

   (d) Use MATLAB to plot the results of the two methods.

2. For the following, do the calculations with using MATALB's tt ode45 solver.

   (a) Use the results of the numerical calculations to determine the period of the large-amplitude pendulum for $\theta(0)$ of 1°, 10° and 100°.

   (b) Compare the periods of the nonlinear numerical solution to the periods of the analytical lineralized soultions (where the approximation of $\sin(\theta) \approx \theta$). How much error is there in the linearized solutions?

   (c) Plot the motion of the pendulum when you start with an initial angular velocity of zero, and an initial displacement such that the pendulum starts close to the top. Also, make a phase space plot of the motion for these initial conditions. In your writeup, explain the funny shape of the motion.

   (d) With an initial displacement of zero (the pendulum hanging straight down), determine the initial angular velocity that just causes the the pendulum to go over the top.

3. Consider a pendulum with an externally applied force (Equation (7) of the class notes on Numerical Solutions to Differential Equations). Let $R = 0$ and $R' = 0$, and let $F_{\text{applied}} = \cos(\omega_0 t)$. Plot the motion for both the non-linear pendulum and the linearized pendulum (where you replace $\sin(\theta)$ by θ). Plot the motion for at least 50 seconds. Explain the differences between the actual and the linearized plots.