EE 451

Butterworth Bandpass Filter Design

$$F_{P1} = 20 \text{ kHz} \quad F_{P2} = 45 \text{ kHz} \quad R_P = 0.5 \text{ dB}$$
$$F_{S1} = 10 \text{ kHz} \quad F_{S2} = 60 \text{ kHz} \quad R_S = 40 \text{ dB}$$

$$s = \Omega_P \frac{\hat{s}^2 + \hat{\Omega}_0^2}{\hat{s} Bw}$$

$$\Omega = -\Omega_P \frac{\hat{\Omega}_0^2 - \hat{\Omega}^2}{\hat{\Omega} Bw}$$

$$\hat{\Omega}_0 = \sqrt{\hat{\Omega}_{P1} \hat{\Omega}_{P2}} = 1.5708 \times 10^5$$

$$Bw = \hat{\Omega}_{P2} - \hat{\Omega}_{P1} = 1.8850 \times 10^5$$

$$\Omega_P = 1$$

$$\Omega_{S1} = -\Omega_P \frac{\hat{\Omega}_0^2 - \hat{\Omega}_{S1}^2}{\hat{\Omega}_{S1} Bw} = -3.2$$

$$\Omega_{S2} = -\Omega_P \frac{\hat{\Omega}_0^2 - \hat{\Omega}_{S2}^2}{\hat{\Omega}_{S2} Bw} = 1.8$$

$$\Omega_S = \min(|\Omega_{S1}|, |\Omega_{S2}|) = 1.8$$

$$\Omega_P = 1 \quad R_P = 0.5 \text{ dB}$$
$$\Omega_S = 1.8 \quad R_S = 40 \text{ dB}$$

$$\delta_p = 1 - 10^{-R_P/20} = 0.0559$$

$$\delta_s = 10^{-R_S/20} = 0.01$$

$$\epsilon = \sqrt{\frac{2\delta_p - {\delta_p}^2}{(1 - \delta_p)^2}} = 0.3493$$

$$\gamma = \sqrt{\frac{1-{\delta_s}^2}{\delta_s^2}} = 99.995$$

$$N = \frac{\log\left(\gamma/\epsilon\right)}{\log\left(\Omega_S/\Omega_P\right)} = 9.6241 \rightarrow 10$$

$$\Omega_c = \Omega_S\left(\gamma\right)^{-1/N} = 1.1357$$

or

$$\Omega_c = \Omega_P\left(\epsilon\right)^{-1/N} = 1.1109$$

$$\Omega_c = 1.1357$$

N poles at

$$p_k = \Omega_c e^{j\left(\frac{\pi}{2}+\frac{(2k+1)\pi}{2N}\right)}, \quad k = 0, 1, \ldots N-1$$

N zeros at infinity

$$H_{LP}(s) = \frac{\Omega_c^N}{\prod_{k=0}^{N}\left(s-p_k\right)}$$

$$s = \Omega_P \frac{\hat{s}^2+\hat{\Omega}_0^2}{\hat{s}Bw}$$

$$\hat{s}^2 - \frac{Bw\, s}{\Omega_P}\hat{s} + \hat{\Omega}_0^2 = 0$$

$$\hat{s} = \frac{Bw\, s}{2\,\Omega_P} \pm \sqrt{\left(\frac{Bw\, s}{2\,\Omega_P}\right)^2 - \hat{\Omega}_0^2}$$

$$\hat{p}_k = \frac{Bw\, p_k}{2\,\Omega_P} \pm \sqrt{\left(\frac{Bw\, p_k}{2\,\Omega_P}\right)^2 - \hat{\Omega}_0^2}$$

$$p_1 = 1.1357e^{j\,\frac{11\pi}{20}}$$
$$\hat{p}_1 = 1.1985\times10^5 e^{-j1.6379},\ 2.9645\times10^5 e^{-j1.6379}$$

$$\hat{z}_k = \frac{Bw\, z_k}{2\,\Omega_P} \pm \sqrt{\left(\frac{Bw\, z_k}{2\,\Omega_P}\right)^2 - \hat{\Omega}_0^2}$$

$$z_1 = \infty$$

$$\hat{z}_1 = 0,\ \infty$$

$$H_{BP}(\hat{s}) = \frac{\Omega_c^N}{\prod_{k=0}^{N}\left(\Omega_P \frac{\hat{s}^2+\hat{\Omega}_0^2}{\hat{s}\,Bw} - p_k\right)}$$

$$H_{BP}(\hat{s}) = \frac{\Omega_c^N \hat{s}^N Bw^N / \Omega_P^N}{\prod_{k=0}^{N}\left(\hat{s}^2 - \frac{p_k}{\Omega_P} + \hat{\Omega}_0^2\right)}$$

$$H_{BP}(\hat{s}) = G\frac{\hat{s}^N}{\prod_{k=0}^{2N}\left(\hat{s} - \hat{p}_k\right)}$$

$$G = \Omega_c^N Bw^N / \Omega_P^N = 3.2655 \times 10^{52}$$