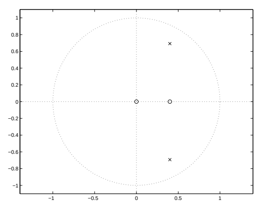
EE 451 - Final Exam

December 11, 2000

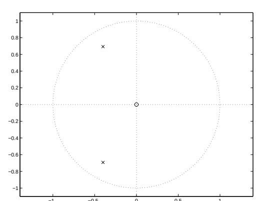
Name: ____

Open book, open notes, and a calculator Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work.

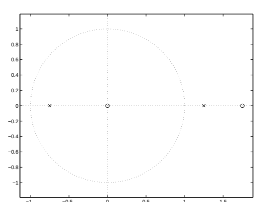
1. There are many ways to represent a discrete-time system. Some of these are: difference equation, pole-zero diagrams, impulse response $h(n)$, system function $H(z)$, frequency response $H(\omega)$, and state-space matrices. Below are the pole-zero diagrams of four systems. This is followed by other other reprenstations of the same systems. Associate the appropriate representations with the pole-zero diagrams. For example, for pole-zero diagram (a), is the system function $H_1(z)$, $H_2(z)$, $H_3(z)$, or $H_4(z)$?

(a) 

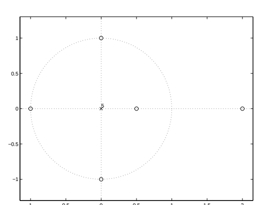
$h(n)$: _______; $H(z)$: _______; $|H(\omega)|$: _______; Diff Eqn: _______; State Space: _______

(b) 

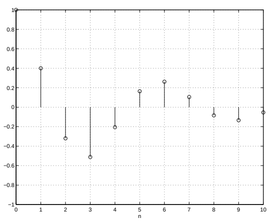
$h(n)$: _______; $H(z)$: _______; $|H(\omega)|$: _______; Diff Eqn: _______; State Space: _______

(c) 

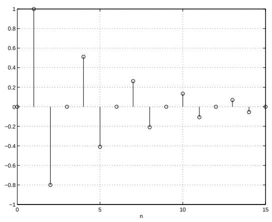
$h(n)$: _______; $H(z)$: _______; $|H(\omega)|$: _______; Diff Eqn: _______; State Space: _______

(d) 

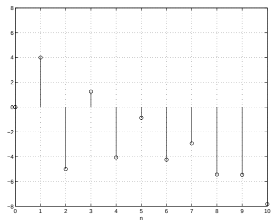
$h(n)$: _______; $H(z)$: _______; $|H(\omega)|$: _______; Diff Eqn: _______; State Space: _______;



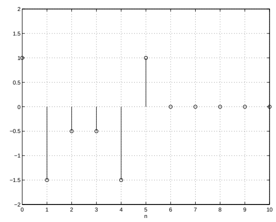
$h_1(n)$



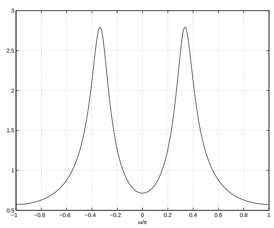
$h_2(n)$



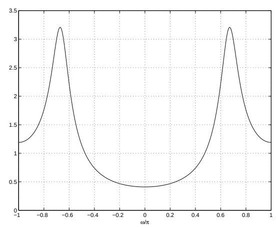
$h_3(n)$



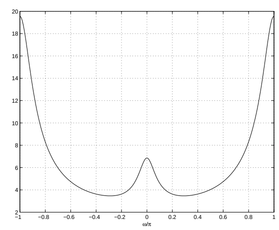
$h_4(n)$



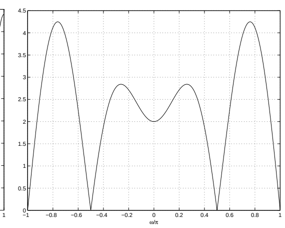
$H_1(\omega)$



$H_2(\omega)$



$H_3(\omega)$



$H_4(\omega)$

$$H_1(z) = \frac{1 - 0.4z^{-1}}{1 - 0.8z^{-1} + 0.64z^{-2}}$$

$$H_2(z) = \frac{z^{-1}}{1 + 0.8z^{-1} + 0.64z^{-2}}$$

$$H_3(z) = \frac{4 - 7z^{-1}}{1 - \frac{1}{2}z^{-1} - \frac{15}{16}z^{-2}}$$

$$H_4(z) = 1 - \frac{3}{2}z^{-1} - \frac{1}{2}z^{-2} - \frac{1}{2}z^{-3} - \frac{3}{2}z^{-4} + z^{-5}$$

$$y_1(n) = x(n) - 0.4x(n-1) + 0.8y_1(n-1) - 0.64y_1(n-2)$$

$$y_2(n) = x(n-1) - 0.8y_2(n-1) - 0.64y_2(n-2)$$

$$y_3(n) = 4x(n) - 7x(n-1) + \frac{1}{2}y_3(n-1) - \frac{15}{16}y_3(n-2)$$

$$y_4(z) = x(n) - \frac{3}{2}x(n-1) - \frac{1}{2}x(n-2) - \frac{1}{2}x(n-3) - \frac{3}{2}x(n-4) + x(n-5)$$

$$\mathbf{F}_1 = \begin{bmatrix} 0.8e^{j\pi/3} & 0 \\ 0 & 0.8e^{-j\pi/3} \end{bmatrix} \quad \mathbf{q}_1 = \begin{bmatrix} 0.5 - j0.2887 \\ 0.5 + j0.2887 \end{bmatrix} \quad \mathbf{g}_1 = \begin{bmatrix} j0.6928 \\ -j0.6928 \end{bmatrix} \quad d_1 = 1$$

$$\mathbf{F}_2 = \begin{bmatrix} 0.8e^{j2\pi/3} & 0 \\ 0 & 0.8e^{-j2\pi/3} \end{bmatrix} \quad \mathbf{q}_2 = \begin{bmatrix} 0.5 - j0.2887 \\ 0.5 + j0.2887 \end{bmatrix} \quad \mathbf{g}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad d_2 = 0$$

$$\mathbf{F}_3 = \begin{bmatrix} -3/4 & 0 \\ 0 & 5/4 \end{bmatrix} \quad \mathbf{q}_3 = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix} \quad \mathbf{g}_3 = \begin{bmatrix} 7.5 \\ -2.5 \end{bmatrix} \quad d_3 = 4$$

$$\mathbf{F}_4 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \mathbf{q}_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{g}_4 = \begin{bmatrix} 1 \\ -3/2 \\ -1/2 \\ -1/2 \\ -3/2 \end{bmatrix} \quad d_4 = 1$$

2. Consider the following signal:

$$x_a(t) = \sin(10\pi t) - 2\sin(30\pi t) + 3\sin(34\pi t)$$

The signal is sampled with a sampling frequency of 200 Hz.

(a) What continuous-time frequencies are present (in Hz) in $x_a(t)$?

(b) What is the discrete-time signal $x(n)$?

(c) What discrete-time frequencies are present in the signal $x(n)$?

(d) What is the output $y_a(t)$? What frequencies are present in $y_a(t)$?

(e) Was the sampling frequency sufficiently high to prevent aliasing? If not, what minimum sampling frequency should have been used?

3. A causal system is described by the difference equation

$$y(n) = x(n) + 0.8y(n-1) - 0.15y(n-2)$$

(a) Find the transfer function $H(z)$.

(b) Draw a pole-zero diagram of $H(z)$.

(c) Find the impulse response $h[n]$.

(d) Find the step response $g(n)$ of the system.

(e) Is the system stable? Explain

(f) Write a state space representation of this system.

4. Data is digitized with a 20 kHz sampling frequency. The only data of interest in the signal is that part with frequency content less that 3 kHz.

   (a) Design a system which changes the sampling rate from 20 kHz to 6 kHz. Draw a block diagram showing how you would change the sampling rate from 20 kHz to 6 kHz. Be sure to specify upsampling and downsampling factors, and filter specifications of the ideal filters.

   (b) Are there any practical problems in implementing the above system? If so, what would you do to make the system easier to implement?

5. Consider the system of Figure X(a), where $H_0(z)$, $H_1(z)$ and $H_2(z)$ are, respectively, ideal linear-phase filters with frequency responses as indicated in Figure X(b). The input $x_a(t)$ has the continuous-time Fourier transform as shown in Figure X(c).

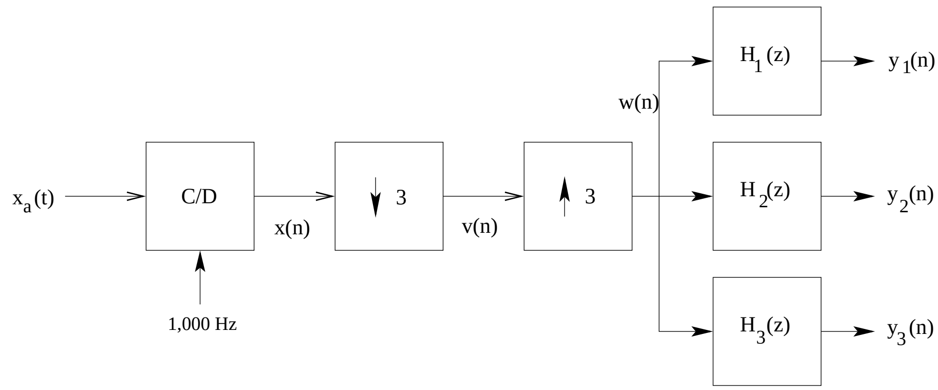


Figure 6(a)

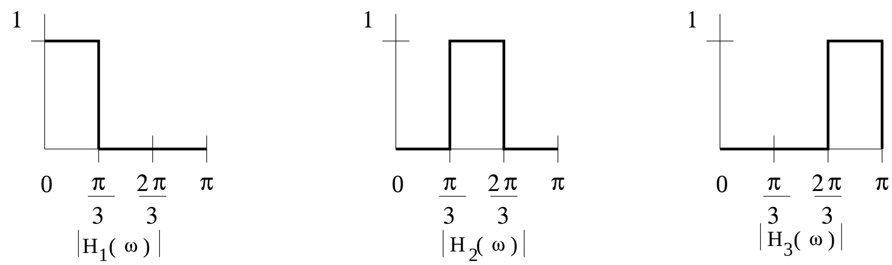


Figure 6(b)

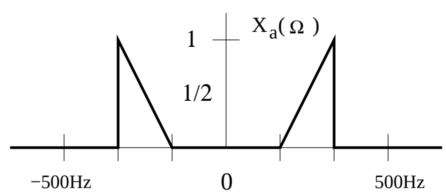


Figure 6(c)

(a) Sketch the discrete-time Fourier-transform of $x(n)$.

(b) Sketch the discrete-time Fourier-transform of $v(n)$.

(c) Sketch the discrete-time Fourier-transform of $w(n)$.

(d) Sketch the discrete-time Fourier-transform of the outputs $y_0(n)$, $y_1(n)$ and $y_2(n)$.

6. A bandlimited continuous-time signal $x_a(t)$ is sampled at a rate of 100 samples per second, generating a discrete-time signal $x(n)$. A 500-point DFT of the signal is taken, using a rectangular window.

   (a) What is the discrete-time frequency resolution (in rad/sample) of the DFT – that is, what is the difference in frequency between $X(k)$ and $X(k+1)$?

   (b) What is the continuous-time frequency resolution (in Hz) of the DFT – that is, what is the difference in frequency between $X(k)$ and $X(k+1)$?

   (c) What should be the stopband edge frequency in Hz of the continuous-time anti-aliasing filter prior to sampling?

   (d) Determine the continuous-time frequency in Hz of the DFT sample $X(31)$.

   (e) Determine the continuous-time frequency in Hz of the DFT sample $X(390)$.

7. You are to design a Butterworth high-pass digital filter satisfying the following specifications using the bilinear transformation:

$$F_{pass} = 5 \text{ kHz} \qquad R_{pass} = 0.3 \text{ dB}$$
$$F_{stop} = 2.5 \text{ kHz} \qquad R_{stop} = 45 \text{ dB}$$

The sampling frequency is 20 kHz.

(a) What are the discrete-time passband (ω_{pass}) and stopband (ω_{stop}) frequencies?

(b) What are the corresponding continuous-time passband (Ω_{pass}) and stopband (Ω_{stop}) frequencies using the bilinear transformation?

(c) What is the order M of the CT filter?

(d) What is the 3 dB frequency Ω_o of the CT filter?

(e) Sketch the s-plane poles and zeros.

(f) Transform one of the s-plane poles to a z-plane pole. Sketch what you thing the z-plane pole-zero diagram would look like if you found all the z-plane poles and zeros..