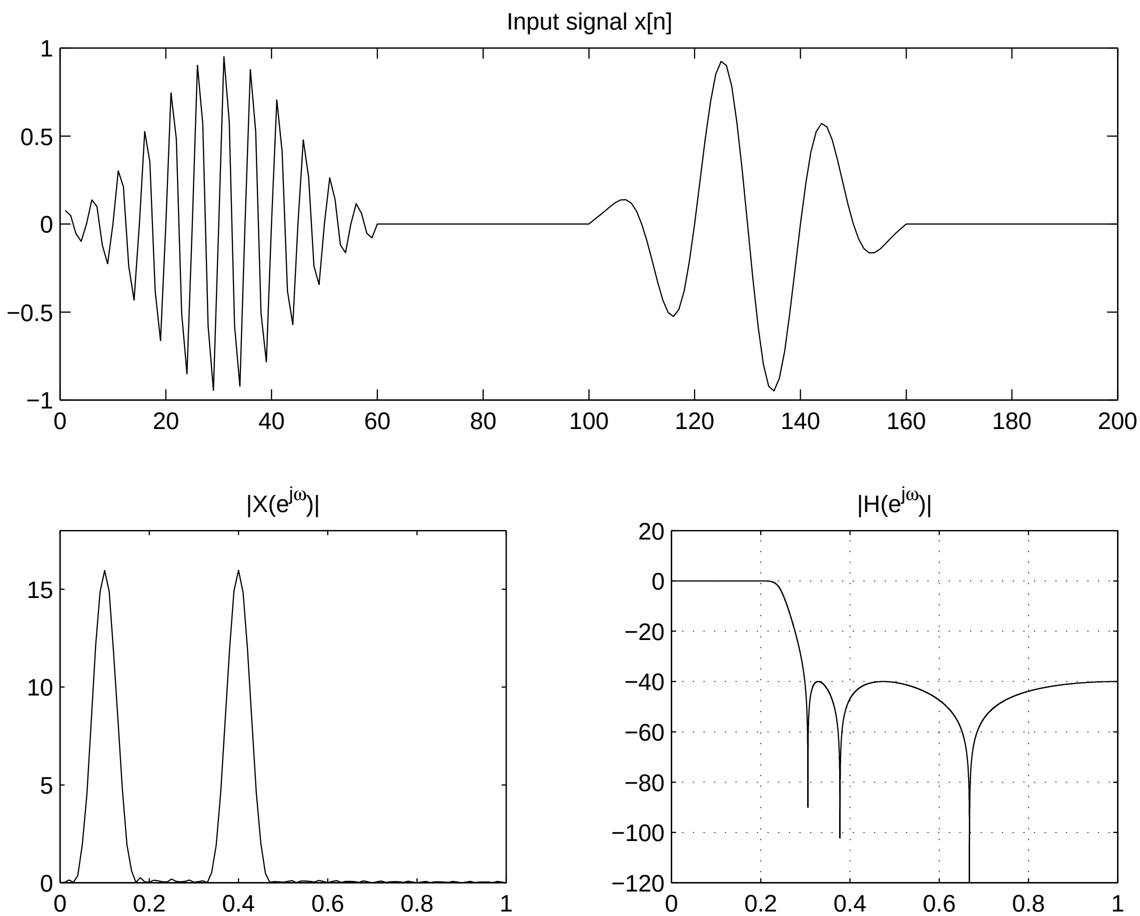
EE 451 - Exam 3

November 19, 1999

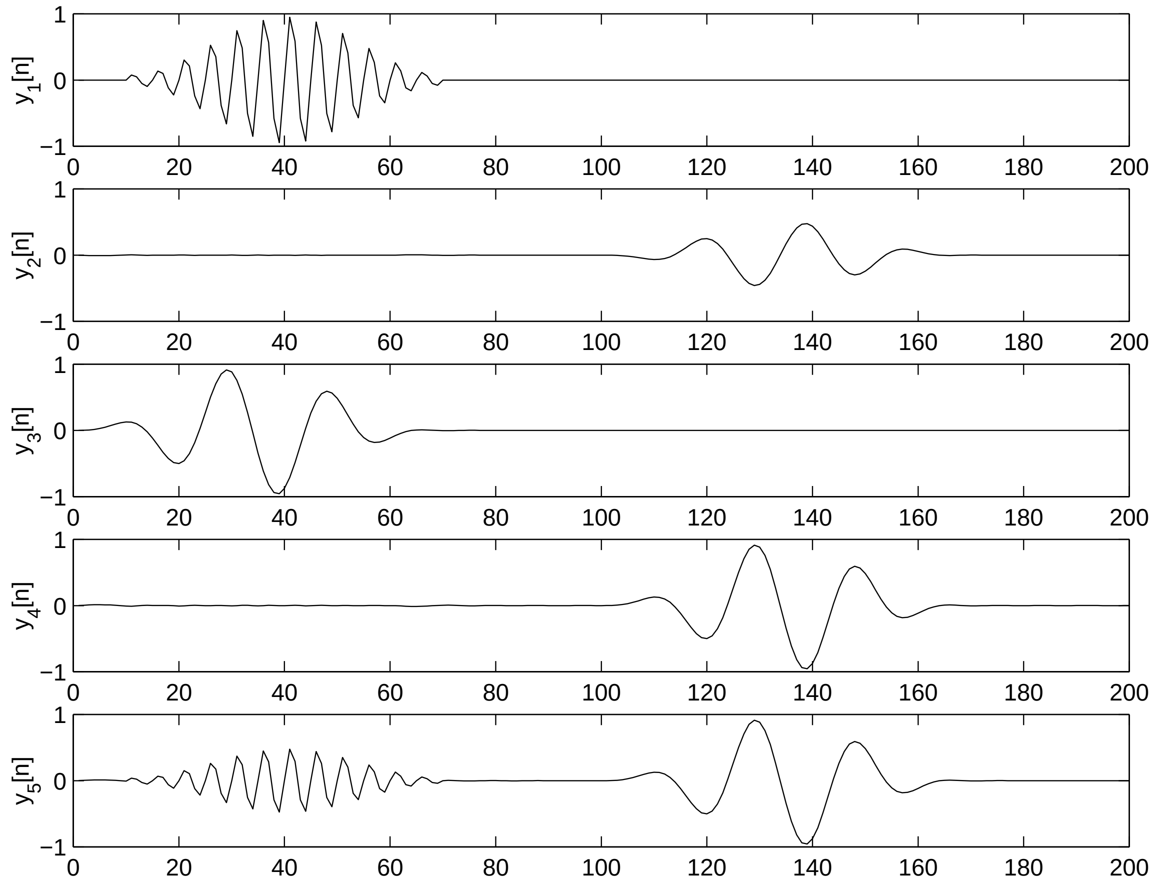
Name: ____

Closed book. You may use a calculator, the sheet on IIR filter design, and one page of notes. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work.

1. The upper panel in the figure below shows a discrete-time signal $x[n]$ and the lower left panel shows its DTFT $X(e^{j\omega})$. Note that the discrete samples of $x[n]$ have been connected by straight lines using the MATLAB `plot` command. The signal $x[n]$ is used as the input to a causal LTI filter with the frequency response magnitude $|H(e^{j\omega})$ shown in the lower right panel. Note that this magnitude is plotted on a dB scale, i.e., $20\log_{10}|H(e^{j\omega})|$.



(a) The figure below shows five possible output signals $y_1[n]$ through $y_5[n]$. Which of these five signals could be the output of the filter when $x[n]$ is the input? For each possible output, justify your answer. **Choices without justification will be assumed to be guesses and will receive no credit.**



i. $y_1[n]$: Yes No Why: ____________________

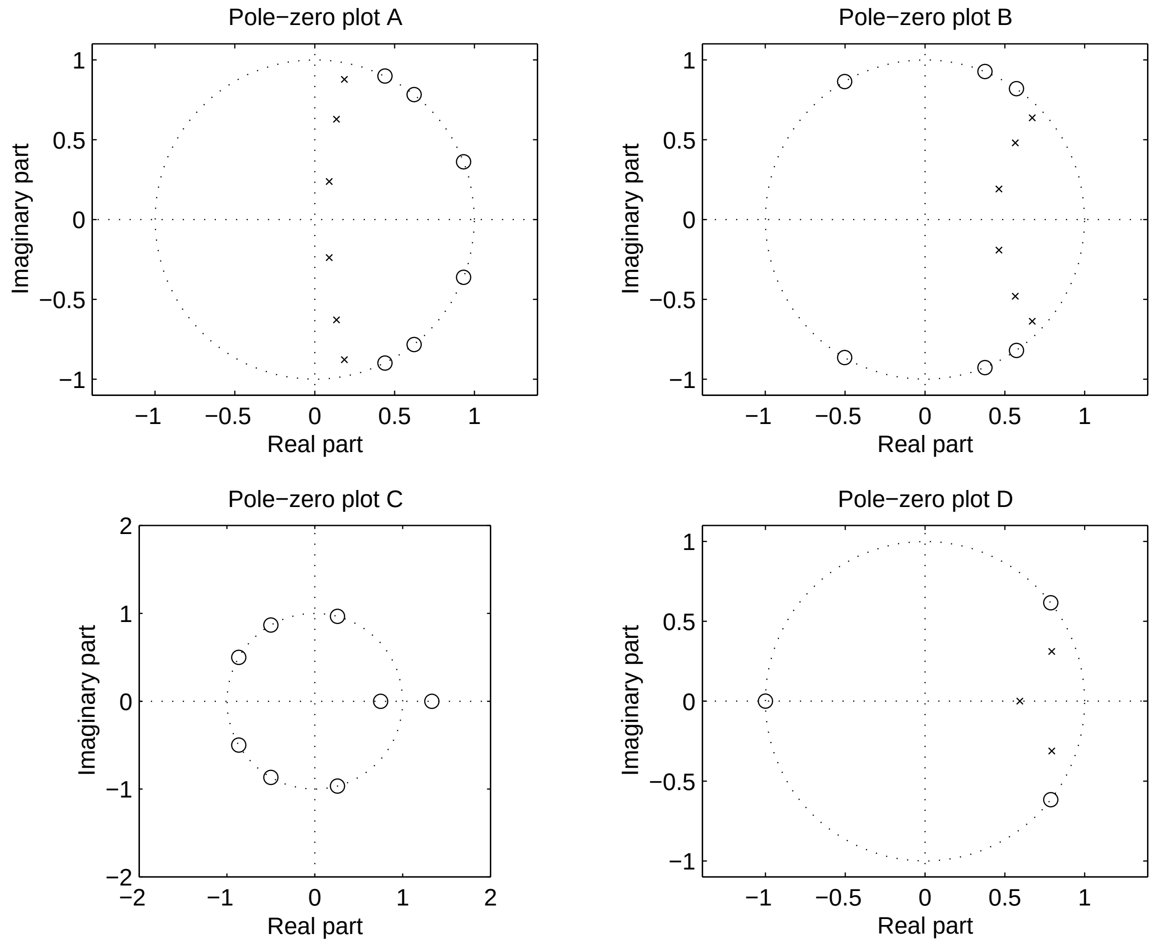
ii. $y_2[n]$: Yes No Why: ____________________

iii. $y_3[n]$: Yes No Why: ____________________

iv. $y_4[n]$: Yes No Why: ____________________

v. $y_5[n]$: Yes No Why: ____________________

(b) The figure below shows four different pole-zero plots. Which of these pole-zero plots could belong to the filter $|H(e^{j\omega})|$? Again, justify your answer for each choice. **Choices without justification will be assumed to be guesses and will receive no credit.**



i. Plot A: `Yes` `No` `Why:` ____________________

ii. Plot B: `Yes` `No` `Why:` ____________________

iii. Plot C: `Yes` `No` `Why:` ____________________

iv. Plot D: `Yes` `No` `Why:` ____________________

2. A discrete-time filter is characterized by the following properties:
   (1) It is high-pass and has one pole and one zero.
   (2) The pole is at a distance $r = 0.9$ from the origin of the z-plane.
   (3) Constant signals do not pass through the system.
   (4) The gain at frequency $\omega = \pi$ is 1.

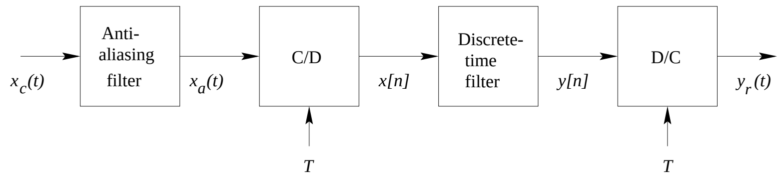
   (a) Plot the pole-zero diagram of the filter.

   (b) Determine its system function $H(z)$.

   (c) Sketch (do not compute exactly) the magnitude response $|H(e^{j\omega})|$.

   (d) Determine the input-output relation (difference equation) of the filter in the time domain.

3. You need to filter a continuous-time signal with a discrete-time system as shown below:



You are to design a Butterworth low-pass digital filter satisfying the following specifications (on the continuous-time signal $x_c(t)$) using the bilinear transformation:

$$f_{pass} = 3 \text{ kHz} \quad A_{pass} = 0.5 \text{ dB}$$
$$f_{stop} = 4 \text{ kHz} \quad A_{stop} = 60 \text{ dB}$$

The sampling frequency is 10 kHz.

(a) What are the discrete-time passband (ω_{pass}) and stopband (ω_{stop}) frequencies?

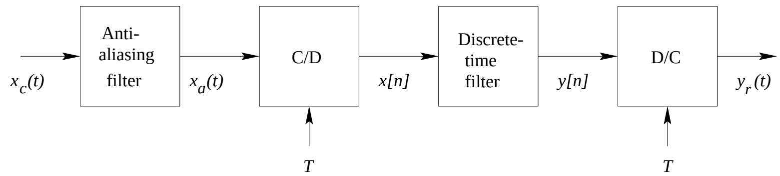
(b) What are the corresponding continuous-time passband (Ω_{pass}) and stopband (Ω_{stop}) frequencies?

(c) What is the order M of the CT filter?

(d) What is the 3 dB frequency Ω_o of the CT filter?

(e) Assume the CT filter has a real pole at $-\Omega_o$. Where will this pole appear in the z-plane after applying the bilinear transformation?

4. You need to filter a continuous-time signal with a discrete-time system as shown below:



You are to design an FIR low-pass digital filter satisfying the following specifications (on the continuous-time signal $x_c(t)$) using the window method:

$$
\begin{array}{ll}
f_{pass} = 3 \text{ kHz} & A_{pass} = 0.5 \text{ dB} \\
f_{stop} = 4 \text{ kHz} & A_{stop} = 60 \text{ dB}
\end{array}
$$

The sampling frequency is 10 kHz.

(a) Which of the common windows we discussed in class will be able to meet the specifications for this filter? Why?

(b) Assume you decide to design the filter using a Kaiser window. What order N of filter will you need?

(c) What is the value of β you will need for the Kaiser filter?

(d) Why might you want to use an FIR filter rather than an IIR filter in a discrete-time system?