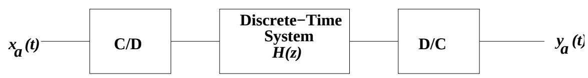
# EE 451

## Homework #9

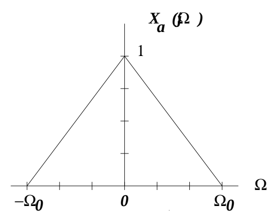
Due October 22, 2001

1. A signal $x_a(t)$ is processed by the system shown in (a) below. The spectrum of $x_a(t)$ is shown in (b) below, where $\Omega_o = 2\pi(1000)$ rad/sec. The discrete-time system $H(z)$ in an ideal lowpass filter with frequency response

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| < \omega_c, \\ 0, & otherwise \end{cases}$$




(a)

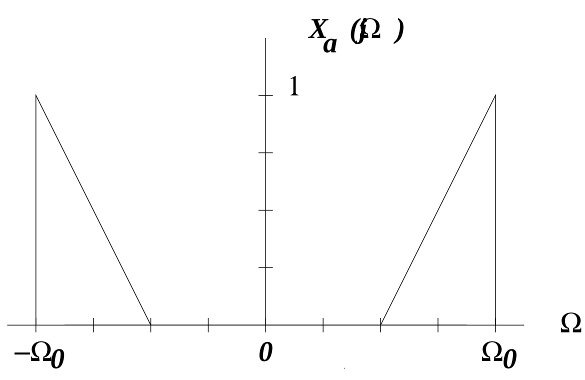



(b)

   (a) What is the minimum sampling frequency $F_S = 1/T$ such that no aliasing occurs in sampling the input?

   (b) If $\omega_c = \pi/2$, what is the minimum sampling frequency such that $y_a(t) = x_a(t)$?

2. A continuous-time signal $x_a(t)$, with Fourier transform $X_a(j\Omega)$ shown below, is sampled with a sampling period $T = 2\pi/\Omega_0$ to form the sequence $x[n] = x_a(nT)$.




   (a) Sketch the Fourier transform $X(e^{j\omega})$ for $|\omega| < \pi$.

   (b) The signal $x[n]$ is to be transmitted across a digital channel. At the receiver, the original signal $x_a(t)$ must be recovered. Draw a block diagram of the recovery system and specify its characteristics. Assume ideal filters are available.

   (c) In terms of $\Omega_0$, for what range of values of $T$ can $x_a(t)$ be recovered from $x[n]$?

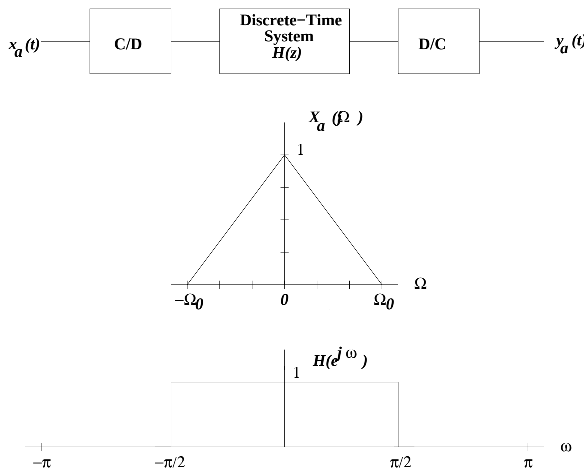
3. In the system below, $X_a(j\Omega)$ and $H(e^{j\omega})$ are as shown. Note that $\Omega_0 = 2\pi \times 5 \times 10^3$. Sketch and label the Fourier transform of $y_a(t)$ for each of the following cases:

   (a) $1/T_1 = 1/T_2 = 10^4$

   (b) $1/T_1 = 1/T_2 = 2 \times 10^4$

   (c) $1/T_1 = 2 \times 10^4, \qquad 1/T_2 = 10^4$

   (d) $1/T_1 = 10^4, \qquad 1/T_2 = 2 \times 10^4$




4. A Butterworth analog lowpass filter is to be designed with the following specifications:

$$F_P = 2\,kHz \quad R_P = 0.5 \text{ dB}$$
$$F_S = 8\,kHz \quad R_S = 30 \text{ dB}$$

   (a) What order filter $N$ is required?

   (b) What 3-dB frequency $\Omega_c$ will you use?

   (c) Find the location of the poles of the filter in the $s$-domain.

   (d) Write down the transfer function $H(s)$ of the filter. Use second-order sections in the denominator so all coefficients are real.

   (e) Plot the frequency response of the filter using the MATLAB `freqs` function. Verify that the filter meets the specifications.

   (f) Check your answers to (a), (b) and (c) using the MATLAB `buttord` and `butter` functions.

5. A Butterworth analog high-pass filter is to be designed with the following specifications:

$$F_P = 8\,kHz \quad R_P = 0.5 \text{ dB}$$
$$F_S = 2\,kHz \quad R_S = 30 \text{ dB}$$

   (a) Using $\Omega_p = 1$ find the specifications of the lowpass transfer function $H_{LP}(s)$ which can be used as a prototype to design the highpass filter.

   (b) What order filter $N$ is required?

   (c) What 3-dB frequency $\Omega_c$ will you use?

(d) Find the location of the poles of the lowpass prototype filter in the $s$-domain.

(e) Write down the transfer function $H_{LP}(s)$ of the filter. Use second-order sections in the denominator so all coefficients are real.

(f) Use Equation 5.59 of the Text to find $H_{HP}(s)$.

(g) Plot the frequency response of the filter using the MATLAB `freqs` function. Verify that the filter meets the specifications.

6. A Butterworth bandpass filter is to be designed with the following specifications:

$$\begin{array}{lll} F_{P1} = 20 \text{ kHz} & F_{P2} = 45 \text{ kHz} & R_P = 0.5 \text{ dB} \\ F_{S1} = 10 \text{ kHz} & F_{S2} = 60 \text{ kHz} & R_S = 40 \text{ dB} \end{array}$$

(a) Find $\Omega_0$, the center frequency of the bandpass filter.

(b) Using $\Omega_p = 1$ find the specifications of the lowpass transfer function $H_{LP}(s)$ which can be used as a prototype to design the bandpass filter.

(c) What order filter $N$ is required?

(d) What 3-dB frequency $\Omega_c$ will you use?

(e) Find the location of the poles of the lowpass prototype filter in the $s$-domain.

(f) Find the locations of the poles and zeros of the bandpass filter from part (e) and Equation 5.61 of the Text.