This tutorial is available as a supplement to the textbook *Fundamentals of Signals and Systems Using Matlab* by Edward Kamen and Bonnie Heck, published by Prentice Hall. It has been altered from its original form (from the ftp site ftp.mathworks.com in pub/books/heck) for use in the Electrical Engineering Department at New Mexico Tech. The tutorial covers basic MATLAB commands that are used in introductory signals and systems analysis. It is meant to serve as a quick way to learn MATLAB and a quick reference to the commands that are used in this textbook. For more detailed information, the reader should consult the official MATLAB documentation. An easy way to learn MATLAB is to sit down at a computer and follow along with the examples given in this tutorial and the examples given in the textbook.

The tutorial is designed for students using either the professional version of MATLAB (ver. 4.0) with the Control Systems Toolbox (ver. 3.0) and the Signal Processing Toolbox (ver. 2.0), or using the Student Edition of MATLAB (ver. 4.0).

The topics covered in this tutorial are:

1. INTRODUCTION

MATLAB is an interactive program for scientific and engineering calculations. The MATLAB family of programs includes the base program plus a variety of toolboxes (collections of special files called M-files that extend the functionality of the base program). Together, the base program and toolboxes provide a powerful tool for engineering design and analysis in a variety of areas.

Most of the statements, functions, and commands are computer platform independent. This means regardless of the particular computer system you are using, your interaction with MATLAB will be basically the same. A typical session will utilize some basic features that allow you to interact with the program: (1) statements and variables, (2) graphics, (3) matrices, and (4) M-files. The goal of this tutorial is to introduce you to these four basic features along with some advanced features for the design and analysis of signals and linear systems.

2. WHERE TO FIND MATLAB AT NEW MEXICO TECH

MATLAB can be found at the Tech Computer Center SPARCstation Room (Spear 14) and in the Electrical Engineering Digital Laboratory (Workman 116).

3. MATLAB BASICS

MATLAB is started by various methods depending on which computer environment you are using. When using the Unix-based SPARCstations in the Tech Computer Center, start a MATLAB session by typing `matlab` at an x-window prompt. The PC's in the Electrical Engineering Digital Laboratory are dual boot meaning they can be brought up in either Windows or Linux. At this time MATLAB will only run under Windows, so make sure you enter this environment upon login. In Windows, start your MATLAB session by double clicking on the MATLAB icon. Once started, MATLAB provides you with an interactive window and the prompt '`>>`' at which you can type commands. MATLAB is ended in all environments by typing `exit` at the prompt or by using the menu option. After each MATLAB command, the "return" or "enter" key must be depressed.

A. Definition of Variables

Variables are alphanumeric names that can be assigned numerical values. The names begin with a letter and are followed by any number of letters and numbers up to 19 characters. Variables are assigned numerical values by typing the expression directly, for example, typing

```
a = 1+2
```

yields:

```
a =
        3
```

The answer will not be displayed when a semicolon is put at the end of an expression, for example type `a = 1+2`;.

MATLAB utilizes the following arithmetic operators:

+	addition
-	subtraction
*	multiplication
/	division
^	power operator
'	transpose

A variable can be assigned using a formula that utilizes these operators and either numbers or previously defined variables. For example, since `a` was previously defined, the following expression is valid

```
b = 2*a;
```

To determine the value of a previously defined quantity, type the quantity by itself:

```
b
```

yields:

```
b =
         6
```

If your expression does not fit on one line, use an ellipsis (three or more periods) at the end of the line) and continue on the next line.

```
c = 1+2+3+...
     5+6+7;
```

There are several predefined variables which can be used at any time, in the same manner as user-defined variables:

`i`	sqrt(-1)
`j`	sqrt(-1)
`pi`	3.1416...

For example,

```
y= 2*(1+4*j)
```

yields:

```
y=
      2.0000 + 8.0000i
```

There are also a number of predefined functions that can be used when defining a variable. Some common functions that are used in this text are:

abs	magnitude of a complex number (absolute value for real numbers)
sqrt	square root of a number
angle	angle of a complex number, in radians
conj	conjugate of a complex number
imag	imaginary part of a complex number
real	real part of a number
cos	cosine function, assumes argument is in radians
acos	arccosine function
sin	sine function, assumes argument is in radians
asin	arcsine function
tan	tangent function, assumes argument in radians
atan	arctangent function
atan2	four quadrant arctangent function (requires two arguments)
exp	exponential function
log	natural logarithm of a number
log10	base 10 logarithm of a number

For example, with `y` defined as above,

```
c = abs(y)
```

yields:

```
c =

    8.2462
```

```
c = angle(y)
```

yields:

```
c =

    1.3258
```

With `a`=3 as defined previously,

```
c = cos(a)
```

yields:

```
c =

   -0.9900
```

```
c = exp(a)
```

yields:

```
c =

   20.0855
```

Note that `exp` can be used on complex numbers. For example, with `y = 2+8i` as defined above,

```
c = exp(y)
```

yields:

```
c =

  -1.0751 + 7.3104i
```

which can be verified by using Euler's formula:

$$c = e^2\cos(8) + je^2\sin(8)$$

B. Definition of Matrices

MATLAB is based on matrix and vector algebra; even scalars are treated as 1x1 matrices. Therefore, vector and matrix operations are as simple as common calculator operations.

Vectors can be defined in two ways. The first method is used for arbitrary elements:

```
v = [1 3 5 7];
```

creates a 1x4 vector with elements 1, 3, 5 and 7. Note that commas could have been used in place of spaces to separate the elements. Additional elements can be added to the vector:

```
v(5) = 8;
```

yields the vector `v = [1 3 5 7 8]`. Previously defined vectors can be used to define a new vector. For example, with `v` defined above

```
a = [9 10];
b = [v a];
```

creates the vector `b = [1 3 5 7 8 9 10]`.

The second method is used for creating vectors with equally spaced elements:

```
t = 0:.1:10;
```

creates a 1x101 vector with the elements 0, .1, .2, .3,...,10. Note that the middle number defines the increment. If only two numbers are given, then the increment is set to a default of 1:

```
k = 0:10;
```

creates a 1x11 vector with the elements 0, 1, 2, ..., 10.

Matrices are defined by entering the elements row by row (rows are separated by semicolons):

```
M = [1 2 4; 3 6 8];
```

creates the matrix

$$M = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 6 & 8 \end{bmatrix}$$

There are a number of special matrices that can be defined:

null matrix:	`M = [];`
nxm matrix of zeros:	`M = zeros(n,m);`
nxm matrix of ones:	`M = ones(n,m);`
nxn identity matrix:	`M = eye(n);`

A particular element of a matrix can be assigned:

```
M(1,2) = 5;
```

places the number 5 in the first row, second column.

In this text, matrices are used only in Chapter 12; however, vectors are used throughout the text. Operations and functions that were defined for scalars in the previous section can also be used on vectors and matrices. For example,

```
a = [1 2 3];
b = [4 5 6];
c = a + b
```

yields:
```
c =
     5 7 9
```

When matrix dimensions are compatible, all the basic matrix operations are defined including addition (+), subtraction (-), multiplication (*), transpose (‘), determinant (det), and inverse (inv).

Functions are applied element by element. For example,

```
t = 0:10;
x = cos(2*t);
```

creates a vector `x` with elements equal to cos(2t) for t = 0, 1, 2, ..., 10.

Operations that need to be performed element-by-element can be accomplished by preceding the operation by a ".". For example, to obtain a vector `x` that contains the elements of x(t) = tcos(t) at specific points in time, you cannot simply multiply the vector `t` with the vector `cos(t)`. Instead you multiply their elements together:

```
t = 0:10;
x = t.*cos(t);
```

C. General Information

Matlab is case sensitive so "`a`" and "`A`" are two different names.

Comment statements are preceded by a "`%`".

On-line help for MATLAB can be reached by typing `help` for the full menu or typing `help` followed by a particular function name or M-file name. For example, `help cos` gives help on the cosine function.

The number of digits displayed is not related to the accuracy. To change the format of the display, type `format short e` for scientific notation with 5 decimal places, `format long e` for scientific notation with 15 significant decimal places and `format bank` for placing two significant digits to the right of the decimal.

The commands `who` and `whos` give the names of the variables that have been defined in the workspace. The `clear` command by itself clears all the variables from your work space while `clear` followed by variable names clears only those variables.

The command `length(x)` returns the length of a vector `x` and `size(x)` returns the dimension of the matrix `x`.

D. M-files

M-files are macros of MATLAB commands that are stored as ordinary text files with the extension "m", that is *filename*.m. An M-file can be either a function with input and output variables or a list of commands. All of the MATLAB examples in this textbook are contained in M-files that are available at the MathWorks ftp site.

The following describes the use of M-files on a PC version of MATLAB. MATLAB requires that the M-file must be stored either in the working directory or in a directory that is specified in the MATLAB path list. For example, consider using MATLAB on a PC with a user-defined M-file stored in a directory called "\MATLAB\MFILES". Then to access that M-file, either change the working directory by typing `cd\matlab\mfiles` from within the MATLAB command window or by adding the directory to the path. Permanent addition to the path is accomplished by editing the \MATLAB\matlabrc.m file, while temporary modification to the path is accomplished by typing `path(path,'\matlab\mfiles`') from within MATLAB.
The M-files associated with this textbook should be downloaded from the MathWorks ftp site and copied to a subdirectory named "\MATLAB\KAMEN" and then this directory should be added to the path. The M-files that come with MATLAB are already in appropriate directories and can be used from any working directory.

As example of an M-file that defines a function, create a file in your working directory named yplusx.m that contains the following commands:

```
function z = yplusx(y,x)
z = y + x;
```

The following commands typed from within MATLAB demonstrate how this M-file is used:

```
x = 2;
y = 3;
z = yplusx(y,x)
```

MATLAB M-files are most efficient when written in a way that utilizes matrix or vector operations. Loops and if statements are available, but should be used sparingly since they are computationally inefficient. An example of the use of the command `for` is

```
for k=1:10,
   x(k) = cos(k);
end
```

This creates a 1x10 vector x containing the cosine of the positive integers from 1 to 10. This operation is performed more efficiently with the commands

```
k = 1:10;
x = cos(k);
```

which utilizes a function of a vector instead of a for loop. An `if` statement can be used to define conditional statements. An example is

```
if(a <= 2),
  b = 1;
elseif(a >= 4)
  b = 2;
else
  b = 3;
end
```

The allowable comparisons between expressions are >=, <=, <, >, ==, and ~=.

Several of the M-files written for this textbook employ a user-defined variable which is defined with the command `input`. For example, suppose that you want to run an M-file with different values of a variable `T`. The following command line within the M-file defines the value:

```
T = input('Input the value of T: ')
```

Whatever comment is between the quotation marks is displayed to the screen when the M-file is running, and the user must enter an appropriate value.

E. Recording your work

The `diary` function is useful for recording and documenting your work in MATLAB. It records everything you enter as well as MATLAB's responses and stores them into a file. To begin recording your MATLAB work, type `diary filename` where filename is the name of the file you want MATLAB to create and store your work in. Once this is done you can perform all of your calculations as usual while everything you are seeing in your MATLAB window is being recorded into your diary file. Once your work is done, type `diary off` to stop recording. You may now edit your file outside of MATLAB (using emacs or the Notepad) and print it out (select `File->Print` in the emacs or Notepad editor menu).

4. FOURIER ANALYSIS

Commands covered:
```
dft
idft
fft
ifft
contfft
```

The `dft` command uses a straightforward method to compute the discrete Fourier transform. Define a vector `x` and compute the DFT using the command

```
X = dft(x)
```

The first element in `X` corresponds to the value of X(0). The function `dft` is available from the MathWorks ftp site and is defined in Figure C.2 of the textbook.

The command `idft` uses a straightforward method to compute the inverse discrete Fourier transform. Define a vector `X` and compute the IDFT using the command

```
x = idft(X)
```

The first element of the resulting vector `x` is x[0]. The function `idft` is available at the MathWorks ftp site and is defined in Figure C.3 of the textbook.

For a more efficient but less obvious program, the discrete Fourier transform can be computed using the command `fft` which performs a Fast Fourier Transform of a sequence of numbers. To compute the FFT of a sequence x[n] which is stored in the vector `x`, use the command

```
X = fft(x)
```

Used in this way, the command `fft` is interchangeable with the command `dft`. For more computational efficiency, the length of the vector `x` should be equal to an exponent of 2, that is 64, 128, 512, 1024, 2048, etc. The vector `x` can be padded with zeros to make it have an appropriate length. MATLAB does this automatically by using the following command where N is defined to be an exponent of 2:

```
X = fft(x,N);
```

The longer the length of `x`, the finer the grid will be for the FFT. Due to a wrap around effect, only the first N/2 points of the FFT have any meaning.

The `ifft` command computes the inverse Fourier transform:

```
x = ifft(X);
```

The FFT can be used to approximate the Fourier transform of a continuous-time signal as shown in Section 6.6 of the textbook. A continuous-time signal x(t) is sampled with a period of T seconds, then the DFT is computed for the sampled signal. The resulting amplitude must be scaled and the corresponding frequency determined. An M-file that approximates the Fourier Transform of a sampled continuous-time signal is available from the ftp site and is given below:

```
function [X,w] = contfft(x,T);
[n,m] = size(x);
if n<m,
  x = x';
end
Xk = fft(x);
N = length(x);
n = 0:N-1;
n(1) = eps;
X = (1-exp(-j*2*pi*n/N))./(j*2*pi*n/N/T).*Xk.';
w = 2*pi*n/N/T;
```

The input is the sampled continuous-time signal x and the sampling time T. The outputs are the Fourier transform stored in the vector `X` and the corresponding frequency vector `w`.

5. CONTINUOUS TIME SYSTEM ANALYSIS

A. Transfer Function Representation

Commands covered:

```
tf2zp
zp2tf
cloop
feedback
parallel
series
```

Transfer functions are defined in MATLAB by storing the coefficients of the numerator and the denominator in vectors. Given a continuous-time transfer function

$$H(s) = \frac{B(s)}{A(s)}$$

where $B(s) = b_M s^M + b_{M-1} s^{M-1} + \ldots + b_0$ and $A(s) = s^N + a_{N-1} s^{N-1} + \ldots + a_0$. Store the coefficients of B(s) and A(s) in the vectors `num` $= [b_M\ b_{M-1}\ \ldots\ b_0]$ and `den` $= [1\ a_{N-1}\ \ldots\ a_0]$. In this text, the names of the vectors are generally chosen to be `num` and `den`, but any other name could be used. For example,

$$H(s) = \frac{2s+3}{s^3+4s^2+5}$$

is defined by

```
num = [2 3];
den = [1 4 0 5];
```

Note that all coefficients must be included in the vector, even zero coefficients.

A transfer function may also be defined in terms of its zeros, poles and gain:

$$H(s) = \frac{k(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

To find the zeros, poles and gain of a transfer function from the vectors `num` and `den` which contain the coefficients of the numerator and denominator polynomials, type

```
[z,p,k] = tf2zp(num,den)
```

The zeros are stored in `z`, the poles are stored in `p`, and the gain is stored in `k`. To find the numerator and denominator polynomials from `z, p,` and `k,` type

```
[num,den] = zp2tf(z,p,k)
```

The overall transfer function of individual systems in parallel, series or feedback can be found using MATLAB. Consider block diagram reduction of the different configurations shown in Figure 1. Store the transfer function G in `numG` and `denG`, and the transfer function H in `numH` and `denH`.

To reduce the general feedback system to a single transfer function, $G_{cl}(s) = G(s)/(1+G(s)H(s))$ type

```
[numcl,dencl] = feedback(numG,denG,numH,denH);
```

For a unity feedback system, let `numH = 1` and `denH = 1` before applying the above algorithm. Alternately, use the command

```
[numcl,dencl] = cloop(numG,denG,-1);
```

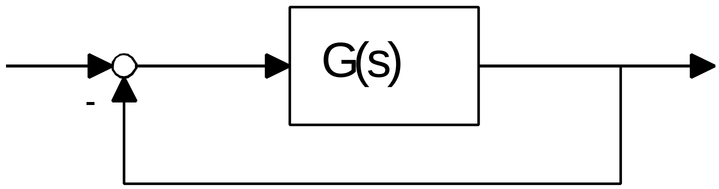
To reduce the series system to a single transfer function, $G_s(s) = G(s)H(s)$ type

```
[nums,dens] = series(numG,denG,numH,denH);
```

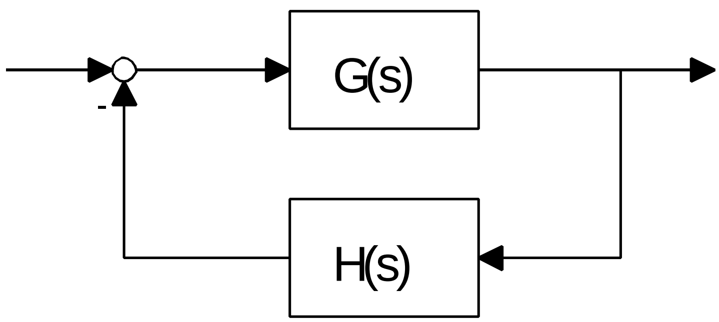
To reduce the parallel system to a single transfer function, $G_p(s) = G(s) + H(s)$ type

```
[nump,denp] = parallel(numG,denG,numH,denH);
```

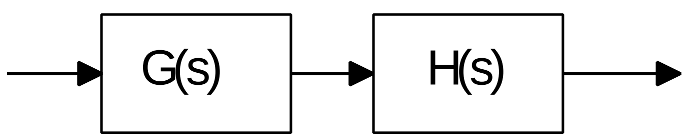
(Parallel is not available in the Student Version.)



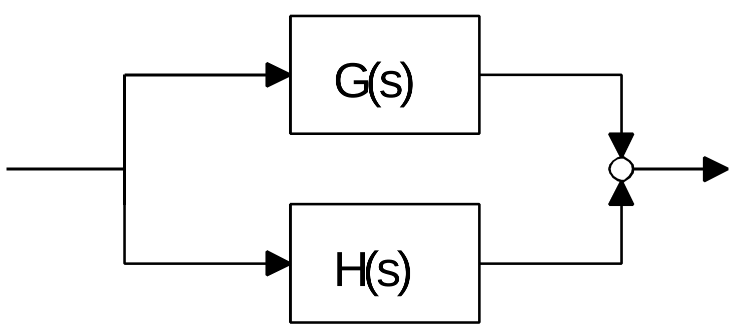
unity feedback



feedback



series



parallel

B. Time Simulations

Commands covered: `residue`, `step`, `impulse`, `lsim`

The analytical method to find the time response of a system requires taking the inverse Laplace Transform of the output Y(s). MATLAB aides in this process by computing the partial fraction expansion of Y(s) using the command `residue`. Store the numerator and denominator coefficients of Y(s) in `num` and `den`, then type

```
[r,p,k] = residue(num,den)
```

The residues are stored in `r`, the corresponding poles are stored in `p`, and the gain is stored in `k`. Once the partial fraction expansion is known, an analytical expression for y(t) can be computed by hand.

A numerical method to find the response of a system to a particular input is available in MATLAB. First store the numerator and denominator of the transfer function in `num` and `den`, respectively. To plot the step response, type

```
step(num,den)
```

To plot the impulse response, type

```
impulse(num,den)
```

For the response to an arbitrary input, use the command `lsim`. Create a vector `t` which contains the time values in seconds at which you want MATLAB to calculate the response. Typically, this is done by entering

```
t = a:b:c;
```

where `a` is the starting time, `b` is the time step and `c` is the end time. For smooth plots, choose `b` so that there are at least 300 elements in `t` (increase as necessary). Define the input `x` as a function of time, for example, a ramp is defined as `x = t`. Then plot the response by typing

```
lsim(num,den,x,t);
```

To customize the commands, the time vector can be defined explicitly and the step response can be saved to a vector. Simulating the response for five to six time constants generally is sufficient to show the behavior of the system. For a stable system, a time constant is calculated as 1/Re(-p) where p is the pole that has the largest real part (i.e., is closest to the origin).

For example, consider a transfer function defined by

$$H(s) = \frac{2}{s+2}$$

The step response y is calculated and plotted from the following commands:

```
num = 2; den = [1 2];
t = 0:3/300:3;    % for a time constant of 1/2
y = step(num,den,t);
plot(t,y)
```

For the impulse response, simply replace the word `step` with `impulse`. For the response to an arbitrary input stored in `x`, type

```
y = lsim(num,den,x,t);
plot(t,y)
```

C. Frequency Response Plots

Commands covered:
```
freqs
bode
logspace
log10
semilogx
unwrap
```

To compute the frequency response H(ω) of a transfer function, store the numerator and denominator of the transfer function in the vectors `num` and `den`. Define a vector w that contains the frequencies for which H(ω) is to be computed, for example `w = a:b:c` where `a` is the lowest frequency, `c` is the highest frequency and `b` is the increment in frequency. The command

```
H = freqs(num,den,w)
```

returns a complex vector `H` that contains the value of H(ω) for each frequency in `w`.

To draw a Bode plot of a transfer function which has been stored in the vectors `num` and `den`, type

```
bode(num,den)
```

To customize the plot, first define the vector `w` which contains the frequencies at which the Bode plot will be calculated. Since `w` should be defined on a log scale, the command `logspace` is used. For example, to make a Bode plot ranging in frequencies from 10^{-1} to 10^{2}, define `w` by

```
w = logspace(-1,2);
```

The magnitude and phase information for the Bode plot can then be found be executing:

```
[mag,phase] = bode(num,den,w);
```

To plot the magnitude in decibels, convert `mag` using the following command:

```
magdb = 20*log10(mag);
```

To plot the results on a semilog scale where the y-axis is linear and the x-axis is logarithmic, type

```
semilogx(w,magdb)
```

for the log-magnitude plot and type

```
semilogx(w,phase)
```

for the phase plot. The phase plot may contain jumps of $\pm 2\pi$ which may not be desired. To remove these jumps, use the command `unwrap` prior to plotting the phase.

```
semilogx(w,unwrap(phase))
```

D. Analog Filter Design

Commands covered:

```
buttap
cheb1ab
zp2tf
lp2lp
lp2bp
lp2hp
lp2bs
```

MATLAB contains commands for various analog filter designs, including those for designing a Butterworth filter and a Type I Chebyshev filter. The commands `buttap` and `cheb1ab` are used to design lowpass Butterworth and Type I Chebyshev filters, respectively, with cutoff frequencies of 1 rad/sec. For an n-pole Butterworth filter, type

```
[z,p,k] = buttap(n)
```

where the zeros of the filter are stored in `z`, the poles are stored in `p` and the gain of the filter is in `k`. For an n-pole Type I Chebyshev filter with `Rp` decibels of ripple in the passband, type

```
[z,p,k] = cheb1ab(n,Rp)
```

To find the numerator and denominator polynomials of the resulting filter from `z`, `p` and `k`; type

```
[b,a] = zp2tf(z,p,k)
```

where `a` contains the denominator coefficients and `b` contains the numerator coefficients.

Frequency transformations from one lowpass filter to another with a different cutoff frequency, or from lowpass to highpass, or lowpass to bandstop or lowpass to bandpass can be performed in MATLAB. These transformations can be used with either the Butterworth filters or the Chebyshev filters. Suppose `b` and `a` store the numerator and denominator of a transfer function of a lowpass filter with cutoff frequency 1 rad/sec. To map to a lowpass filter with cutoff frequency `Wo` and numerator and denominator coefficients stored in `b1` and `a1`, type

```
[b1,a1] = lp2lp(b,a,Wo)
```

To map to a highpass filter with cutoff frequency `Wo`, type

```
[b1,a1] = lp2hp(b,a,Wo)
```

To map to a bandpass filter with bandwidth `Bw` centered at the frequency `Wo`, type

```
[b1,a1] = lp2bp(b,a,Wo,Bw)
```

To map to a bandstop filter with stopband bandwidth `Bw` centered about the frequency `Wo`, type

```
[b1,a1] = lp2bs(b,a,Wo,Bw)
```

E. Control Design

Commands covered: `rlocus`

Consider a feedback loop as shown in Figure 1 where G(s)H(s) = KP(s) and K is a gain and P(s) contains the poles and zeros of the controller and of the plant. The root locus is a plot of the roots of the closed loop transfer function as the gain is varied. Suppose that the numerator and denominator coefficients of P(s) are stored in the vectors `num` and `den`. Then the following command computes and plots the root locus:

```
rlocus(num,den)
```

To customize the plot for a specific range of K, say for K ranging from 0 to 100, then use the following commands:

```
K = 0:100;
r = rlocus(num,den,K);
plot(r,'.')
```

The graph contains dots at points in the complex plane that are closed loop poles for integer values of `K` ranging from 0 to 100. To get a finer grid of points, use a smaller increment when defining `K`, for example, `K = 0:.5:100`. The resulting matrix `r` contains the closed poles for all of the gains defined in the vector `K`. This is particularly useful to calculate the closed loop poles for one particular value of `K`. Note that if the root locus lies entirely on the real axis, then using `plot(r,'.')` gives inaccurate results.

F. State Space Representation

Commands Covered: `step`
`lsim`
`ss2tf`
`tf2ss`
`ss2ss`

The standard state space representation is used in MATLAB, i.e.,

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

where x is nx1 vector, u is mx1, y is px1, A is nxn, B is nxm, and C is pxn. The response of a system to various inputs can be found using the same commands that are used for transfer function representations: `step`, `impulse`, and `lsim`. The argument list contains the A, B, C, and D matrices instead of the numerator and denominator vectors. For example, the step response is obtained by typing:

```
[y,x,t] = step(A,B,C,D);
```

The states are stored in `x`, the outputs in `y` and the time vector, which is automatically generated, is stored in `t`. The rows of `x` and `y` contain the states and outputs for the time points in `t`. Each column of `x` represents a state. For example, to plot the second state versus time, type

```
plot(t,x(:,2))
```

To find the response of an arbitrary input or to find the response to initial conditions, use `lsim`. Define a time vector `t` and an input matrix `u` with the same number of rows as in `t` and the number of columns equaling the number of inputs. An optional argument is the initial condition vector `x0`. The command is then given as

```
[y,x] = lsim(A,B,C,D,u,t,x0);
```

You can find the transfer function for a single-input/single-output (SISO) system using the command:

```
[num,den] = ss2tf(A,B,C,D);
```

The numerator coefficients are stored in `num` and the denominator coefficients are stored in `den`.

Given a transformation matrix `T`, the `ss2ss` function will perform the similarity transform. Store the state space model in `A`, `B`, `C` and `D` and the transformation matrix in `T`.

```
[Abar,Bbar,Cbar,Dbar]=ss2ss(A,B,C,D,T);
```

performs the similarity transform z=Tx resulting in a state space system that is defined as:

$$\dot{x} = \bar{A}x + \bar{B}u$$
$$y = \bar{C}x + \bar{D}u$$

where $A = TAT^{-1}$, $B = TB$, $C = CT^{-1}$, $D = D$.

6. DISCRETE-TIME SYSTEM ANALYSIS

A. Convolution

Commands covered: `conv`
`deconv`

To perform discrete time convolution, x[n]*h[n], define the vectors x and h with elements in the sequences x[n] and h[n]. Then use the command

```
y = conv(x,h)
```

This command assumes that the first element in `x` and the first element in `h` correspond to n=0, so that the first element in the resulting output vector corresponds to n=0. If this is not the case, then the output vector will be computed correctly, but the index will have to be adjusted. For example,

```
x = [1 1 1 1 1];
h = [0 1 2 3];
y = conv(x,h);
```

yields `y = [0 1 3 6 6 6 5 3]`. If `x` is indexed as described above, then y[0] = 0, y[1] = 1, ... In general, total up the index of the first element in `h` and the index of the first element in `x`, this is the index of the first element in `y`. For example, if the first element in `h` corresponds to n = -2 and the first element in `x` corresponds to n = -3, then the first element in `y` corresponds to n = -5.

Care must be taken when computing the convolution of infinite duration signals. If the vector `x` has length q and the vector `h` has length r, then you must truncate the vector `y` to have length min(q,r). See the comments in Problem 3.7 of the textbook for additional information.

The command `conv` can also be used to multiply polynomials: suppose that the coefficients of a(s) are given in the vector `a` and the coefficients of b(s) are given in the vector `b`, then the coefficients of the polynomial a(s)b(s) can be found as the elements of the vector defined by `ab = conv(a,b)`.

The command `deconv` is the inverse procedure to the convolution. In this text, it is used as a means of dividing polynomials. Given a(s) and b(s) with coefficients stored in `a` and `b`, then the coefficients of c(s) = b(s)/a(s) are found by using the command `c = deconv(b,a)`.

B. Transfer Function Representation

For a discrete-time transfer function, the coefficients are stored in descending powers of z or ascending powers of z^{-1}. For example,

$$H(z) = \frac{2z^2+3z+4}{z^2+5z+6} = \frac{2+3z^{-1}+4z^{-2}}{1+5z^{-1}+6z^{-2}}$$

then define the vectors as

```
num = [2 3 4];
den = [1 5 6];
```

C. Time Simulations

Commands Covered: `recur`
`conv`
`dstep`
`dimpulse`
`filter`

There are three methods to compute the response of a system described by the following recursive relationship

$$y[n] + \sum_{i=1}^{N} a_i y[n-i] = \sum_{i=0}^{M} b_i x[n-i]$$

The first method uses the command `recur` and is useful when there are nonzero initial conditions. This command is available from the MathWorks ftp site and a shortened version is given in Figure C.5 of the textbook. The inputs to the function are the coefficients a_i and b_i stored in the vectors $a = [a_1\ a_2\ \ldots\ a_N]$ and $b = [b_0\ b_1\ \ldots\ b_M]$, the initial conditions on x and on y are stored in the vectors $x0 = [x[n_0-M],\ x[n_0-M+1], \ldots, x[n_0-1]]$ and $y0 = [y[n_0-N],\ y[n_0-N+1], \ldots, y[n_0-1]]$, and the time indices for which the solution needs to be calculated are stored in the vector `n` where n_0 represents the first element in this vector. To use `recur`, type

```
y = recur(a,b,n,x,x0,y0);
```

The output is a vector `y` with elements y[n]; the first element of `y` corresponds to the time index n_0. For example, consider the system described by

$$y[n] - 0.6y[n-1] + 0.08y[n-2] = x[n-1]$$

where x[n] = u[n] and with initial conditions y[-1] = 2, y[-2] = 1, and x[-1] = x[-2] = 0. To compute the response y[n] for n = 0, 1,...,10, type

```
a = [-0.6 0.08]; b = [0 1];
x0 = 0; y0 = [1 2];
n = 0:10;
x = ones(1,11);
y = recur(a,b,n,x,x0,y0);
```

The vector `y` contains the values of y[n] for n = 0,1,...,10.

The second method to compute the response uses convolution and is useful when the initial conditions on y are zero. This method involves first finding the impulse response of the system, h[n], and then convolving h[n] with x[n] as discussed in Section 4.A. For example, consider the system described above with zero initial conditions, that is, y[-1]=y[-2]=0. The impulse response for this system is $h[n] = 5[(0.4)^n-(0.2)^n]u[n]$. The commands to compute y[n] are

```
n = 0:10;
x = ones(1,11);
h = 5*(0.4).^n - 5*(.02).^n;
y = conv(x,h);
y = y(1:length(n));
```

The vector `y` contains the values of y[n] for n = 0,1,...,10. Note that the vector was truncated to `length(n)` because both x[n] and h[n] are infinite duration signals. See the comments in Section 4.A regarding the convolution of infinite duration signals.

The third method of solving for the response requires that the transfer function of the system be known. The commands `dstep` and `dimpulse` compute the unit step response and the unit impulse response, respectively while the command `filter` computes the response to initial conditions and to arbitrary inputs. The denominator coefficients are stored as `den = [1` a_1 a_2 `...` a_N`]` and the numerator coefficients are stored as `num = [`b_0 b_1 `...` b_M`, 0 ... 0]` where there are N-M zeros padded on the end of the coefficients. For example, consider the system given above with initial conditions y[-1] = y[-2] = 0. To compute the step response for n=0 to n=10, type the commands

```
n = 0:10;
num = [0 1 0]; den = [1 -0.6 0.08];
y = dstep(num,den,length(n));
```

The response can then be plotted using the `stem` plot. To compute the impulse response, simply replace `dstep` with `dimpulse` in the above commands.

To compute the response to an arbitrary input, store the input sequence in the vector `x`. The command

```
y = filter(num,den,x);
```

is used to compute the system response. If the system has nonzero initial conditions, the initial conditions can be stored in a vector v0. For a first order system where N=M=1, define `zi = [`b_1`*x[-1]-a*y[-1]]`. For a second order system where N=M=2, define `zi = [`b_1`*x[-1]+`b_2`*x[-2]-`a_1`*y[-1]-`a_2`*y[-2], `b_1`*x[-1]-`a_2`*y[-1]]`. To compute the response with nonzero initial conditions, type

```
y = filter(num,den,x,zi);
```

For example, consider the previous system with the initial conditions y[-1] = 2 and y[-2] = 1 and input x[n] = u[n]. Type the following commands to compute y[n].

```
n = 0:10;  x = ones(1:11);
num = [0 1 0]; den = [1 -0.6 0.08];
zi = [0.6*2-0.08*1, -0.08*2];
y = filter(num,den,x,zi);
```

D. Frequency Response Plots

Commands covered: `freqz`

The DTFT of a system can be calculated from the transfer function using `freqz`. Define the numerator and the denominator of the transfer function in `num` and `den`. The command

```
[H,Omega] = freqz(num,den,n,'whole');
```

computes the DTFT for n points equally spaced around the unit circle at the frequencies contained in the vector `Omega`. The magnitude of `H` is found from `abs(H)` and the phase of `H` is found from `angle(H)`. To customize the range for Ω, define a vector `Omega` of desired frequencies, for example `Omega = -pi:2*pi/300:pi` defines a vector of length 301 with values that range from -π to π. To get the DTFT at these frequencies, type

```
H = freqz(num,den,Omega);
```

E. Digital Filter Design

Commands covered:

```
bilinear
butter
cheby1
hamming
hanning
```

The analog prototype method of designing IIR filters can be done by first designing an analog filter with the desired characteristics as shown in Section 3.D, then mapping the filter to the discrete-time domain. Store the numerator and denominator of the analog filter, H(s), in the vectors `num` and `den`, and let `T` be the sampling period. Then the numerator and denominator of the digital filter $H_d(z)$ is found from the following command

```
[numd,dend] = bilinear(num,den,1/T)
```

Alternately, the commands `butter` and `cheby1` automatically design the analog filter and then use the bilinear transformation to map the filter to the discrete-time domain. Lowpass, highpass, bandstop, and bandpass filters can be designed using this method. The digital cutoff frequencies must be specified; these should be normalized by π. To design a digital lowpass filter based on the analog Butterworth filter, use the commands

```
[num,den] = butter(n,Omegac)
```

where `n` is the number of poles and `Omegac` is the normalized digital cutoff frequency, $\Omega_c = \omega_c T/\pi$. To design a highpass filter with cutoff frequency `Omegac`, use the commands

```
[num,den] = butter(n,Omegac,'high')
```

To design a bandpass filter with passband from `Omega1` to `Omega2`, define `Omega = [Omega1,Omega2]` and use the command

```
[num,den] = butter(n,Omega)
```

To design a bandstop filter with stopband from `Omega1` to `Omega2`, define `Omega = [Omega1, Omega2]` and use the command

```
[num,den] = butter(n,Omega,'stop')
```

The design for an n^{th} order Type I Chebyshev filter is accomplished using the same methods as for `butter` except that "`butter`" is replaced by "`cheby1`":

```
[num,den] = cheby1(n,Omegac);  % for a lowpass filter
[num,den] = cheby1(n,Omegac,'high'); % for a highpass filter
```

If `Omega` has two elements,

```
[num,den] = cheby1(n,Omega);  % for a bandpass
[num,den] = cheby1(n,Omega,'stop');  % for a bandstop
```

The windows used in FIR filter design are given by

```
w = boxcar(N)     % rectangular window
w = hamming(N)
w = hanning(N)
```

These commands are used to truncate the infinite impulse response of an ideal digital filter with the result being an FIR filter with length N.

The Signal Processing Toolbox also provides commands for computing the FIR filter directly. To obtain an FIR filter with length N and cutoff frequency Omegac (normalized by π) use the command

```
hd = fir1(N-1,Omegac)
```

The vector `hd` contains the impulse response of the FIR where `hd(1)` is the value of $h_d[0]$. A length N highpass filter with normalized cutoff frequency Omegac is designed by using the command

```
hd = fir1(N-1,Omegac,'high')
```

A bandpass with passband from `Omega1` to `Omega2` is obtained by typing

```
hd = fir1(N-1,Omega)
```

where `Omega = [Omega1,Omega2]`. A bandstop filter with stopband from `Omega1` to `Omega2` is obtained by typing

```
hd = fir1(N-1,Omega,'stop')
```

where `Omega = [Omega1,Omega2]`. The `fir1` command uses the Hamming window by default. Other windows are obtained by adding an option of `'hanning'` or `'boxcar'` to the arguments; for example,

```
hd = fir1(N-1,Omegac,'high',boxcar(N))
```

creates a highpass FIR filter with cutoff frequency `Omegac` using a rectangular window.

F. Digital Control Design

Commands covered: `bilinear`
`c2dm`
`hybrid`

An analog controller $G_c(s)$ can be mapped to a digital controller $G_d(z)$ using the bilinear transformation or the step response matching method. Store the numerator and denominator of $G_c(s)$ in `num` and `den`. Then the numerator and denominator of $G_d(z)$ is found from the bilinear transformation using the commands

```
[numd,dend] = bilinear(num,den,1/T)
```

where `T` is the sampling frequency. To use the step invariant method, use the commands

```
[numd,dend] = c2dm(num,den,T,'zoh')
```

To simulate the response of a continuous-time plant with a digital controller, use the command `hybrid`, which is available at the MathWorks ftp site. Consider the block diagram in Figure 11.25. The numerator and denominator coefficients of the plant are stored in `NGp` and `DGp`; the numerator and denominator coefficients of the controller are stored in `NGd` and `DGd`; the reference input signal is stored in `r`; and the sampling time is stored in `T`. The increments in the time vector should selected to be the sampling time divided by an integer, for example, `t = 0:b:Tend` where there is some integer m such that bm=T. The command is used as

```
[y,ud] = hybrid(NGp,DGp,NGd,DGd,T,t,r);
```

The outputs of the command are the system response, `y`, and the control signal that is input to the plant, `ud`. The M-file contains a loop which computes the discrete-time control and then simulates the continuous-time plant for T seconds with the constant control. The process repeats for the next T second interval. The commands for `hybrid` are given below:

```
function [Y,UD] = hybrid(Np,Dp,Nd,Dd,T,t,U);
[Ac,Bc,Cc,Dc]=tf2ss(Np,Dp);
[Ad,Bd,Cd,Dd]=tf2ss(Nd,Dd);
nsam = T/(t(2)-t(1));    % # of integration pts per sample

% initialize
Y = 0;
UD = 0;
[ncr,ncc] = size(Ac);
xc0 = zeros(ncr,1);
[ndr,ndc] = size(Ad);
xdk = zeros(ndr,1);
kmax = fix(t(length(t))/T);  % # of complete samples in t
for k = 0:kmax-1

  % calculate control and output of zoh
  ek = U(k*nsam+1) - Y(k*nsam+1);
  xd = Ad*xdk + Bd*ek;
  zoh = Cd*xdk + Dd*ek;
  xdk = xd;

  % integrate continuous-time plant with input
  %   of zoh for T seconds
  udi = zoh*ones(nsam+1,1);
  ti = t(k*nsam+1:(k+1)*nsam+1);
  [yi,xi] = lsim(Ac,Bc,Cc,Dc,udi,ti,xc0);
  xc0 = xi(nsam+1,:);

  % augment vectors
  Y = [Y;yi(2:nsam+1)];
  UD = [UD;udi(2:nsam+1)];
end

if(kmax*nsam+1 < length(t))
% compute tail of simulation from t(kmax*nsam)
%   to t_end
  k = kmax;

% calculate control and output of zoh
  ek = U(k*nsam+1) - Y(k*nsam+1);
  xd = Ad*xdk + Bd*ek;
  zoh = Cd*xdk + Dd*ek;

% integrate continuous-time plant with input of zoh
  ti = t(k*nsam+1:length(t));
  udi = zoh*ones(length(ti),1);
  [yi,xi] = lsim(Ac,Bc,Cc,Dc,udi,ti,xc0);
```

```
% augment vectors
  Y = [Y;yi(2:length(yi))];
  UD = [UD;udi(2:length(udi))];
end
```

G. State Space Representation

Commands Covered: `dlsim` `dstep` `dimpulse`

Most of the commands for the continuous time state space representation also work for the discrete time state space. For example, `ss2tf`, `tf2ss`, and `ss2ss` for discrete time are used exactly the same way as for the continuous time case discussed in Section 3.F. There is a discrete time version of the command `lsim`, which is used as follows:

```
[y,x] = dlsim(A,B,C,D,u,n);
```

where the output is stored in `y`, the states are stored in `x`, the input is stored in `u` and the time index is stored in `n`.

7. PLOTTING

Commands covered: `plot` `xlabel` `ylabel` `title` `grid` `axis` `stem` `subplot`

The command most often used for plotting is `plot`, which creates linear plots of vectors and matrices; `plot(t,y)` plots the vector `t` on the x-axis versus vector `y` on the y-axis. There are options on the line type and the color of the plot which are obtained using plot(t,y,'option'). The linetype options are '-' solid line (default), '--' dashed line, '-.' dot dash line, ':' dotted line. The points in `y` can be left unconnected and delineated by a variety of symbols: + . * o x. The following colors are available options:

r	red
b	blue
g	green
w	white
k	black

For example, `plot(t,y,'--')` uses a dashed line, `plot(t,y,'*')` uses * at all the points defined in `t` and `y` without connecting the points, and `plot(t,y,'g')` uses a solid green line. The options can also be used together, for example, `plot(t,y,'g:')` plots a dotted green line.

To plot two or more graphs on the same set of axes, use the command `plot(t1,y1,t2,y2)`, which plots `y1` versus `t1` and `y2` versus `t2`.

To label your axes and give the plot a title, type

```
xlabel('time (sec)')
ylabel('step response')
title('My Plot')
```

Finally, add a grid to your plot to make it easier to read. Type

```
grid
```

The problem that you will encounter most often when plotting functions is that MATLAB will scale the axes in a way that is different than you want them to appear. You can easily override the autoscaling of the axes by using the `axis` command after the plotting command:

```
axis([xmin xmax ymin ymax]);
```

where `xmin, xmax, ymin,` and `ymax` are numbers corresponding to the limits you desire for the axes. To return to the automatic scaling, simply type `axis`.

For discrete-time signals, use the command `stem` which plots each point with a small open circle and a straight line. To plot y[k] versus k, type `stem(k,y)`. The command can be modified to plot circles which are filled in. Specifically, the following line in `stem`

```
h = plot(x,y,'o',xx(:),yy(:),linetype);
```

can be replaced with

```
h = plot(x,y,'.',xx(:),yy(:),linetype);
set(h,'markersize',18);
```

to create closed circles.

To plot more than one graph on the screen, use the command `subplot(mnp)` which partitions the screen into an mxn grid where p determines the position of the particular graph counting the upper left corner as p=1. For example,

```
subplot(211),semilogx(w,magdb);
subplot(212),semilogx(w,phase);
```

plots the bode plot with the log-magnitude plot on top and the phase plot below. Titles and labels can be inserted immediately after the appropriate `semilogx` command or `plot` command. To return to a full screen plot, type `subplot(111)`.

8. LOADING AND SAVING DATA

When using MATLAB, you may wish to leave the program but save the vectors and matrices you have defined. To save the file to the working directory, type

```
save filename
```

where "`filename`" is a name of your choice. To retrieve the data later, type

```
load filename
```