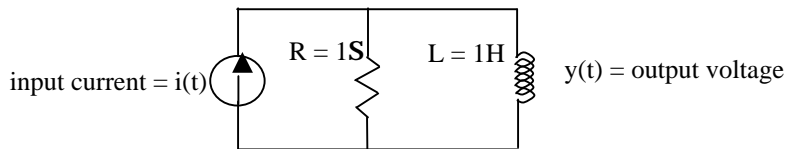


1. P8.5-3 (note: $y[k] - y[k-1]$. $Tf[k]$ is also a valid approximation of an integrator)
2. P8.5-4a
3. P9.1-1 (solve by hand, first three terms $\Rightarrow y[0], y[1], y[2]$)
4. P9.1-2 (solve by hand, first three terms $\Rightarrow y[0], y[1], y[2]$)
5. For the difference equations given, determine their order and if they are linear, time-invariant, and causal. Note $y[k]$ is the output and $f[k]$ the input.
 - a. $y[k + 1] = y[k]f[k - 1]$
 - b. $y[k + 2] - 5y[k + 1] + 7y[k] + 1 = 4f[k + 1] - 2f[k]$
 - c. $y[k] + 2^k y[k - 1] + 3y[k - 3] = 1.5^{k-1} f[k + 1] + f[k]$
 - d. $3.3y[k + 1] + y[k] - 1.2y[k - 1] = 2f[k + 1] + 2.1f[k] + 2.2f[k - 1]$
6. Consider the RL circuit shown.



- a. Compute the output voltage $y(t)$ (express in analytical form) for all $t \geq 0$ when $y(0^-) = 0$ and $i(t) = u(t) - u(t-1)$ where $u(t)$ is the unit step function.
 - b. Using Euler's approximation of derivatives with T arbitrary and input $i(t)$ arbitrary, derive a difference equation model for the RL circuit.
 - c. Use your answer to part b with $T = 0.1$ sec, $i(t) = u(t) - u(t-1)$ (that needs to be discretized), and matlab to recursively solve for and plot the approximation of $y(t)$ for $0 \text{sec} \leq t \leq 5 \text{sec}$. Plot the exact solution from part a on the same graph and compare the results.
7. Consider the differential equation $\frac{d^2 y(t)}{dt^2} + 3 \frac{dy(t)}{dt} + 2y(t) = 0, y(0^-) = 1, \dot{y}(0^-) = 0$.
- a. Compute $y(t)$ and express in analytical form, then plot $y(t)$ for $0 \text{sec} \leq t \leq 10 \text{sec}$.
 - b. Using Euler's approximation to the derivative with T arbitrary, derive a difference equation model from the differential equation.
 - c. Recursively solve your difference equation in part b using $T = 0.4$ and then $T = 0.1$ for $0 \text{sec} \leq t \leq 10 \text{sec}$ and plot your results. Compare these numerical solutions to the exact solution plotted in part a. Which time interval T gives a better approximation? Why?
8. Consider the differential equation model of a high-speed vehicle on a horizontal surface

$$12 \frac{dv(t)}{dt} + 0.9v(t) + 0.6v^2(t) = f(t)$$

$$v(0^-) = 7$$

$$f(t) = 9t(u(t) - u(t - 5))$$

where $v(t)$ is the vehicle velocity and $f(t)$ is the drive/brake force.

- a. Solve for $v(t)$ as an analytical expression.
- b. Using Euler's approximation to the derivative with T arbitrary, derive a difference equation model from the differential equation.
- c. Using the answer in part b with $T = 0.2$ sec, compute and plot the approximation to $v(t)$ using recursion for $0 \text{sec} \leq t \leq 20 \text{sec}$.
- d. Using the answer in part b with $T = 2.0$ sec, compute and plot the approximation to $v(t)$ using recursion for $0 \text{sec} \leq t \leq 20 \text{sec}$.
- e. Are you more confident in your approximation from part c or d? Why?