EE342 Problem Set 4 Due: W 02/17/99

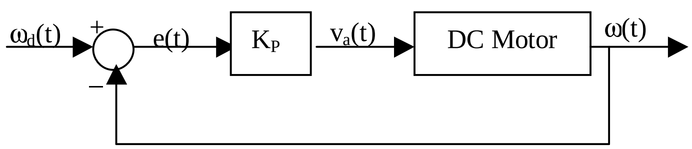
The following problems deal with simulating the motion control of a differential drive mobile robot. The end objective is a DC motor powered mobile robot capable of accurate left-wall following. Each DC motor will be assumed to have the transfer function model shown below where $\omega(t)$ is the motor speed (output) in krpm and $v_a(t)$ is the armature voltage (motor input) in volts.

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_T}{LJs^2 + (RJ + \beta L)s + \beta R + K_T K_b}$$

As is the case with real DC motors their parameters will vary from motor to motor. Sample values for two motors (the left and right) with loads are shown.

	Left DC Motor	Right DC Motor
K_T (oz•in/A)	3.9	4.0
L (mH)	1.55	1.50
J (oz•in•m•m /krot)	2.0	2.1
R (Ω)	1.92	1.84
β (oz•in/krpm)	10.4	13.3
K_b (V/krpm)	3.1	3.0

1. Consider proportional control of each motor as shown in block diagram form below where $\omega_d(t)$ represents the desired speed and e(t) is the velocity error.



   a) Convert the motor transfer function of one motor into a differential equation and then approximate with a difference equation relating $\omega[n]$ and $v_a[n]$ as found in example 7. Write the difference equation approximation for the motor in general without substituting in specific motor values.
   b) Write the proportional control equation for one motor in difference equation form relating $v_a[n]$, $\omega_d[n]$, and $\omega[n]$.
   c) Assuming zero initial conditions, numerically simulate both motors (via recursion) under proportional control using $\omega_d(t) = u(t)$ krpm, $K_P = 200$, and sample-time T=0.001. Plot simulated response of $\omega(t)$ for both motors for $0.0 \le t \le 0.2$ seconds.

2. We now wish to add in a way for ensuring the mobile robot goes straight (motor speeds are equal) by varying $\omega_d(t)$ for each motor. One way of accomplishing this is to use an integral type outer-loop controller to ensure both motors rotate at the same velocities. The outer loop control law looks as follows:

$$\omega_d = \omega_{ref} \pm K_I \int_0^t (\omega_R - \omega_L)\, dt$$

   Where ω_{ref} is the velocity we'd like the motors to rotate at when it goes straight; ω_R, ω_L are the right and left motor velocities, respectively, K_I is a controller gain, and the sign is determined by which motor is being considered.

   a) Discretize the outer-loop controller and write its difference equation.
   b) Add the outer-loop controller to your simulation with both motors included and recursively simulate the two-motor system for $K_I = 50$, $\omega_{ref} = 1$krpm, and all other parameters as previously defined. Plot your motor velocities for $0.0 \le t \le 0.2$ seconds and comment on whether they become the same.