EE443L Lab 2: Modeling a DC Motor

Introduction:

In this lab we will develop and validate a basic model of a permanent magnet DC motor (Yaskawa Electric MINI-Minertia motor). This model will be utilized throughout the semester to assist in the development of controllers for the various DC motor actuated experiments. The specific input/output relationship, which we are interested in determining, is the manner in which the input (the armature voltage) affects the motor's speed and position. Developing a suitable model of a system is typically the first step towards controlling its behavior.

Prelab:

The DC motor is a common actuator in control systems that converts electrical energy into rotational mechanical energy. A representation of the electrical and mechanical components is shown in the following figure

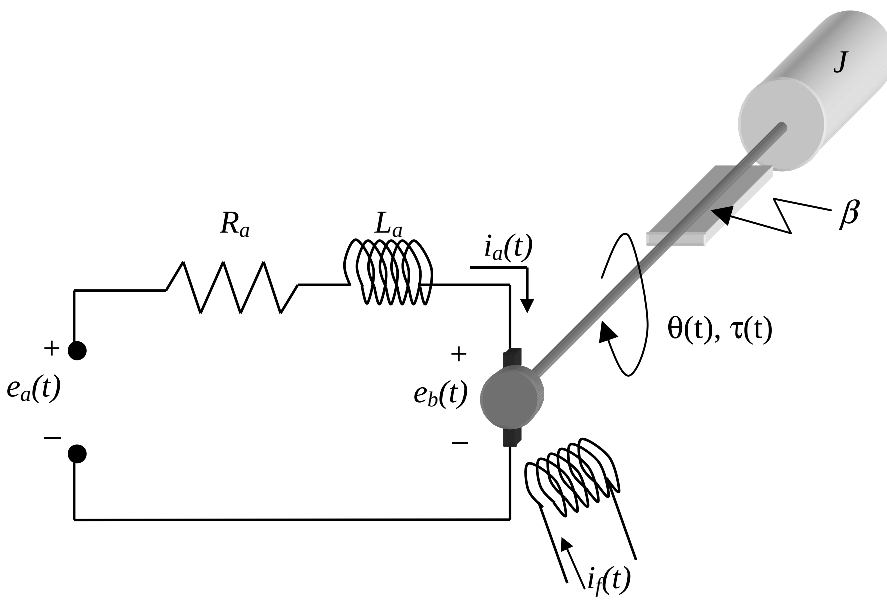

Figure 1: An electromechanical model of a DC motor.

where the motor parameters are defined as follows:

R_a	= armature resistance,	L_a	= armature inductance,
$i_a(t)$	= armature current,	$e_a(t)$	= input voltage,
$e_b(t)$	= back emf (induced) voltage,	$i_f(t)$	= field current,
$\theta(t)$	= shaft angle,	$\tau(t)$	= torque,
J	= moment of inertia,	β	= viscous damping coefficient,
k_m	= torque constant,	k_b	= back-emf constant.

Here we'll discuss a permanent magnet DC motor whose stator consists of a permanent magnet. In this case, we can take the field current to be constant (*i.e.*, a constant magnetic field) and it can be shown that the torque is proportional to the armature current (shown here using the Laplace Transform)

$$\tau(s) = k_m I_a(s) \qquad (1)$$

and that the back-emf voltage is proportional to the shaft speed

$$E_b(s) = k_b \, s\,\theta(s) \qquad (2)$$

Summing the loop voltages suggests that:

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$$

$$E_a(s) = R_a I_a(s) + L_a s I_a(s) + E_b(s) \qquad (3)$$

Summing the angular forces present at the motor's output shaft yields:

$$J\ddot{\theta}(t) + \beta\dot{\theta}(t) = \tau(t)$$

$$Js^2\theta(s) + \beta s\,\theta(s) = \tau(s) \qquad (4)$$

Equations 1-4 can be used to create the following block diagram representation of the motor. Fill in the empty blocks.

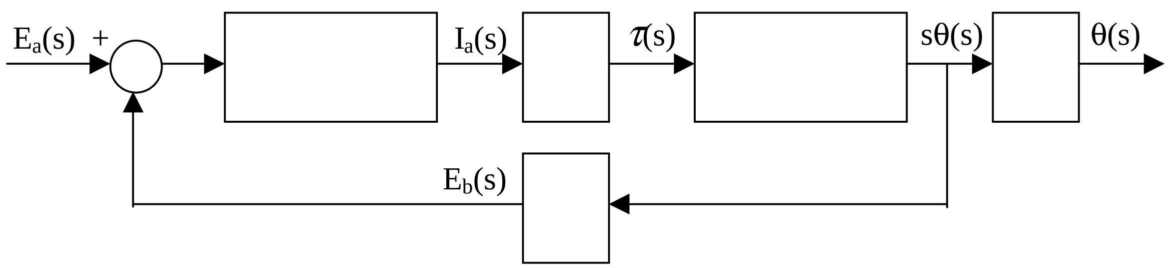

Figure 2: A block diagram of a permanent magnet DC motor.

Prelab Questions:

1) From your block diagram, considering $E_a(s)$ as the input and the motor speed $s\theta(s)$ the output:

   a) Determine the forward path TF.
   $G(s) =$

   b) Determine the feedback TF.
   $H(s) =$

   c) Determine the closed-loop velocity TF.
   $G_{clv}(s) = s\theta(s)/\, E_a(s) =$

2) From your block diagram, determine the closed-loop position TF.
   $G_{clp}(s) = \theta(s)/\, E_a(s) =$

3) Choosing the state vector as $x(t) = [\dot{\theta}(t),\, i_a(t)]^T$, develop a state-space model of the system with input $u(t) = e_a(t)$ and output $\vec{y} = \dot{\theta}(t)$.

Laboratory Procedure:

Through the following steps, motor parameters for the Yaskawa Electric MINI-Minertia motor will be determined and compared to those provided by the data sheet found on the EE443L web page.

1. **Download VI and add conversion factors.** Copy the LabVIEW VI *lab2.vi* and its associated subVI *Altera_Clock_Source.vi* from the network directory *N:\ee443l\Lab 2* into your working directory. Look through this VI to get a good idea of its operation before setting the power supply to 7V, ensuring interface board is powered, and running it. Note the signals plotted have not been converted to useful units; therefore, unit conversions will need to be added utilizing the following specifications and the included wiring diagram. Determine the four conversion factors.
   - Motor voltage is measured with a voltage divider to ensure DAQ card analog input voltages do not exceed ±10V limits. Plot motor voltage with units of Volts.
   - Motor current is measured using the voltage over a power resistor in series with motor. Plot motor current with units of Amps.
   - Motor speed is measured by two methods. Frequency to voltage converters are connected to one channel of the 500 count motor encoders and tuned to 2.1V for an input frequency of 8kHz. The second method of determining speed is an Altera counter that resets at 488.3Hz and utilizes the output of a quadrature encoder to counter interface chip that provides 2000 counts per revolution of the motor. Plot speeds with units of radians/second.

2. **Determine armature resistance (R_a).** The armature resistance can be directly measured by simply measuring the resistance across the motor terminals. After turning off the power supply, carefully turn over the motor box, switch off the board power, unplug the white board to motor connector, and take many readings (>4) from the connector at different shaft positions. Compute the average value with units of Ω and compare to the data sheet value. Reconnect the motor and switch the board power on.

3. **Determine armature inductance (L_a).** Recalling the back-emf voltage and motor velocity relationship $e_b(t) = k_b\dot{\theta}(t)$ suggests that the electrical loop equation can be rewritten as:

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$$

$$= R_a i_a(t) + L_a \frac{di_a(t)}{dt} + k_b\dot{\theta}(t) \qquad (5)$$

If we "lock the rotor" (*i.e.*, stop the motor such that $\dot{\theta}(t) = 0$), then

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt}$$

Solving for the armature current using a step input $e_a(t) = e_{a,const}u(t)$, gives:

$$i_a(t) = \frac{e_{a,const}}{R_a}[1 - e^{-(R_a/L_a)t}]$$

Using the composite disks provided, lock the motor rotor and plot the locked rotor current versus time for power supply voltages of 4V, 6V, and 8V. Zoom in to get a good view of these step responses.

WARNING: Be careful not to overheat the motor by leaving it on too long (longer than 5 seconds) or by using a supply voltage higher than 10V while the rotor is locked.

Determine the time constant ($\frac{L_a}{R_a}$) for each of the three plots from which a corresponding armature inductance can be determined. Compute the average value of the inductance with units of H and compare to the data sheet value. What can you deduce from the steady state value of your locked rotor current step response?

4. **Determine the back emf constant (k_b).** With the composite disk attached to the motor rotor, allow the rotor to spin freely. From equation (5) in steady state ($\frac{di_a(t)}{dt} = 0$) with the armature voltage held constant ($e_a(t) = e_{a,const}$) we see that:

$$e_{a,const} = R_a i_a(t) + k_b \dot{\theta}(t)$$

$$i_a(t) = \frac{-k_b}{R_a}\dot{\theta}(t) + \frac{e_{a,const}}{R_a} \qquad (i.e., \text{a line equation } y = mx + c)$$

Keeping $e_a(t)$ constant (try $e_a(t) = e_{a,const} = 10V$), apply a constant load on the motor via friction and obtain at least 4 steady state readings of $\dot{\theta}(t)$ and $i_a(t)$. Plot these points and fit to a straight line from which k_b with units of V/(rad/sec) can be determined from the slope. Compare this k_b with the data sheet value.

5. **Use the motor data sheet to obtain the remaining motor parameters J, k_m, β in SI units**.

6. **Evaluate your model.** Use Simulink to simulate the block diagram of figure 2 with motor speed as the output using the parameters that you previously obtained in lab. For step inputs of 10V and 15V, plot the step response and compare these simulation results with the results obtained from actual motor tests for the same voltage values.