EE443L Lab 7: Ball & Beam System Modeling, Simulation, and Control

Introduction:

System modeling and simulation provide useful and safe mechanisms for initial controller design. The ball and beam system shown below in figure 1 has the control objective of placing the ball anywhere along the beam by varying the motor voltage. With this objective in mind, the system will be investigated and controlled through various modeling, simulation, and controller design steps.
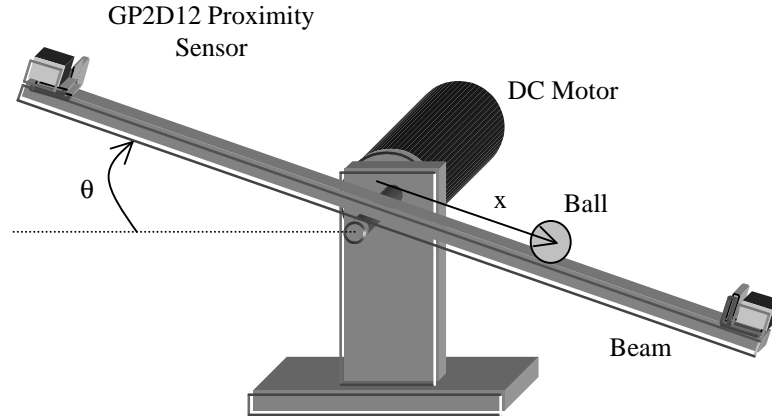

Figure 1: Ball and Beam Apparatus

Prelab:

None.

Laboratory Procedure:

1. For the ball and beam system shown determine the transfer function relationship $X(s)/q(s)$ between ball position $x(t)$ and motor position $q(t)$. Assume the ball is light with mass m much smaller than the mass M of the beam, the beam will stay close to level when in operation, the rotational effects of the ball are negligible, and the friction of the ball on the beam is negligible.

2. Add the ball position transfer function $X(s)/q(s)$ to the DC motor model obtained in earlier labs to achieve the motor/beam/ball system shown in figure 2, change the motor inertia J to be that of the motor plus beam $J_{TOTAL} = J_{MOTOR} + J_{BEAM}$, and determine the system transfer function $X(s)/V_a(s)$ in general.
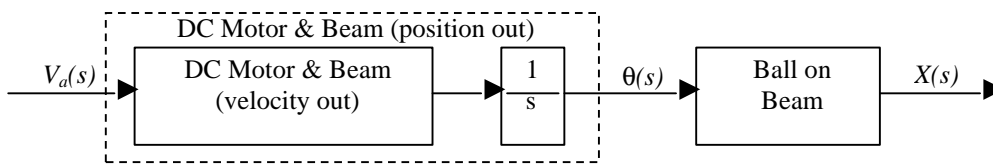

Figure 2: Block Diagram of Motor, Beam, and Ball

3. PID controllers of the type implemented in last week's lab are quite common. Let's investigate the use of a PID ball position controller to place the ball at the desired position $x_d(t)$ along the beam by varying the motor voltage $v_a(t)$. The PID control scheme is shown in figure 3. Note that the error $e(t) = x_d(t) - x(t)$ is the difference between desired ball position and actual ball position.
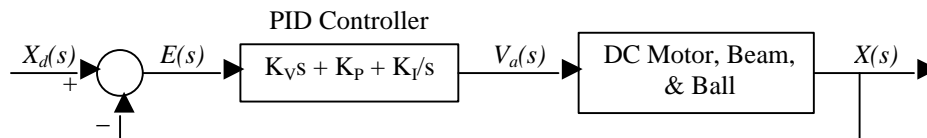

Figure 3: PID Control of Ball Position

a. Determine the transfer function $X(s)/X_d(s)$, in general, for the closed-loop PID controlled system shown in figure 3.
b. Look carefully at the closed-loop characteristic equation, what does the characteristic equation indicate about the control system's stability? Is the PID control scheme a good one to choose?
c. Use matlab to plot the root-locus of the PID controlled system assuming ratios of gains similar to those used in previous labs, i.e., $K_P = K_I$ and $K_V = 0.01K_P = 0.01K_I$. Assume the beam has mass $M = 0.25$kg and length $L = 80$cm. Does the root-locus confirm your stability conclusion above? Which open-loop poles appear to the problem as the root-locus varies? Print out the resulting root-locus.

4. The multiple open-loop ball/beam/motor system poles at zero imply that we probably shouldn't use a PID controller since it adds another pole at zero and makes our pole placement even more difficult. One solution is to use two PD controllers (beam angle and ball position) that effectively give us two additional zeros to pull existing unstable poles to the left without adding poles. This approach is shown in figure 4 where $x_d(t)$, $x(t)$ are the desired and actual ball position, respectively, and $q_d(t)$, $q(t)$ are the desired and actual beam angle, respectively.
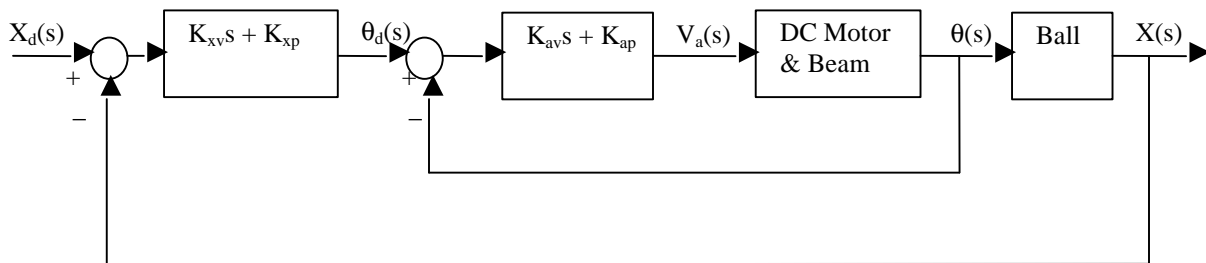

Figure 4: Proposed Ball and Beam Control System

5. Determine the beam angle control system transfer function $q(s)/q_d(s)$ and plot its root locus using our past ratio between the proportional gain K_{ap} and the derivative gain K_{av} noting that this ratio may need to be changed. Pick the gains so that a settling time of approximately 1.0 second is achieved. These gains should yield a reasonable response time without large values. Simulate the step response of the beam angle control system in matlab or simulink for your chosen gains to verify stability and performance. Print out this response.

6. Plot the root locus of the total ball position control system transfer function $X(s)/X_d(s)$ starting with approximately the same ratio between the proportional gain K_{xp} and the derivative gain K_{xv}. Pick gains so that a settling time of approximately 2.0 seconds is achieved and verify a stable and accurate step response of the closed-loop ball/beam control system through a matlab or simulink simulation remembering the desired ball position must be less that 0.4m. Plot this response and use the *ballbeam.m* animation to view the system's response. Show me this animation.