

EE443L Lab 9: Simulated State Variable Feedback Control of Ball and Beam System

Introduction:

State variable feedback is introduced in this lab with the objective of stabilizing the ball and beam system depicted in figure 1 to the origin, *i.e.*, $\mathbf{q} = \dot{\mathbf{q}} = x = \dot{x} = 0$. State space (state variable) representation of a system is a matrix/vector notation containing exactly the same system information found in transfer functions. Advantages of state space design primarily result from the design flexibility afforded by many available linear algebra tools. This week a model, controller, and simulation will be developed for the ball and beam experiment using state space techniques. Next week the design will be experimentally implemented on the physical system.

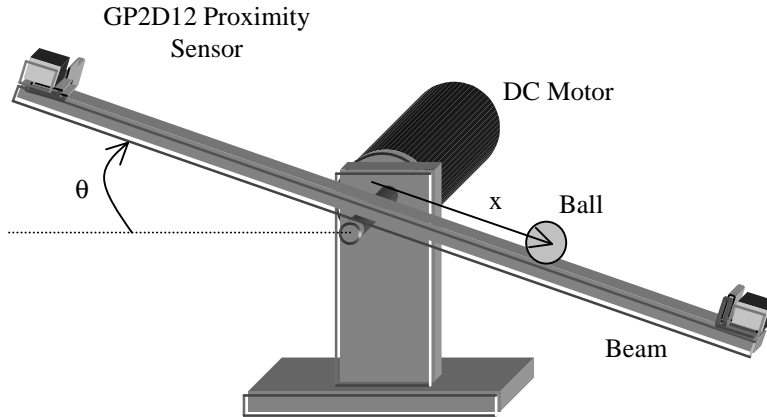


Figure 1: Ball and Beam Apparatus

Procedure:

The ball and beam apparatus shown in figure 1 has been modeled in previous labs by the following coupled transfer functions

$$\frac{\mathbf{q}(s)}{V_a(s)} = \frac{K_m}{s(LJs^2 + (Lb + JR)s + bR + K_m K_b)},$$

$$\frac{X(s)}{\mathbf{q}(s)} = \frac{g}{s^2}$$

where the system parameters are defined in table 1.

Table 1: Parameters for the Ball and Beam	
PARAMETER	DESCRIPTION
J	beam and motor rotor inertia (N m sec ²)
L	motor armature inductance (H)
K_m	motor torque constant (Nm/A)
K_b	motor back emf constant (V/rad/sec)
R	motor armature resistance (Ω)
β	viscous friction of motor (N/m/sec)
g	gravity acceleration (9.81m/sec ²)
v_a	motor voltage (V)
x	ball position (m)
\mathbf{q}	beam angle from horizontal (rad)

1. Check the two transfer functions given to ensure they match those you've determined in previous labs.

- Assume the motor armature inductance is negligible, *i.e.*, $L = 0$ and rewrite the two transfer functions. This assumption is valid as the electrical response time constant is typically much faster than that of the mechanical response. The rationale for wanting to remove inductance from the transfer function is that motor current or equivalently acceleration measurements will not be needed in our state variable feedback controller.
- Change each transfer function with neglected inductance into its corresponding differential equation; one will relate beam angle $q(t)$ to armature voltage $v_a(t)$ and the other relates ball position $x(t)$ to beam angle $q(t)$.
- The primary goal of a state space representation is to reduce a system of higher order differential equations into a system of only first order differential equations. Write the differential equations found above in the state space form $\dot{\bar{x}} = A\bar{x} + Bu$, $\bar{y} = C\bar{x} + Du$ where $\bar{x} = [x \quad \dot{x} \quad q \quad \dot{q}]^T$ is the system state vector, x, q are the system outputs, the system input is $u = v_a$, and A, B, C, D are the system matrices. Write the state space model in general (without parameter values inserted).
- Enter your state space model into matlab with motor and beam parameters determined in previous labs and use the `ctrb()` function to construct the controllability matrix. Is the system controllable? Why is controllability important?
- Define the state feedback controller $u = -K\bar{x} = -[k_1 \quad k_2 \quad k_3 \quad k_4]\bar{x}$ with K the gain vector and the k_i scalar gains, and apply it to your system using matlab's symbolic functions as shown below to find the characteristic equation in terms of the controller gains.

```
% A,B,C,D previously defined as numerical matrices
syms s k1 k2 k3 k4 K;           % create symbolic variables
K = [k1 k2 k3 k4];             % controller gains
char_poly = vpa(simple(det(s*eye(4,4)-(A-B*K))),5)% system char poly
```

- Given the performance specifications of a settling time around 4 sec and a damping ratio of approximately 0.7. Find four poles in general that will satisfy these criteria recalling that the design process will go nicely if two poles dominate the response. Use matlab's `poly()` and `poly2str()` functions to turn these poles into a characteristic polynomial.
- Equate coefficients of the control system characteristic polynomial found in part 6 with the coefficients of the characteristic polynomial containing the desired pole locations determined in part 7 to find the corresponding controller gains.
- Noting that the state space representation of the controlled ball and beam can be written as $\dot{\bar{x}} = (A - BK)\bar{x}$, $\bar{y} = C\bar{x} + Du$, use matlab's `initial()` function as shown below to simulate the controlled system for various initial conditions on the order. Note that the gain matrix K should now be numerical. Verify that your design specifications were met, resulting gains are reasonable in size (say less than 10), and that motor voltages applied are practically achievable. Print these responses with appropriate labels including gains used.

```
% A,B,C,D,K are defined as numerical matrices/vectors
X0 = [0.15; 0; 0; 0];           % initial configuration
[Y,X,T] = initial(A-B*K, zeros(size(B)), C, D, X0, 15);
subplot(2,2,1); plot(T,Y(:,1)); % ball position
subplot(2,2,2); plot(T,Y(:,2)); % beam angle
subplot(2,2,3); plot(T,-K*X'); % motor voltage
```

- View the system's response with the `ballbeam()` function and show me this animation.