EE443L: Intermediate Control Lab

Lab2: Modeling a DC motor

<u>Introduction</u>:

In this lab we will develop and validate a basic model of a permanent magnet DC motor (Yaskawa Electric, Mini-series, Minertia motor). The specific input/output relationship, which we are interested in determining, is the manner in which the input (the armature Voltage) affects the motor's speed and position. Developing a suitable model of a system is typically the first step towards controlling its behavior.

<u>Part 1 (PRELAB)</u>: Developing the motor model.

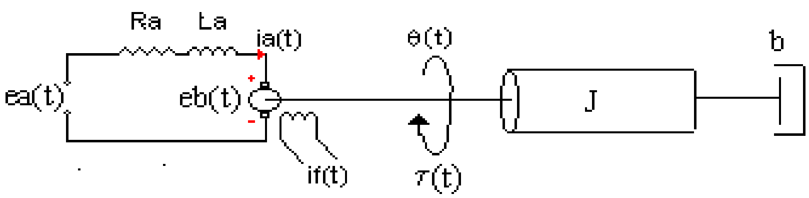

Figure 1: An electromechanical model of a DC motor.

The labeled items in figure 1. refer to the following:

R_a = armature resistance
L_a = armature inductance
$i_a(t)$ = armature current
$e_a(t)$ = input voltage
$e_b(t)$ = back emf
$i_f(t)$ = field current
$q(t)$ = shaft angle
$t(t)$ = torque
J = moment of inertia
b = viscous damping coeff.
k_m = torque constant
k_b = back-emf constant

Furthermore, in the case of a permanent magnet DC motor, the field current is constant (*i.e.* a constant magnetic field). It can be shown that the torque is proportional to the armature current

$$t(s) = k_m I_a(s) \qquad (1)$$

and the back-emf is proportional to the shaft speed

$$E_b(s) = k_b\, s\,\Theta(s) \qquad (2)$$

Summing the loop voltages suggests that:

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$$

$$\Rightarrow \; E_a(s) = R_a I_a(s) + L_a s I_a(s) + E_b(s) \qquad (3)$$

Summing the angular forces present at the motor's output shaft, gives:

$$J\ddot{\theta}(t) + b\dot{\theta}(t) = \tau(t)$$

$$\Rightarrow \; Js^2\Theta(s) + bs\Theta(s) = \tau(s) \qquad (4)$$

– Fill in the empty block diagram blocks below (Hint: use equations 1-4).

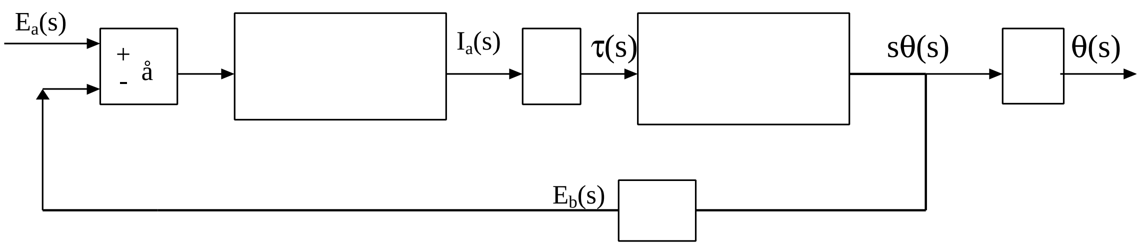

Figure 2. A block diagram of a permanent magnet DC motor

Questions:

1) From your block diagram, considering *$E_a(s)$* as the input and the motor speed $\dot{\theta}(t)$ the output:

(a) Determine the feed-forward TF.
$G_H(s)$ =

(b) Determine the feedback TF.
$G_{fb}(s)$ =

(c) Determine the closed-loop TF.
$G_{cl}(s)$ =

(d) Choosing the state vector as $\vec{x}(t) = [\dot{\theta}(t), i_a(t)]^T$, develop a state-space model of the system

Part 2: Determine the motor's parameters

The armature resistance (R_a):

The armature resistance can be directly measured by simply measuring the resistance across the motor terminals. Be sure to take many readings (>4) at different shaft positions. Compute the average value and compare to the data sheet values.

R_a = ______________ Ω

The armature inductance (L_a):

Recalling that, $e_b(t) = k_b\dot{q}(t)$ suggests that:

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t)$$

$$= R_a i_a(t) + L_a \frac{di_a(t)}{dt} + k_b\dot{q}(t) \qquad (11)$$

If we "lock the rotor" (*i.e.* $\dot{q}(t) = 0$), then

$$e_a(t) = R_a i_a(t) + L_a \frac{di_a(t)}{dt}$$

Solving for the armature current using a step input ($e_a(t) = e_{a,const}u(t)$), gives:

$$i_a(t) = \frac{e_{a,const}}{R_a}[1 - e^{-({R_a}/{L_a})t}]$$

Plot the locked rotor current vs time for step input voltages of 4v, 6v, and 8v each in different rotor positions (be sure to use a suitable low PWM frequency).

--WARNING, be careful not to overheat the motor by leaving the power on too long (>1 min.) or by using a higher voltage (> 12V).

Determine the time constant ($\frac{L_a}{R_a}$) from each of the three plots and select the armarture inductance as the average value:

L_a= __________ mH

- What can you deduce from the steady state value of your step response?

The back emf constant (k_b):

Allow the rotor to spin freely, and attach the composite disk provided. From eq. (11) in steady state ($\frac{di_a(t)}{dt} = 0$) if the armature voltage is held constant ($e_a(t) = e_{a,const}$) we see that:

$$e_{a,const} = R_a i_a(t) + k_b \dot{q}(t)$$

$$i_a(t) = \frac{-k_b}{R_a}\dot{q}(t) + \frac{e_{a,const}}{R_a} \qquad (i.e. \quad y = mx + c)$$

Keeping e_a constant (try $e_a = 10V$) apply a constant load on the motor via friction and obtain at least 4 steady state readings of $\dot{q}(t)$ and $i_a(t)$. Plot these points and fit to a straight line.

K_b = ___________________ $\frac{V/Rad}{Sec.}$

Use the supplied motor data sheet to obtain the parameters (be sure to convert to SI units).

J = ___________ , k_m = ___________, b = ___________.

Part 3: Evaluating the desired model:

Use **Simulink** to simulate the block diagram of figure 2 with motor speed as the output using the parameters that you previously obtained in lab. For step inputs of **5V, 10V, 15V** and **20V** plot the step response. Compare these simulation results with the results obtained from actual motor tests for the same voltage values (*i.e* include these plots in your lab book).