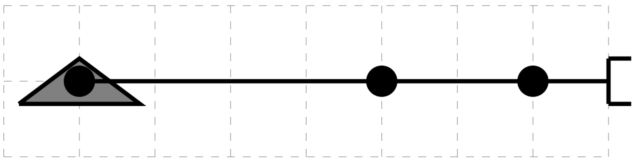
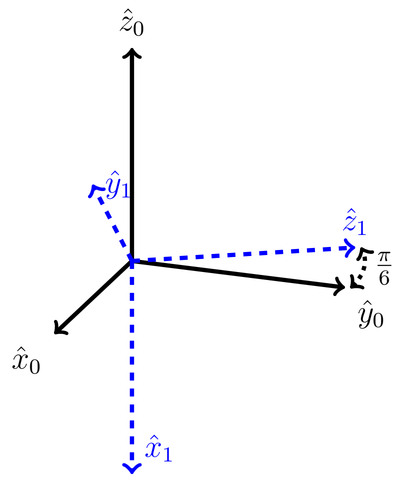
1. Sketch the workspace and dexterous workspace for the planar, three-link, RRR manipulator shown. Assume all rotational joints (denoted by dots) have a full range (360°) of motion.



2. Given frames 0 and 1 as shown in the figure below, find the rotation matrices R_1^0 and R_0^1.



3. Consider the following sequence of rotations that takes a frame initially aligned with the fixed frame and rotates it to a new orientation:

   (a) Rotate by angle α about z-axis.

   (b) Rotate by angle β about current y-axis.

   (c) Rotate by angle γ about fixed x-axis.

   (d) Rotate by angle δ about current z-axis.

   Write out the matrix product that yields the resulting rotation matrix describing the final orientation of the frame with respect to the fixed frame. Do not perform the multiplication.

4. A coordinate frame is rotated away from a fixed frame using the following sequence of rotations:
   (a) $\frac{\pi}{2}$ about the z-axis,
   (b) $\frac{\pi}{2}$ about the current y-axis,
   (c) $\frac{\pi}{2}$ about the current x-axis.

   Find the rotation matrix that represents the composition of these rotations, and sketch the initial/fixed and final frames.

5. A coordinate frame is rotated away from a fixed frame using the following sequence of rotations:
   (a) $\frac{\pi}{2}$ about the z-axis,
   (b) $\frac{\pi}{2}$ about the fixed y-axis,
   (c) $\frac{\pi}{2}$ about the fixed x-axis.

   Find the rotation matrix that represents the composition of these rotations, and sketch the initial/fixed and final frames.

6. Three coordinate frames (0, 1, 2) have been defined, and the orientations of frames 1 and 2 with respect to fixed frame 0 are given by the rotation matrices

$$R_1^0 = \begin{bmatrix} \frac{2}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}, \quad R_2^0 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}.$$

   Find the following:
   (a) rotation matrix R_2^1 that describes orientation of frame 2 with respect to frame 1,
   (b) ZYZ Euler angles $(\varphi, \vartheta, \psi)$ that correspond to the orientation described by R_1^0,
   (c) ZYZ Euler angles $(\varphi, \vartheta, \psi)$ that correspond to the orientation described by R_2^0,
   (d) RPY angles $(\varphi, \vartheta, \psi)$ that correspond to the orientation described by R_1^0,
   (e) RPY angles $(\varphi, \vartheta, \psi)$ that correspond to the orientation described by R_2^0,
   (f) angle and axis (ϑ and $\hat{r}$) that correspond to the orientation described by R_1^0,
   (g) angle and axis (ϑ and $\hat{r}$) that correspond to the orientation described by R_2^0,
   (h) quaternion (η and $\vec{\epsilon}$) that correspond to the orientation described by R_1^0.
   (i) quaternion (η and $\vec{\epsilon}$) that correspond to the orientation described by R_2^0.