Differential (Forward) Kinematics and Trajectory Planning

This project will expand upon your previous projects that compute angle/axis and Euler ZYZ angles from rotation matrices, direct/forward (position) kinematics, and visualization of a manipulator in three dimensions with DH frames. The abilities to compute and display differential kinematics as well as generate trajectories will be added.

1. Implement the following making use of your previous projects that compute and display forward kinematics, and compute representations of orientation from rotation matrices.

   (a) Jacobian via the direct method along with the ability to compute and display the end-effector's (frame n's) linear and angular velocity on the 3D visualization.

   (b) Trajectory planning via a quintic polynomial and Linear Segments with Parabolic Blends (LSPB) that satisfy given constraints on initial and final configurations.

   (c) Capability to plot the following versus time

      i. positions $\vec{q}$ and velocities $\dot{\vec{q}}$ of joints;

      ii. end-effector's position $\vec{o}_n^0 = [o_x, o_y, o_z]^T$ and linear velocity $\dot{\vec{o}}_n^0$; and

      iii. end-effector's orientation as Euler ZYZ angles (ϕ, θ, ψ) or angle/axis $\vec{r} = [r_x, r_y, r_z]^T = [\theta k_x, \theta k_y, \theta k_z]^T$, and angular velocity $\vec{\omega}_{0,n}^0 = [\omega_x, \omega_y, \omega_z]^T$.

2. The project should make use of functions and take the following as user-defined inputs:

   (a) DH table with parameters;

   (b) specifications for joint variables' trajectories to include initial and final times, initial and final positions, initial and final velocities, and initial and final accelerations; and

   (c) trajectory type (quintic or LSPB).

3. The program should have the options and ability to output/show:

   (a) 3D visualization of robot using lines, cylinders or fancier objects to represent joints and links;

   (b) display frames 0 to n on robot's visualization;

   (c) display linear and angular velocity vectors at/for the end-effector;

   (d) values of homogeneous transformation matrix representing the end-effector's pose;

   (e) values of Jacobian;

(f) values of the end-effector's position, linear velocity, orientation as Euler ZYZ angles or angle/axis, and angular velocity; and

(g) plots (versus time) of joint variables, joint velocities, and the end-effector's position, linear velocity, orientation as Euler ZYZ angles or angle/axis, and angular velocity.

4. Use your resulting program/functions to perform the following.

   (a) Simulate and visualize the Stanford arm using quintic trajectories for the joint variables. Let $\vec{q}(t_o = 0) = -\vec{1}$, $\vec{q}(t_f = 10) = \vec{1}$ with units of distance or radians, $\dot{\vec{q}}(t_o) = \dot{\vec{q}}(t_f) = \ddot{\vec{q}}(t_o) = \ddot{\vec{q}}(t_f) = \vec{0}$. Provide values of the forward kinematics T_n^0 and Jacobian J at the initial and final times; and plot (versus time) the joint variables, joint velocities, and the end-effector's position, linear velocity, orientation as Euler ZYZ angles and angle/axis, and angular velocity.

   (b) Simulate and visualize the Puma 260 using quintic trajectories for the joint variables. Let $\vec{q}(t_o = 0) = -\vec{1}$, $\vec{q}(t_f = 10) = \vec{1}$ with units of radians, $\dot{\vec{q}}(t_o) = \dot{\vec{q}}(t_f) = \ddot{\vec{q}}(t_o) = \ddot{\vec{q}}(t_f) = \vec{0}$. Provide values of the forward kinematics T_n^0 and Jacobian J at the initial and final times; and plot (versus time) the joint variables, joint velocities, and the end-effector's position, linear velocity, orientation as Euler ZYZ angles and angle/axis, and angular velocity.

   (c) Simulate and visualize the Adept SCARA using LSPB trajectories for the joint variables. Let $\vec{q}(t_o = 0) = [-1, -1, 0, -1]^T$, $\vec{q}(t_f = 10) = [1, 1, 300, 1]^T$ with units of distance or radians, $\dot{\vec{q}}(t_o) = \dot{\vec{q}}(t_f) = \vec{0}$. Provide values of the forward kinematics T_n^0 and Jacobian J at the initial and final times; and plot (versus time) the joint variables, joint velocities, and the end-effector's position, linear velocity, orientation as Euler ZYZ angles and angle/axis, and angular velocity.

5. Turn in your programs, sample visualizations (maybe bread-crumbs, initial/final configurations, ...), values of initial and final forward kinematics and Jacobian, and plots of values versus time for the three robots and scenarios.