Due W 10/01

1. Consider the transfer function of a second-order system as shown in equation 3 on page 30.
   (a) Solve for the step response of the system as given in equation 4 on page 30. It might be useful to define the attenuation/damping factor $\sigma = \zeta\omega_n$ and damped natural frequence $\omega_d = \omega_n\sqrt{1-\zeta^2}$.
   (b) Common design specifications for a system's underdamped step response are shown in figure 3 on page 26. For an underdamped, second-order system (or a system than can be approximated by a second-order system) these specifications can be (approximately) related to the characteristic equation and pole locations as given in equations 9-11 on page 31. Using the step response found in the previous part derive the three relationships. Approximating rise time can be simplified by finding the time it takes to go from 0% to 100% of the steady-state value (versus 10% to 90%) and assuming $0.4 < \zeta < 0.8$ achieves responses with acceptable overshoot and response time.

2. For the purpose of sketching a root-locus, the transfer function of a PID-controller can be written as

$$C_{PID}(s) = \frac{k_D s^2 + k_P s + k_I}{s} = \frac{k_D(s^2 + \frac{k_P}{k_D}s + \frac{k_I}{k_D})}{s} = \frac{k_D(s-z_1)(s-z_2)}{s}$$

   where z_1, z_2 are zeros introduced into the open-loop transfer function by the controller as is a pole at the origin.
   (a) Solve for the gains k_P, k_I in terms of the zeros z_1, z_2.
   (b) Solve for the zeros z_1, z_2 in terms of the gains k_P, k_I.
   (c) What are the numeric values of the zeros z_1, z_2 that correspond to the gains you used for the pendulum and magnetic-ball in the last homework?
   (d) Using the PID-controller gains from your last homework and k_D as the parameter to be varied, sketch the root-locus for the pendulum linearized about $\theta = 0$rad. Use the guidelines discussed in class to sketch by hand and then plot with Matlab. Note the roots that correspond to your gains and comment on how they relate to the system's response you observed in the last homework.
   (e) Using the PID-controller gains from your last homework and k_D as the parameter to be varied, sketch the root-locus for the pendulum linearized about $\theta = \frac{\pi}{2}$rad. Use the guidelines discussed in class to sketch by hand and then plot with Matlab. Note the roots that correspond to your gains and comment on how they relate to the system's response you observed in the last homework.
   (f) Using the PID-controller gains from your last homework and k_D as the parameter to be varied, sketch the root-locus for the magnetic-ball linearized about $y = 0.5$m. Use the guidelines discussed in class to sketch by hand and then plot with Matlab. Note the roots that correspond to your gains and comment on how they relate to the system's response you observed in the last homework.