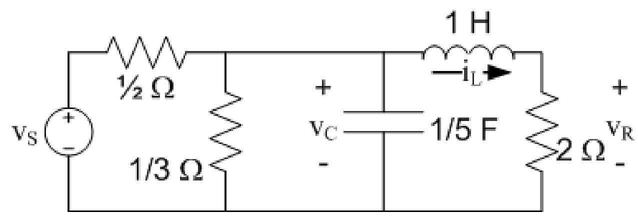
EE 554 Homework Assignment 1
September 1, 2009

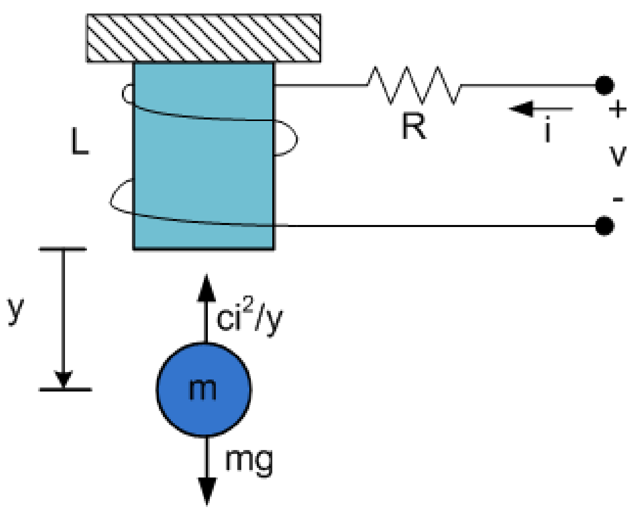
1. **Model a circuit**

   (a) Find a state-space description of the RLC circuit shown. Recall inductor currents and capacitor voltages are typical variables for which initial conditions will be known, thus they make good choices for state variables. Let the states be $x_1 = v_C$, $x_2 = i_L$, input $u = v_S$, and output $y = v_R$.



   (b) Find the transfer function $H(s) = \frac{Y(s)}{U(s)} = \frac{V_R(s)}{V_S(s)}$.

2. **Linearization of magnetic-ball-suspension system**
   Consider the magnetic-ball-suspension system shown below where the objective is to vary the input voltage v such that the ball is suspended at a fixed distance y.



The dynamic equations are

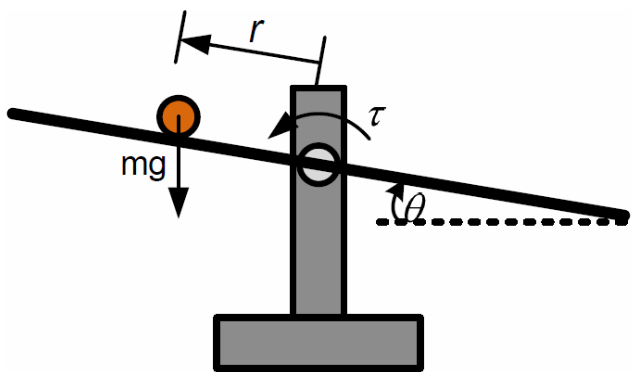
$$
\begin{aligned}
m\frac{d^2y(t)}{dt^2} &= mg - \frac{ci^2(t)}{y(t)} \\
L\frac{di(t)}{dt} &= v(t) - Ri(t)
\end{aligned}
$$

and the parameters and variables are as follows:

$v(t)$ = input voltage (V)	$y(t)$ = ball position (m)
$i(t)$ = winding current (A)	$c = 1$ = proportionality constant
$R = 1\Omega$ = winding resistance	$L = 0.01$H = winding inductance
$m = 1$kg = mass of ball	$g = 9.81m/s^2$ = gravitational acceleration

Linearize the system about the (fixed) distance $y = 0.5$m and find the linear, state-space equations. Find the characteristic equation and poles/eigenvalues of the linear system. Comment on stability.

3. **Linearization of ball-and-beam system**
   Consider the ball-and-beam system shown below where the objective is to vary the input torque τ on the beam such that the ball can be placed at any location r along the beam.



Assuming no slipping, one representation of the system are the state-space equations:

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{-mg\sin(x_3) + mx_1x_4^2}{m + \frac{J_b}{R^2}} \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{\tau - mgx_1\cos(x_3) - 2mx_1x_2x_4}{mx_1^2 + J + J_b}
\end{aligned}
$$

where the states are defined as $x_1 = r$, $x_2 = \dot{r}$, $x_3 = \theta$ and $x_4 = \dot{\theta}$, and parameters and variables are as follows:

$\tau(t)$ = input torque (N·m)	$r(t)$ = position of the ball (m)
$\theta(t)$ = beam angle (rad)	m = mass of the ball (kg)
$g = 9.81m/s^2$ = gravitational acceleration	J_b = moment of inertia of ball
R = radius of the ball	J = moment of inertia of beam

Linearize the system about the equilibrium point where $x_1 = r_0$ (a fixed distance) and $x_2 = x_3 = x_4 = 0$ (horizontal beam at equilibrium) and find the linear, time-invariant, state-space equations.