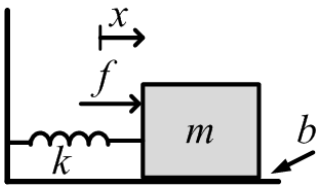
EE 554 Homework Assignment 2
September 10, 2009

1. Consider the linear spring-mass-damper system shown where x (m) is the position of the mass measured from the point where the spring applies no force, k (N/m) is the spring constant, m (kg) is the mass of the object, b (N·s/m) is the coefficient of viscous friction and f (N) is the input force.



   (a) Find a state-space description of the system when the output is the position ($y = x$) and the input is the applied force ($u = f$).

   (b) Find the transfer function $H(s) = \frac{Y(s)}{U(s)}$.

   (c) Use values $m = 1$kg, $k = 1$N/m and $b = 0,\ 1,\ 2,\ 3$N·s/m, and for each value of b use Matlab to

      i. plot the poles and zeros of the system in the complex plane,

      ii. plot the frequency response of the system on semi-log axes with magnitude in dB and phase in degrees,

      iii. plot the unit-step response ($f = 1$N with initial conditions of zero), and

      iv. thoroughly discuss the relationships between the three "views" of the system's behavior.

2. The system described by the state-space equations

$$\begin{aligned} \dot{\vec{x}} &= \begin{bmatrix} -d_1 & -d_2 & -d_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} 0 & n_1 & n_2 \end{bmatrix} \vec{x} \end{aligned}$$

   will have the transfer function

$$\frac{Y(s)}{U(s)} = \frac{n_1 s + n_2}{s^3 + d_1 s^2 + d_2 s + d_3}$$

   (a) Show that this transfer function comes from the state-space equations given.

   (b) Choose two of the poles to be $-2 \pm j$; $n_1,\ n_2$ to keep a DC gain of 0dB; and investigate the impact of the additional pole and zero on the frequency and unit-step responses. Note where they do and do not have noticeable effect as compared to the second-order system without them.