EE 581 Power Systems

Admittance Matrix: Development, Direct and Iterative solutions

Overview and HW # 8

- Chapter 2.4
- Chapter 6.4
- Chapter 6.1-6.3

- Homework:
- Special Problem 1 and 2 (see handout)

Overview and HW # 8

- Homework: Special problem 1
- Solve for the Y_{bus} of the given system

Find θ_2, $|V_3|$, θ_3, S_{G1}, and Q_{G2} for the system shown in Figure E10.6. In the transmission system all the shunt elements are capacitors with an admittance $y_C = j0.01$, while all the series elements are inductors with an impedance of $z_L = j0.1$.

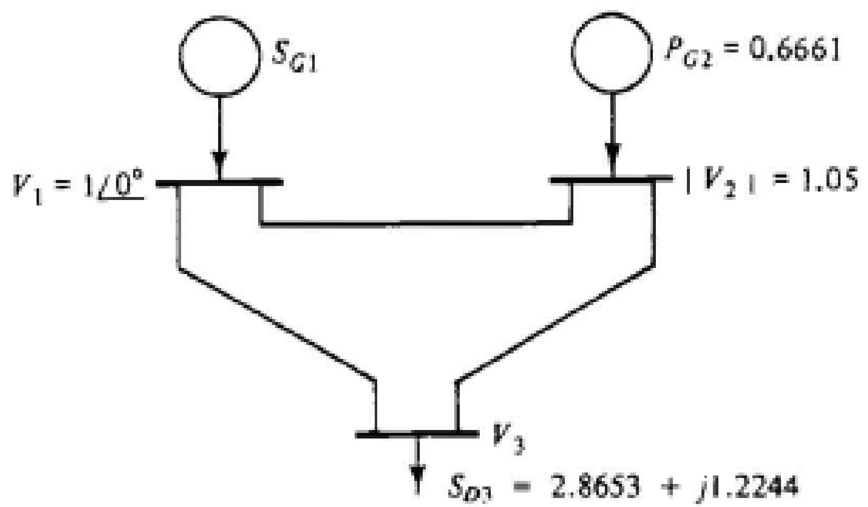
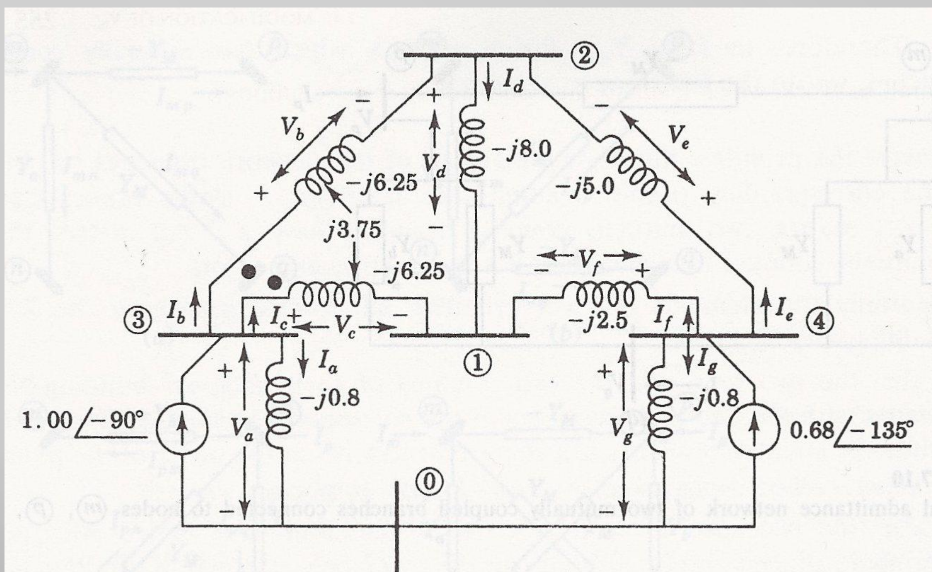
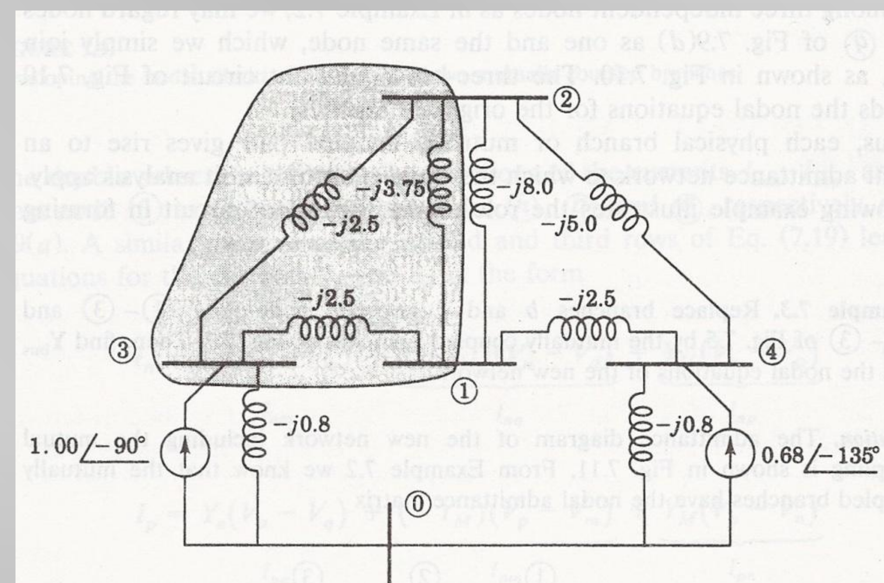


Figure E10.6

Overview and HW # 8

- Homework: Special problem 2
- Solve for the Y_{bus} of the given system (note that the right image shows the equivalent mutual inductance)





HW # 8 Solutions

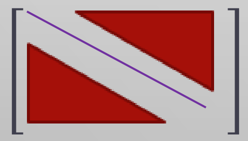
- Special Problem 1

- $Y_{bus} = \begin{bmatrix} -j19.98 & j10 & j10 \\ j10 & -j19.98 & j10 \\ j10 & j10 & -j19.98 \end{bmatrix}$

HW # 8 Solutions

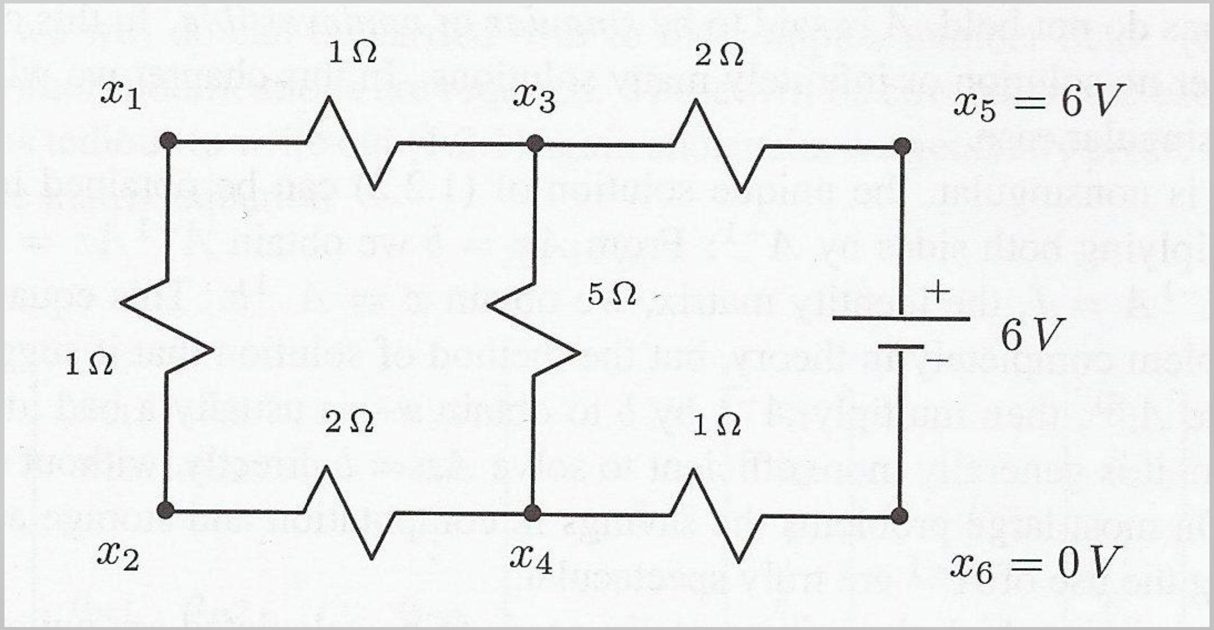
- Special Problem 2
- $Y_{bus}\vec{v} = \vec{I}$ where
- $$Y_{bus} = \begin{bmatrix} -j16.75 & j11.75 & j2.5 & j2.5 \\ j11.75 & -j19.5 & j2.5 & j5 \\ j2.5 & j2.5 & -j5.8 & 0 \\ j2.5 & j5 & 0 & -j8.3 \end{bmatrix}$$
- $$\vec{v} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \qquad \vec{I} = \begin{bmatrix} 0 \\ 0 \\ 1.00\angle-90^\circ \\ 0.68\angle-135^\circ \end{bmatrix}$$

Chapter 2.4: Network Equations

- Development:
- Models can be solved using either nodal (bus) analysis or port analysis
  - Nodal analysis much easier to use than loop equations
  - Nodal analysis used by computer programs
- For ease of calculations, admittance is used (Y)
  - Ohm's Law: V=IR
  - Complex: V=IZ
  - $Y = \frac{1}{Z} \Rightarrow \ \ I = YV$ (Siemens)
  - ***Y* is symmetric** $(Y = Y^T) \Rightarrow$ only have to solve for either the upper or lower triangular part of the matrix!!
  - 

Chapter 2.4: Network Equations

- Simple Example
- Solve for voltages ($x_i \; i = 1..6$) using KCL and develop the impedance matrix



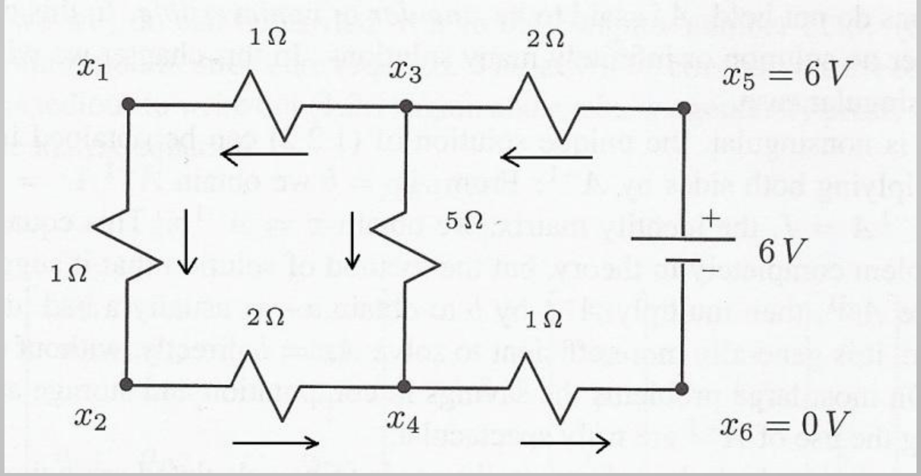
Chapter 2.4: Review of Nodal Analysis

- Node 3: $\frac{1}{2}(x_3 - 6V) = (x_1 - x_3) + \frac{1}{5}(x_4 - x_3)$

  $\Rightarrow -x_1 + 1.7x_3 - 0.2x_4 = 3V$

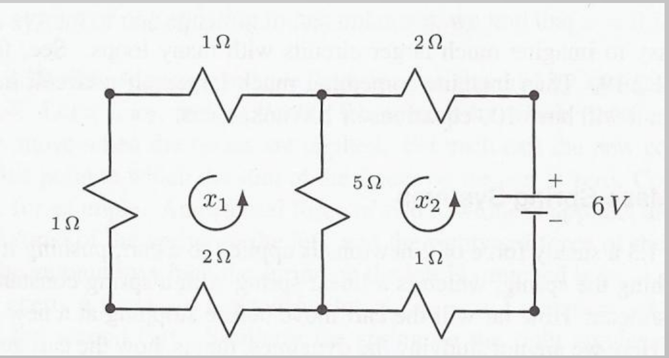
- System of network equations:
  - $2x_1 - x_2 - x_3 = 0$
  - $x_1 - 1.5x_2 + 05.x_4 = 0$
  - $-x_1 + 1.7x_3 - 0.2x_4 = 3$
  - $-0.5x_2 - 0.2x_3 + 17.x_4 = 0$

- $Ax = B$



Chapter 2.4: Review of Nodal Analysis

- Similarly, can solve for the currents such that:
- $Ax = B$ such that we have $ZI = V$ (note Z is real here)
- $\begin{bmatrix} 9 & -5 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$

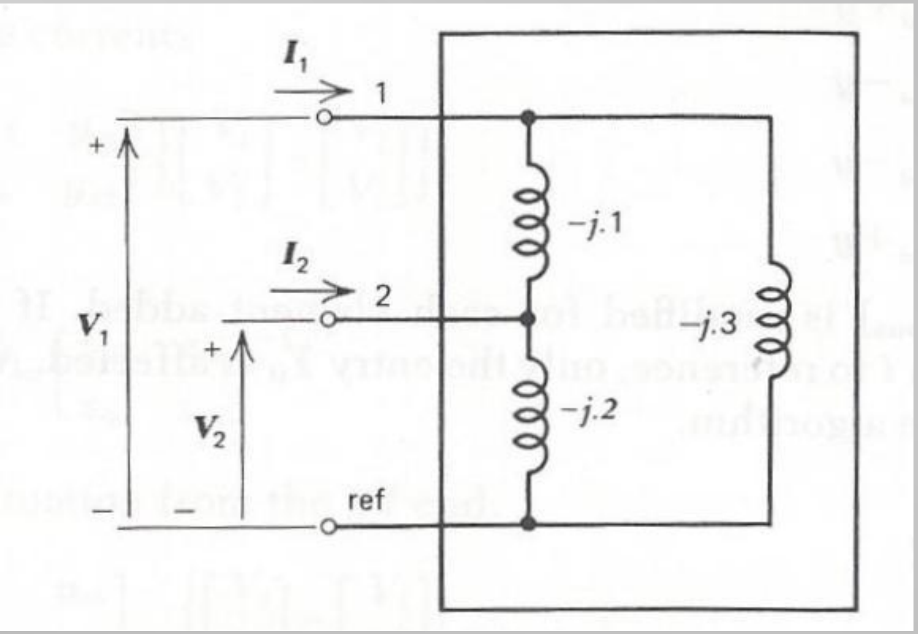


Chapter 2.4: Network Equations

- Outline of steps
- Step 1: Define a reference bus (node), then define all voltages with respect to the reference bus.
- Step 2a: Source transform any voltage sources in parallel to an equivalent current sources.
- Step 2b: Convert all impedance values (Z) to admittance values (Y).
- Step 3: Write the equations found from nodal analysis in a matrix format
- Step 4: Solve the problem using direct or indirect methods.

Chapter 2.4: Network Equations

- Simple transmission network model (values in admittance)
- Solve using
  - 1) a two port network
  - 2) again using nodal analysis



Chapter 2.4: Network Equations

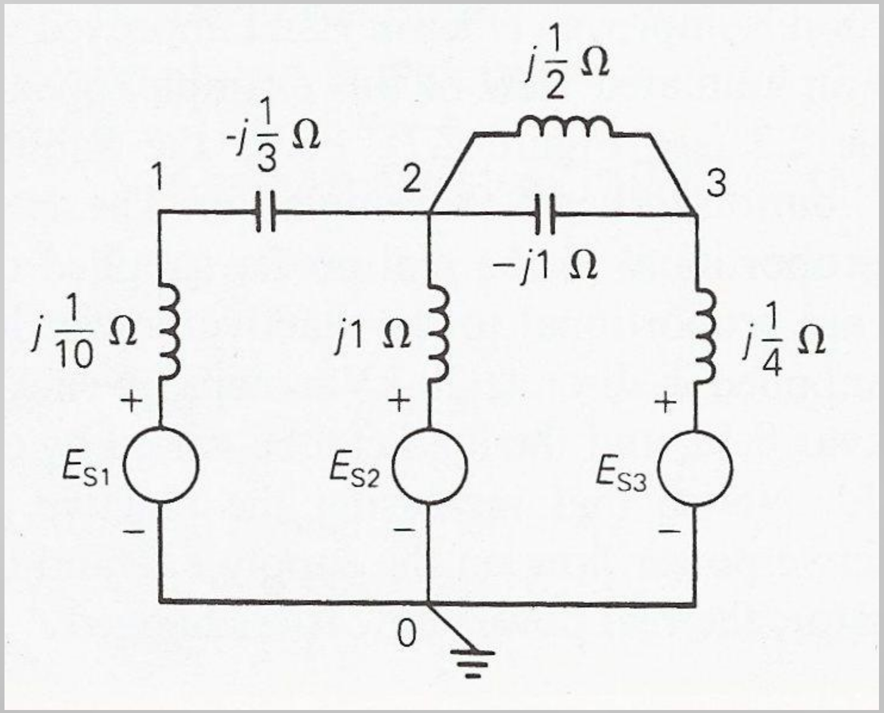
- Two port method:
- $Y_{11} = \frac{I_1}{V_1}|_{V_2=0} = \frac{(-0.1j-0.3j)V_1}{V_1} = -0.4j$
- $Y_{12} = \frac{I_1}{V_2}|_{V_1=0} = \frac{-(-0.1j)V_2}{V_2} = 0.1j$
- $Y_{21} = Y_{12} = 0.1j$
- $Y_{22} = \frac{I_2}{V_2}|_{V_1=0} = \frac{(-0.1j-0.2j)V_2}{V_2} = -0.3j$
- $Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -0.4j & 0.1j \\ 0.1j & -0.3j \end{bmatrix}$

Chapter 2.4: Network Equations

- Nodal analysis method:
- $I_1 = (-0.1j - 0.3j)V_1 - (-0.1j)V_2$
- $I_2 = -(-0.1j)V_1 + (-0.1j - 0.2j)V_2$
- Matrix format: YV=I $\therefore$
- $YV = \begin{bmatrix} -0.4j & 0.1j \\ 0.1j & -0.3j \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
  - Hence both solutions yield the same answer and are valid

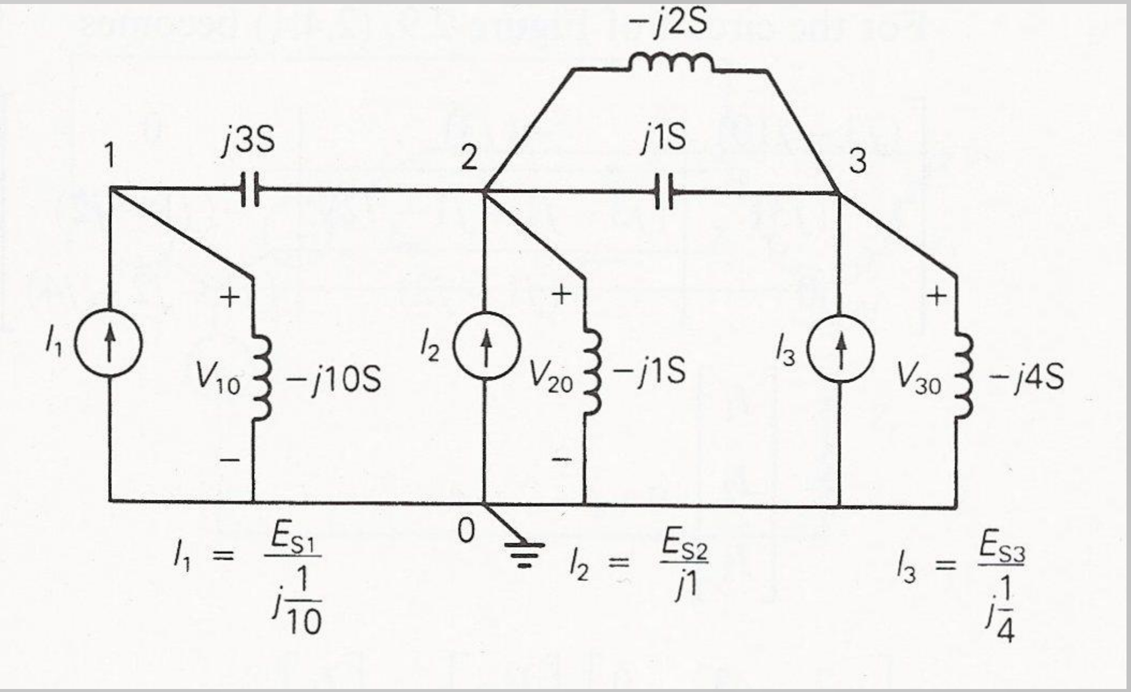
Chapter 2.4: Network Equations

- Example from 2.4:
- Solve the model using nodal analysis:



Chapter 2.4: Network Equations

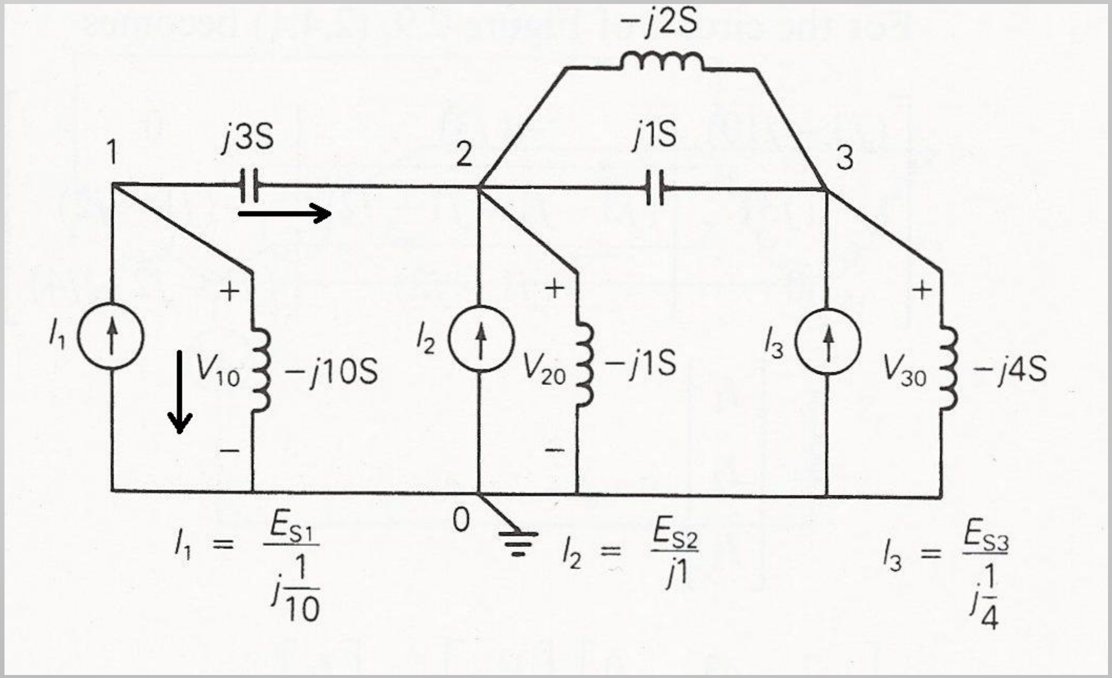
- Step 1 and step 2:



Chapter 2.4: Network Equations

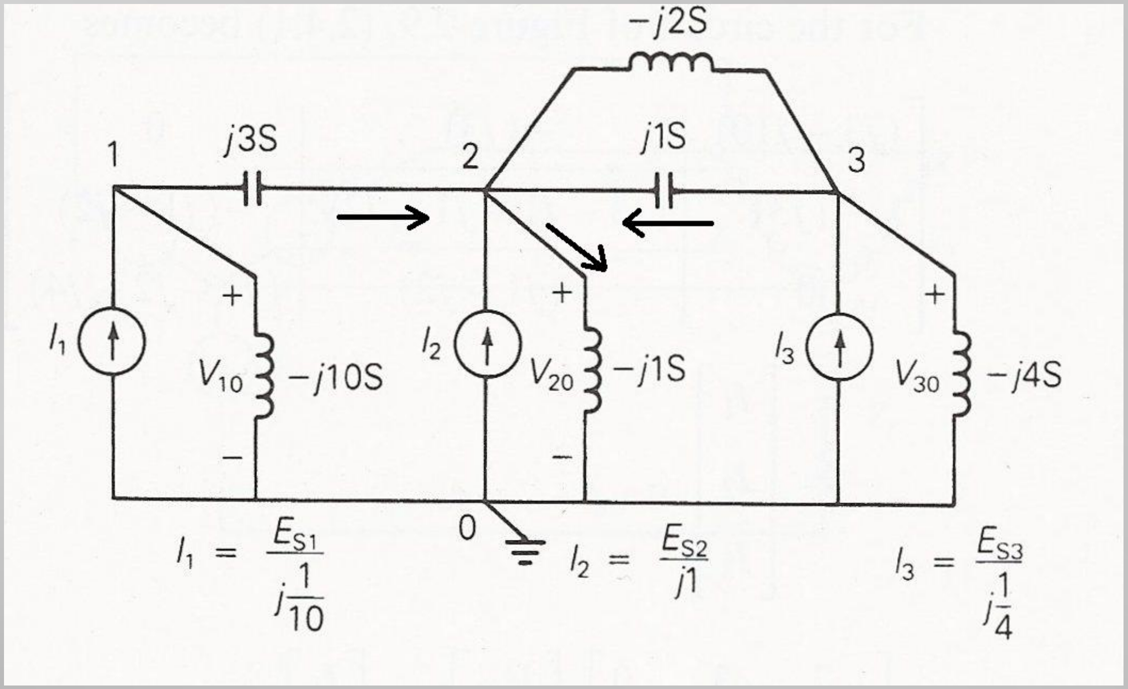
- Step 3:
- $I_1 = (-10j)V_{10} + 3j(\mathrm{V}_{10} - \mathrm{V}_{20})$

  $= (j3 - j10)V_{10} - (j3)V_{20}$



Chapter 2.4: Network Equations

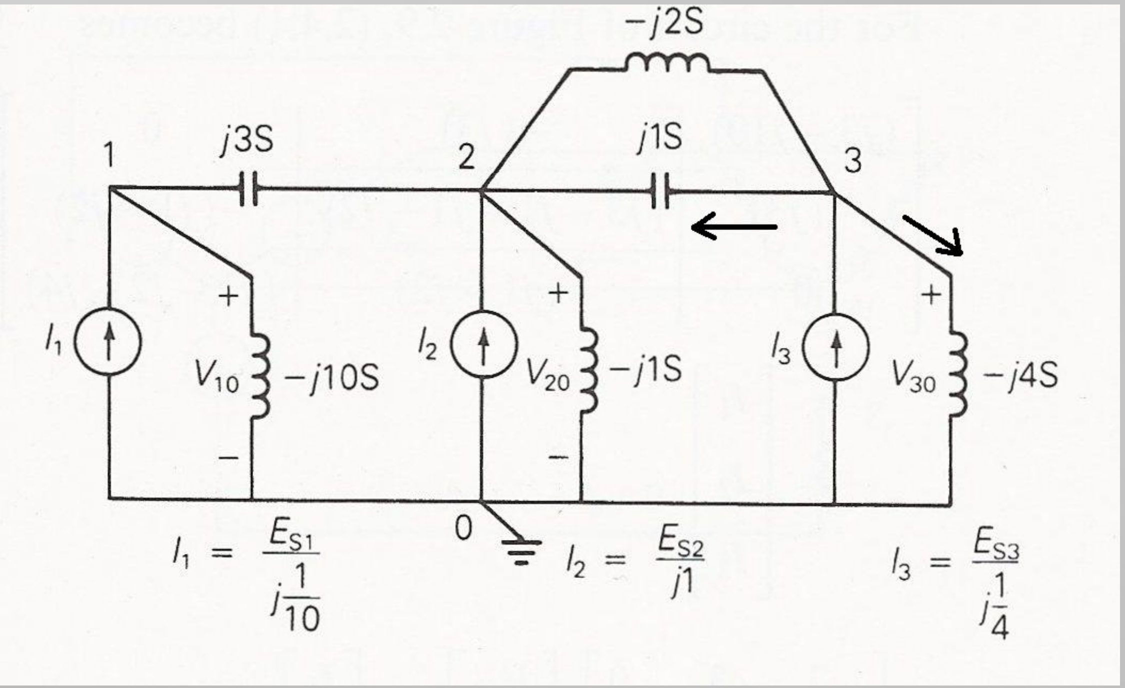
- Step 3:
- $I_2 + 3j(V_{10} - V_{20}) + j(V_{30} - V_{20}) + (-2j)(V_{30} - V_{20}) = -jV_{20}$
- $I_2 = (j3 + j - j2 - j)V_{20} + j3(-V_{10}) + (j - 2j)(-V_{30})$



Chapter 2.4: Network Equations

- Step 3:
- $I_3 = j(V_{30} - V_{20}) + (-2j)(V_{30} - V_{20}) + (-4j)V_{30}$

  $= (j - 2j)(-V_{20}) + (j - 2j - 4j)V_{30}$



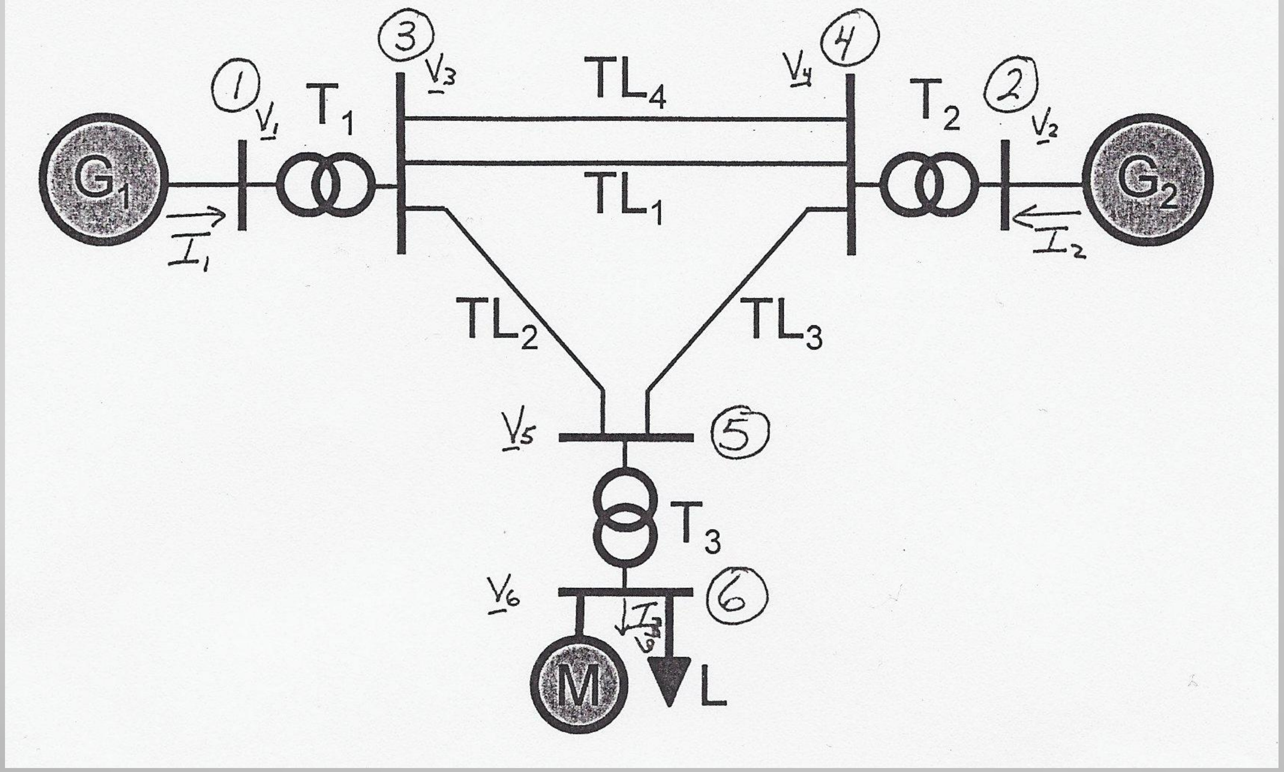
Chapter 2.4: Network Equations

- Step 3:
- Combining equations
  - $(j3 - j10)V_{10} - (j3)V_{20} = I_1$
  - $(j3 + j - j2 - j)V_{20} + j3(-V_{10}) + (j - 2j)(-V_{30}) = I_2$
  - $(j - 2j)(-V_{20}) + (j - 2j - 4j)V_{30} = I_3$
- $$j\begin{bmatrix} -7 & -3 & 0 \\ -3 & 1 & 1 \\ 0 & 1 & -5 \end{bmatrix}\begin{bmatrix} V_{10} \\ V_{20} \\ V_{30} \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

  $Y \qquad V \qquad I$

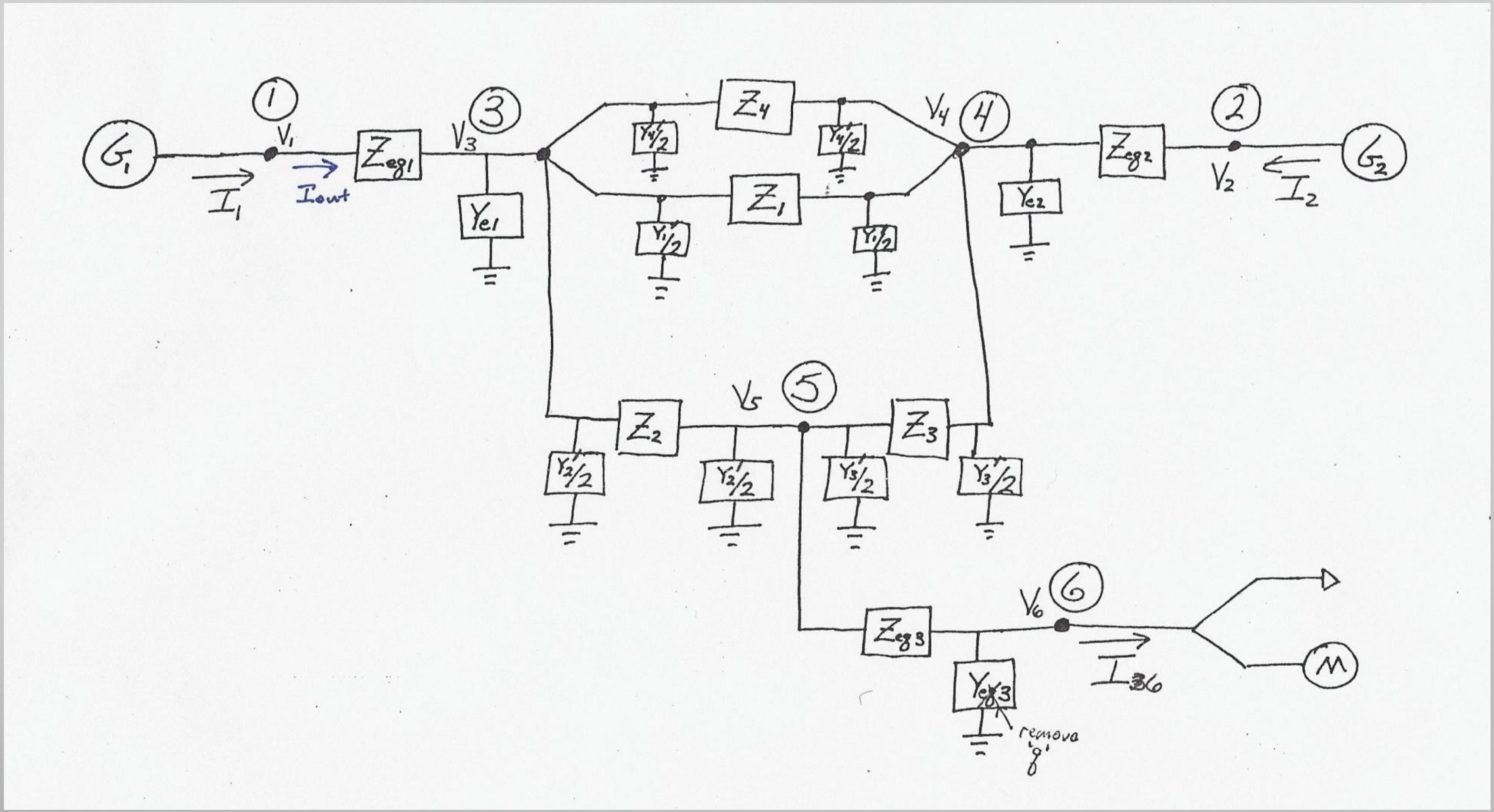
Chapter 2.4: Network Equations

- A Wedeward level example:
- Solve the following 6 bus



Chapter 2.4: Network Equations

- Circuit equivalent model (using the transmission line π equivalent circuit)



Chapter 2.4: Network Equations

- Given parameters:
- Line Data:

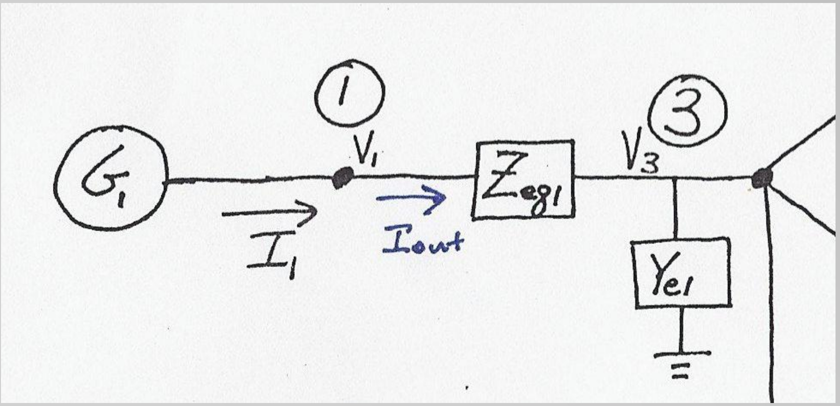
Name	From Bus	To Bus	Z' (p.u.)	Y'/2 (p.u.)
TL1	3	4	0.077 + 0.31j	0.16j
TL2	3	5	0.039 + 0.15j	0.78j
TL3	4	5	0.039 + 0.15j	0.78j
TL4	3	4	0.077 + 0.31j	0.16j

- Transformer Data:

Name	From Bus	To Bus	Zeq (p.u.)	Ye (p.u.)
T1	1	3	0.17j	0
T2	2	4	0.4j	0
T3	5	6	0.4j	0

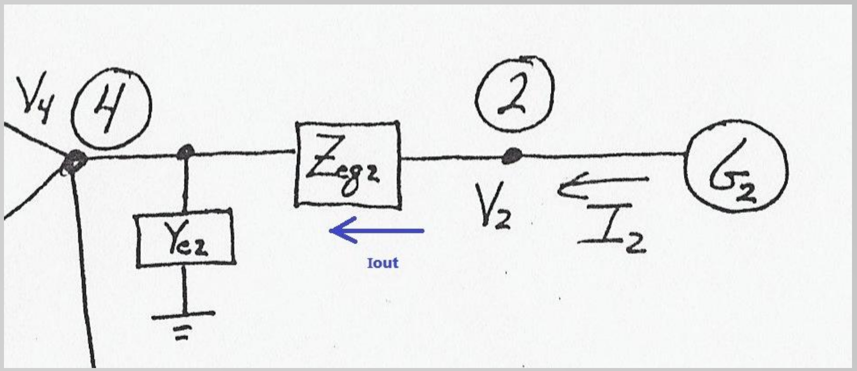
Chapter 2.4: Network Equations

- Bus 1:
- $I_1 = \left(\frac{1}{Z_{eq1}}\right)(V_1 - V_3) = Y_{eq1}V_1 - Y_{eq1}V_3$



Chapter 2.4: Network Equations

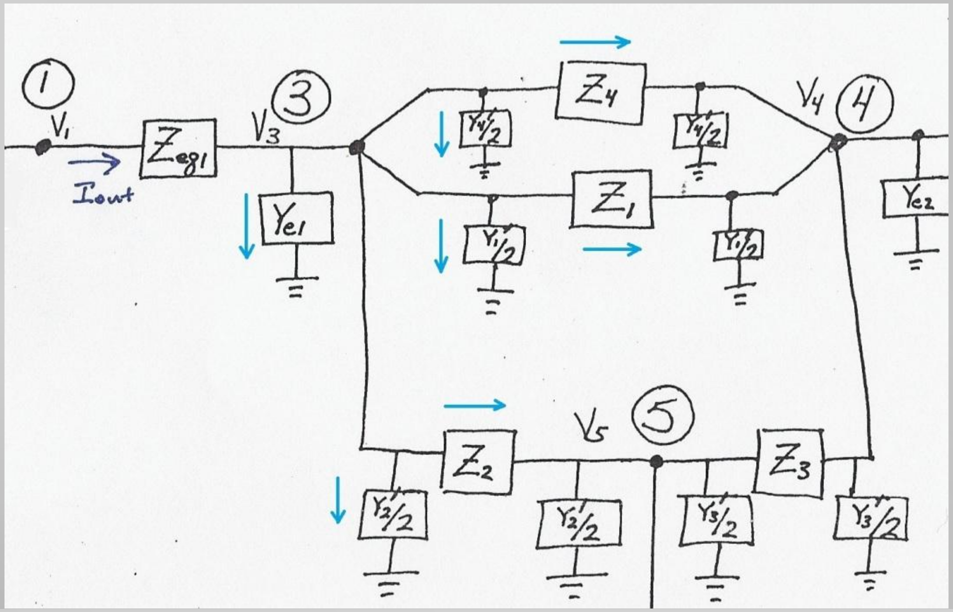
- Bus 2:
- $I_2 = \left(\frac{1}{Z_{eq2}}\right)(V_2 - V_4) = Y_{eq2}V_2 - Y_{eq2}V_4$



Chapter 2.4: Network Equations

- Bus 3:
- $0 = -\left(\frac{1}{Z_{eq1}}\right)(V_1 - V_3) + Y_{e1}V_3 + \left(\frac{Y_2'}{2}\right)V_3 + \left(\frac{1}{Z_2}\right)(V_3 - V_5) + \left(\frac{Y_1'}{2}\right)V_3 + \left(\frac{1}{Z_1}\right)(V_3 - V_4) + \left(\frac{Y_4'}{2}\right)V_3 + \left(\frac{1}{Z_4}\right)(V_3 - V_4)$

$$= -Y_{eq1}V_1 + \left(Y_{eq1} + Y_{e1} + \frac{Y_2'}{2} + Y_2 + \frac{Y_1'}{2} + Y_1 + \frac{Y_4'}{2} + Y_4\right)V_3 - (Y_1 + Y_4)V_4 - Y_2V_5$$



Chapter 2.4: Network Equations

- Bus 4 (similarly):
- $0 = -Y_{eq2}V_2 + \left(Y_4 + \frac{Y_4'}{2} + Y_1 + \frac{Y_1'}{2} + Y_3 + \frac{Y_3'}{2} + Y_{e2} + Y_{eq2}\right)V_4 - (Y_1 + Y_4)V_3 - Y_3V_5$

- Bus 5:
- $0 = -Y_2V_3 - Y_3V_4 + \left(Y_2 + \frac{Y_2'}{2} + Y_{eq3} + Y_3 + \frac{Y_3'}{2}\right) - Y_{eq3}V_6$

- Bus 6:
- $-I_6 = Y_{eq3}V_5 + \left(Y_{eq3} + Y_{e3}\right)V_6$

Chapter 2.4: Network Equations

- Formation of the Admittance Matrix:

→ in matrix-vector form $\vec{I} = Y_{bus}\vec{V}$

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ 0 \\ 0 \\ -I_6 \end{bmatrix} = \begin{bmatrix} Y_{eq1} & 0 & -Y_{eq1} & 0 & 0 & 0 \\ 0 & Y_{eq2} & 0 & -Y_{eq2} & 0 & 0 \\ -Y_{eq1} & 0 & \begin{matrix} Y_{eq1} + Y_{e1} + Y_2'/2 + Y_2 \\ + Y_1'/2 + Y_1 + Y_4'/2 + Y_4 \end{matrix} & -Y_1 - Y_4 & -Y_2 & 0 \\ 0 & -Y_{eq2} & -Y_1 - Y_4 & \begin{matrix} Y_4 + Y_4'/2 + Y_1 + Y_1'/2 \\ + Y_3 + Y_3'/2 + Y_{e2} + Y_{eq2} \end{matrix} & -Y_3 & 0 \\ 0 & 0 & -Y_2 & -Y_3 & \begin{matrix} Y_2 + Y_2'/2 + Y_{eq3} \\ + Y_3'/2 + Y_3 \end{matrix} & -Y_{eq3} \\ 0 & 0 & 0 & 0 & -Y_{eq3} & Y_{eq3} + Y_{e3} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix}$$

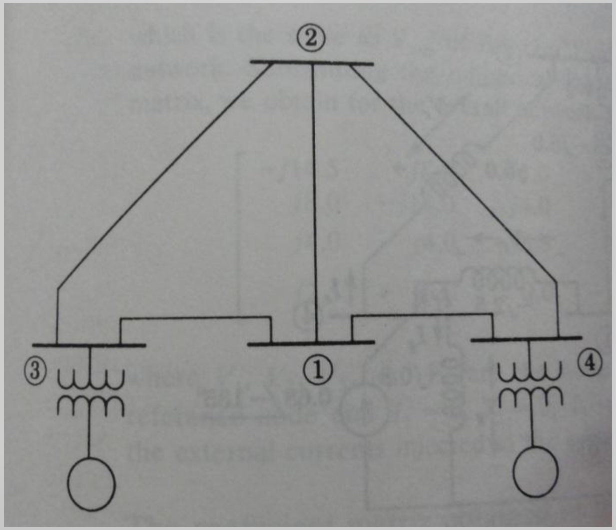
- Note that:
  - $Y_{kk} =$ admittances connected directly to k^th bus
  - $Y_{kn} = -$admittances connected between buses k and n ($k \neq n$)

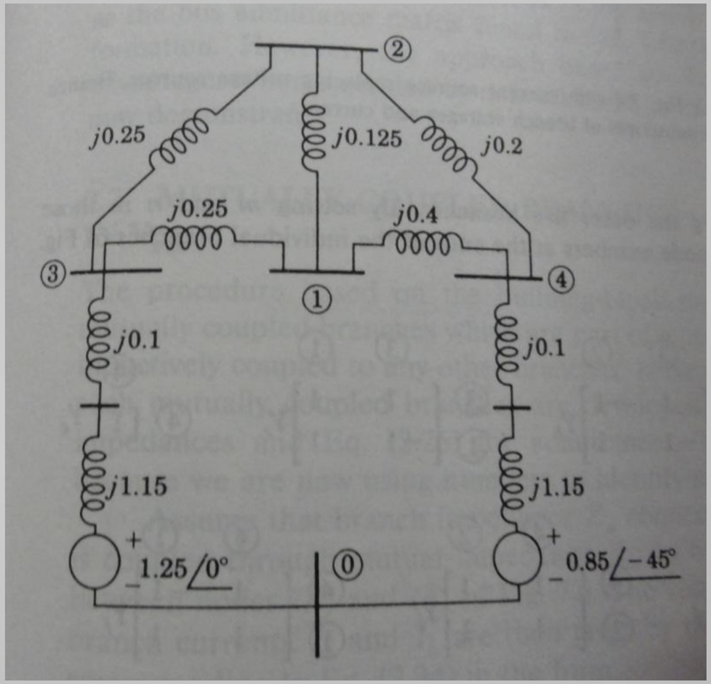
Chapter 2.4: Network Equations

- Admittance matrix formation via inspection:

- Single-line four bus

Equiv. Reactance diagram

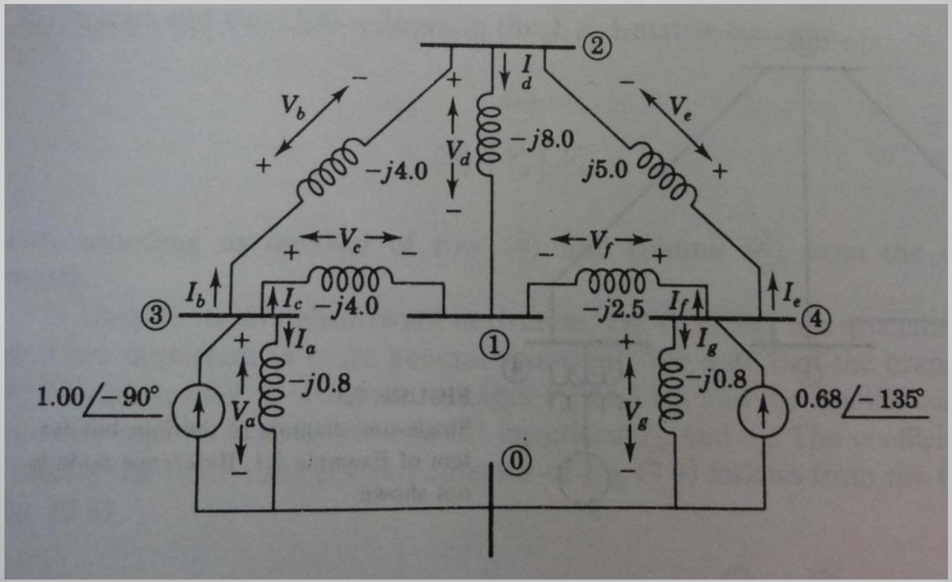




Chapter 2.4: Network Equations

- Find Y_{bus} via inspection of the per-unit diagram (converted to current sources):

- $$\begin{bmatrix} (Y_c + Y_d + Y_f) & -Y_d & -Y_c & -Y_f \\ -Y_d & (Y_b + Y_d + Y_e) & -Y_b & -Y_e \\ -Y_c & -Y_b & (Y_a + Y_b + Y_c) & 0 \\ -Y_f & -Y_e & 0 & (Y_e + Y_f + Y_g) \end{bmatrix} \vec{V} = \vec{I}$$

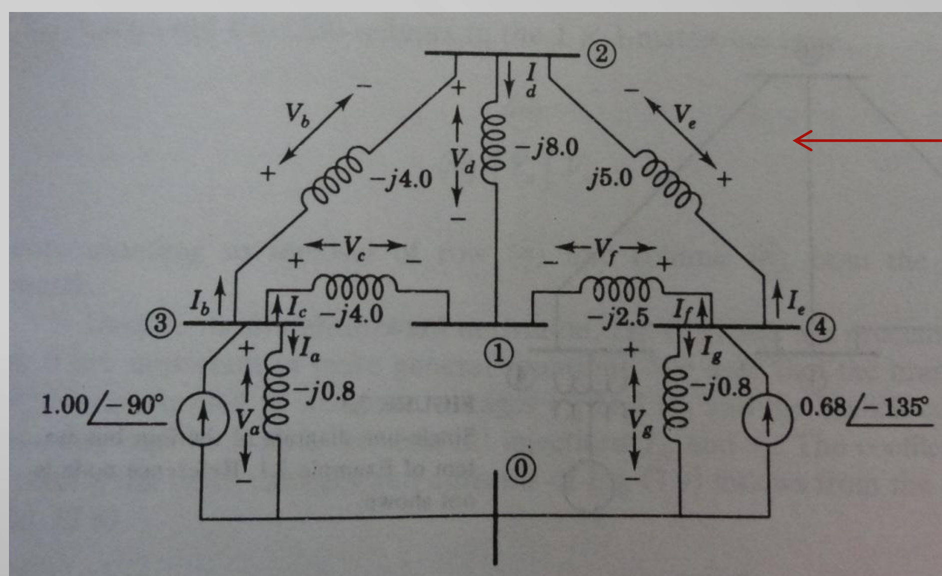


Y_{kk} = admittances connected directly to k[th] bus

Y_{kn} = −admittances connected between buses k and n ($k \neq n$)

Chapter 2.4: Network Equations

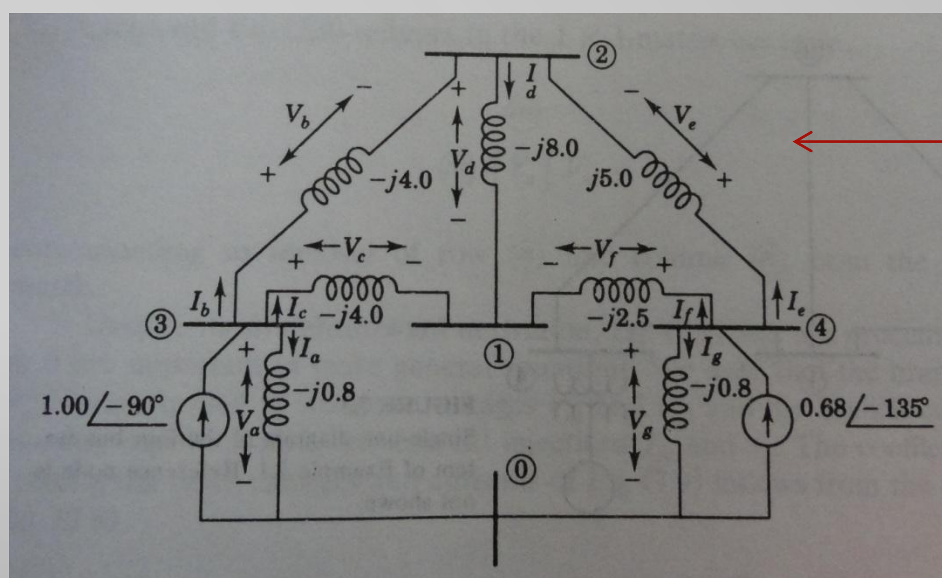
- Find Y_{bus} via inspection of the per-unit diagram (converted to current sources):
- $$\begin{bmatrix} j(-4-8-2.5) & j8 & j4 & j2.5 \\ j8 & j(-4-8-5) & j4 & j5 \\ j4 & j4 & j(-0.8-4-4) & 0 \\ j2.5 & j5 & 0 & j(-5-2.5-0.8) \end{bmatrix} \vec{V} = \vec{I}$$



Note: book should have $Y_e = -j5.0$

Chapter 2.4: Network Equations

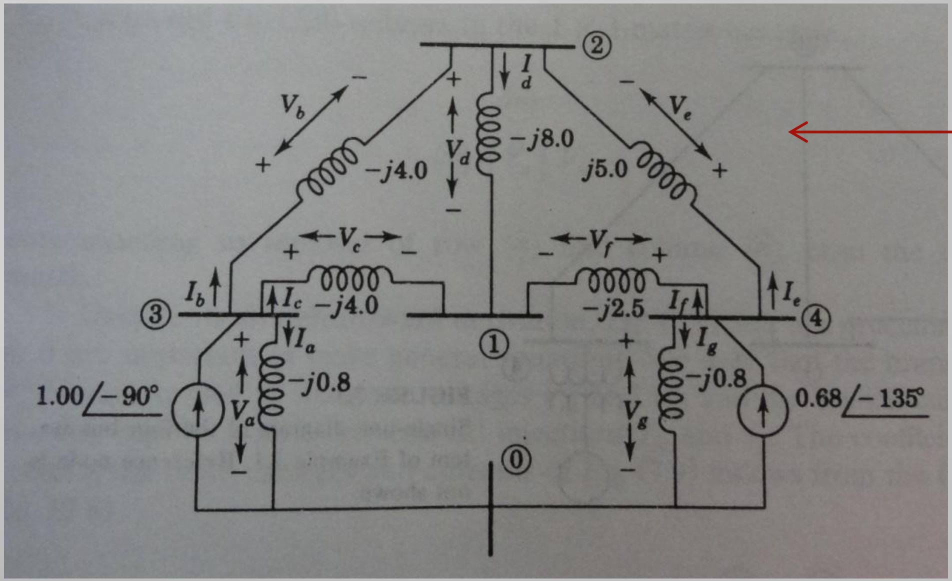
- Find Y_{bus} via inspection of the per-unit diagram (converted to current sources):
- $$\begin{bmatrix} j(-4-8-2.5) & j8 & j4 & j2.5 \\ j8 & j(-4-8-5) & j4 & j5 \\ j4 & j4 & j(-0.8-4-4) & 0 \\ j2.5 & j5 & 0 & j(-5-2.5-0.8) \end{bmatrix} \vec{V} = \vec{I}$$



Note: book should have $Y_e = -j5.0$

Chapter 2.4: Network Equations

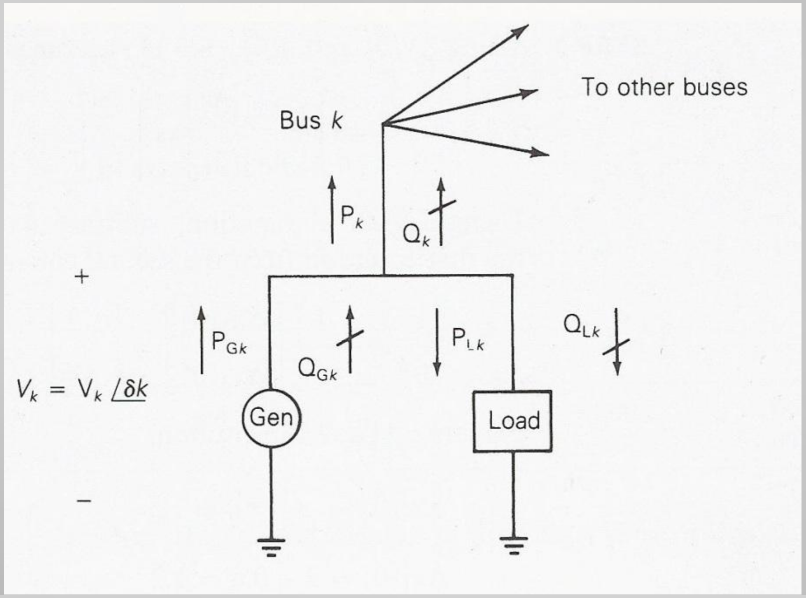
- Find Y_{bus} via inspection of the per-unit diagram (converted to current sources):
- $$\begin{bmatrix} -j14.5 & j8 & j4 & j2.5 \\ j8 & -j17 & j4 & j5 \\ j4 & j4 & -j8.8 & 0 \\ j2.5 & j5 & 0 & -j8.3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1.00\angle-90^\circ \\ 0.68\angle-135^\circ \end{bmatrix}$$



Note: book should have $Y_e = -j5.0$

Chapter 6.4: Power Flow and Bus Type

- Four bus variables per bus:
  - V_k the voltage magnitude
  - δ_k the phase angle
  - P_k the net real power, where $P_k = P_{Generator,k} + P_{Load,k}$
  - Q_k the net reactive power, where where $Q_k = Q_{Generator,k} + Q_{Load,k}$
- Bus k



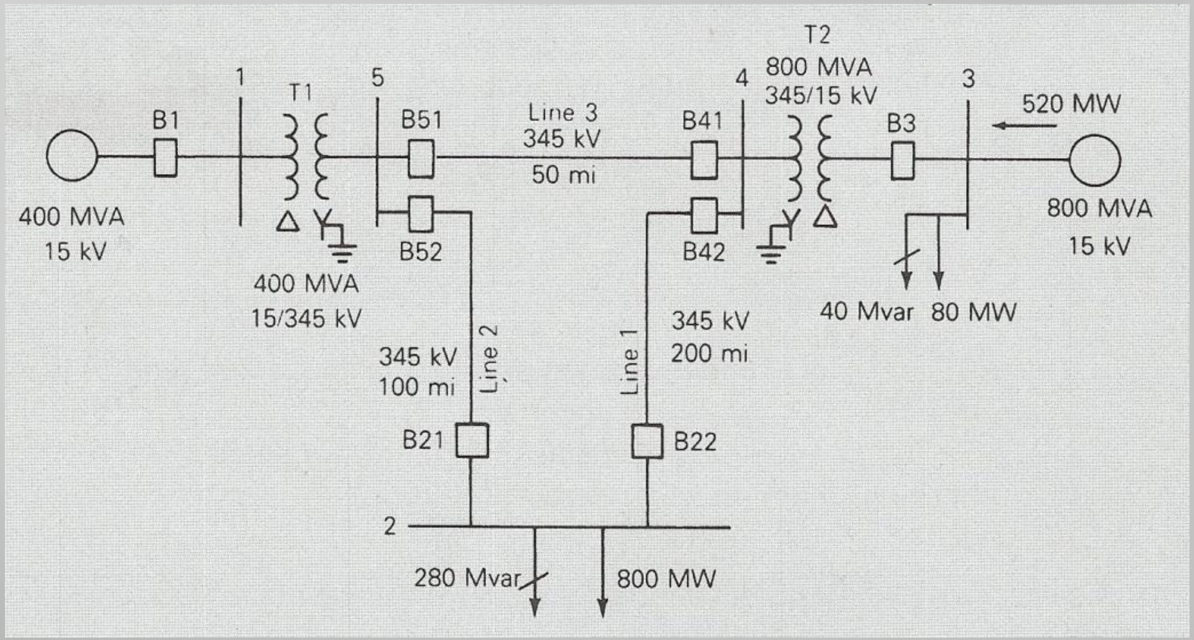
Chapter 6.4: Power Flow and Bus Type

- Bus Types:

1) Swing Bus – The only reference bus (notated as $k = 1$ for $\mathrm{V}_k \angle \delta_k$)
   - Input (known) data: $\mathrm{V}_1 \angle \delta_1 = 1.00 \angle 0^{\circ}\ \mathrm{p.u.}$ (typically)
   - Computed variables: P_1 and Q_1
2) Load (PQ) Bus – The most common type
   - Input (known) data: P_k and Q_k
   - Computed variables: V_k and δ_k
   - If no generation: $P_k = -P_{load}, k$; if the load is purely inductive: $Q_k = -Q_{Load,k}$
3) Voltage controlled (PV) bus
   - Input (known) data: P_k and V_k
   - Computed variables: Q_k and δ_k

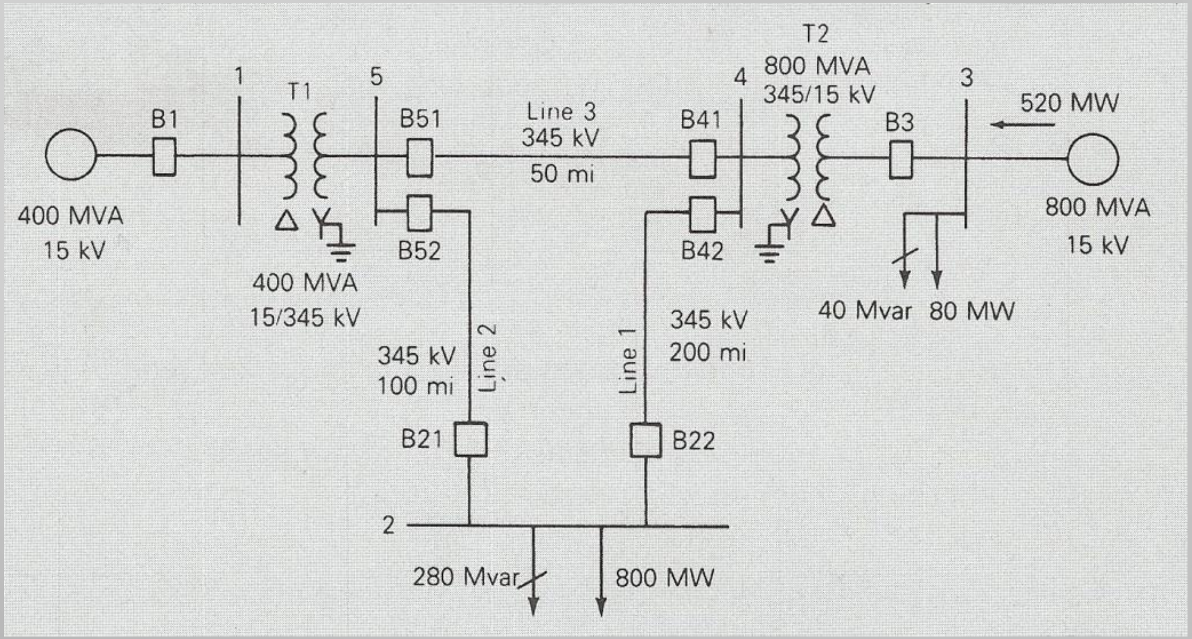
Chapter 6.4: Power Flow and Bus Type

- Example 6.9
- Bus 1 – Swing (reference) bus
- Buses 2, 4, 5 – load bus
- Bus 3 – PV bus



Chapter 6.4: Power Flow and Bus Type

- Example 6.9 continued: Y_{bus} is symmetric, so solve for upper triangular
- Buses 1 and 5 connected $\Rightarrow Y_{11}\ and\ Y_{15}\ \neq 0$, else $Y_{1i} = 0$
- Buses 2, 4, and connected $\Rightarrow Y_{22}, Y_{24},\ and\ Y_{25} \neq 0$ else $Y_{2,i+1}\ = 0$
- Buses 3 and 4 connected $\Rightarrow Y_{33}\ and\ Y_{33}\ \neq 0$, else $Y_{3,i+2} = 0$
- Etc…



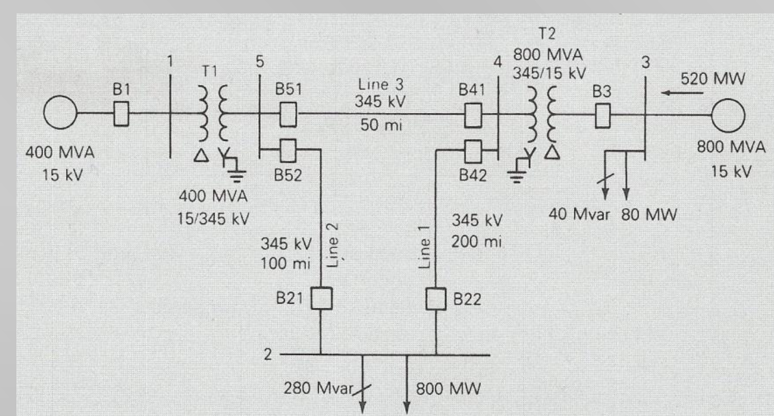
Chapter 6.4: Power Flow and Bus Type

- Example 6.9 continued: Y_{bus} is symmetric, so solve for upper triangular

- $$Y_{bus}(upper) = \begin{bmatrix} Y_{11} & 0 & 0 & 0 & Y_{15} \\ & Y_{22} & 0 & Y_{24} & Y_{25} \\ & & Y_{33} & Y_{34} & 0 \\ & & & Y_{44} & Y_{45} \\ & & & & Y_{55} \end{bmatrix}$$

- Since Y is symmetric

- $$Y_{bus} = \begin{bmatrix} Y_{11} & 0 & 0 & 0 & Y_{15} \\ 0 & Y_{22} & 0 & Y_{24} & Y_{25} \\ 0 & 0 & Y_{33} & Y_{34} & 0 \\ 0 & Y_{24} & Y_{34} & Y_{44} & Y_{45} \\ Y_{15} & Y_{25} & 0 & Y_{45} & Y_{55} \end{bmatrix}$$



Chapter 6.4: Power Flow and Bus Type

- Example 6.9 continued: Y_{bus} is symmetric, so solve for upper triangular
- $Y_{11} = \frac{1}{Z_{11}} = \frac{1}{R_{11}+jX_{11}} = \frac{1}{0.0015+j0.02} = 3.729 - j49.72;\ Y_{15} = -Y_{11}$
- $Y_{22} = \frac{1}{R_{24}+jX_{24}} + j\frac{B_{24}}{2} + \frac{1}{R_{25}+jX_{25}} + j\frac{B_{25}}{2} = 2.68 - j28.6$
- $Y_{24} = -1 * \left(\frac{1}{R_{24}+jX_{24}}\right) = -1 * \left(\frac{1}{0.009+j0.1}\right) = -0.89 + j9.9$ ← $Y_{km} = -\sum Admittances$
- $Y_{25} = -1 * \left(\frac{1}{R_{25}+jX_{25}}\right) = -1 * \left(\frac{1}{0.0045+j0.05}\right) = -1.78 + j19.9$ ←
- $Y_{33} = \frac{1}{R_{34}+jX_{34}} = \frac{1}{0.00075+j0.01} = 7.49 - j99.4$
- $Y_{34} = -Y_{33} = -7.49 + j99.4$ ←
- $Y_{44} = \frac{1}{Z_{24}} + j\frac{B_{24}}{2} + \frac{1}{Z_{45}} + j\frac{B_{45}}{2} + \frac{1}{Z_{34}} = 11.9 - j147.96$

Chapter 6.4: Power Flow and Bus Type

- Example 6.9 continued:
- $Y_{45} = -1 * \left(\frac{1}{R_{45}+jX_{45}} + j\frac{B_{45}}{2}\right) = -1 * \left(\frac{1}{0.009+j0.1}\right) = -3.6 + j39.5$ ← $Y_{km} = -\sum Admit.$
- $Y_{55} = \frac{1}{Z_{15}} + \frac{1}{Z_{45}} + j\frac{B_{45}}{2} + \frac{1}{Z_{25}} + j\frac{B_{25}}{2} = 9.0 - j108.6$
- Final matrix (using symmetric property of Y_{bus}):
- $Y_{bus} =$

$$\begin{bmatrix} 3.7 - j49.7 & 0 & 0 & 0 & -3.7 + j49.7 \\ 0 & 2.7 - j28.5 & 0 & -0.9 + j9.9 & -1.8 + j19.8 \\ 0 & 0 & 7.5 - j99.4 & -7.5 + j99.4 & 0 \\ 0 & -0.9 + j9.9 & -7.5 + j99.4 & 11.9 - j147.96 & -3.6 + j39.6 \\ -3.7 + j49.7 & -1.8 + j19.8 & 0 & -3.6 + j39.5 & 9.0 - j108.6 \end{bmatrix}$$

Chapter 6: Solving Network Equation

- Solutions to the matrices:
- Direct method
  - Gaussian Elimination
  - LU Decomposition
- Indirect methods
  - Jacobi
  - Gauss-Seidel
  - Newton-Raphson