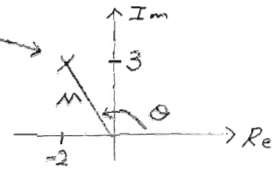
ES332 HW 10 Soln

P4.1 (c) $-2 + j3$



$M = \sqrt{(-2)^2+(3)^2} = \sqrt{13}$

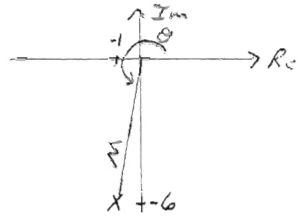
$\theta = \tan^{-1}(3/-2) = 2.16 \text{ rad} = 123.7°$

↑ be careful of quadrant

$\sqrt{13}\, e^{j2.16}$

$= 3.61 \angle 123.7°$

(d) $-1 - j6$



$M = \sqrt{(-1)^2 + (-6)^2} = \sqrt{37}$

$\theta = \tan^{-1}(-6/-1) = -1.736 \text{ rad} = -99.5°$

careful ↑ of quadrant

$\sqrt{37}\, e^{-j1.736}$

$= 6.08 \angle -99.5°$

can add 2π, 360° to angles to get 4.55 rad, 260.5°

P4.2 (c) $5e^{-j60°} = 5(\cos(-60°) + j\sin(-60°))$

$= 5(1/2 + j(-\sqrt{3}/2))$

$\frac{5}{2} - j\frac{5\sqrt{3}}{2} = 2.5 - j4.33$

(d) $4e^{-j150°} = 4(\cos(-150°) + j\sin(-150°))$

$= 4(-\sqrt{3}/2 - j1/2)$

$= -2\sqrt{3} - j2$

$-2\sqrt{3} - j2 = -3.46 - j2$

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P4.3 (c) $\hat{A}_1\hat{A}_2 = 5e^{-j60°}2e^{j120°} = 10e^{j60°}$
$= 10(\cos 60° + j\sin 60°)$
$= 10(1/2 + j\sqrt{3}/2)$
$= \underline{5 + j5\sqrt{3} = 5 + j8.66}$

(d) $\hat{A}_1\hat{A}_2 = 4e^{j45°}2e^{-j90°} = 8e^{-j45°}$
$= 8(\cos(-45°) + j\sin(-45°))$
$= 8(\sqrt{2}/2 - j\sqrt{2}/2)$
$= \underline{4\sqrt{2} - j4\sqrt{2} = 5.66 - j5.66}$

P4.4 (c) $\frac{\hat{A}_1}{\hat{A}_2} = \frac{5e^{-j60°}}{2e^{j120°}} = \frac{5}{2}e^{-j180°} = \frac{5}{2}(\cos(-180°) + j\sin(-180°))$
$= \frac{5}{2}(-1 + j0)$
$= \underline{\frac{-5}{2} + j0 = -2.5}$

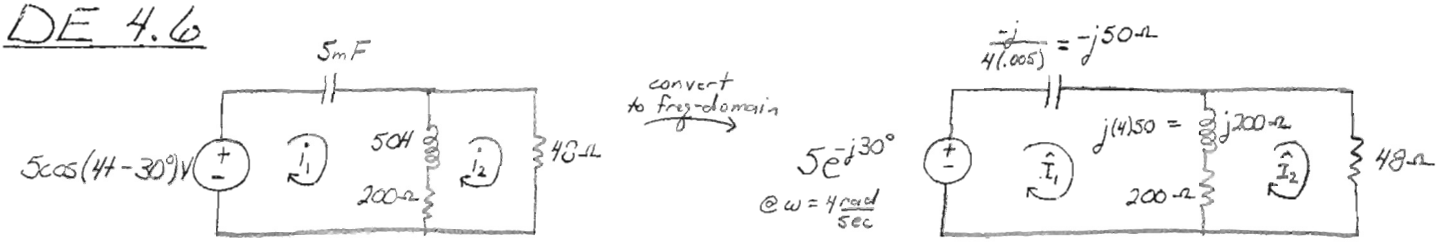
(d) $\frac{\hat{A}_1}{\hat{A}_2} = \frac{4e^{j45°}}{2e^{-j90°}} = \frac{4}{2}e^{j135°} = 2(\cos 135° + j\sin 135°)$
$= 2(-\sqrt{2}/2 + j\sqrt{2}/2)$
$= \underline{-\sqrt{2} + j\sqrt{2} = -1.414 + j1.414}$

P4.5 (c) $\hat{A}_1 + \hat{A}_2 = 5e^{-j60°} + 2e^{j120°}$
$= 5(\cos(-60°) + j\sin(-60°)) + 2(\cos(120°) + j\sin(120°))$
$= \frac{5}{2} - j\frac{5\sqrt{3}}{2} + 2(-\frac{1}{2}) + j2(\frac{\sqrt{3}}{2})$
$= \underline{\frac{3}{2} - j\frac{3\sqrt{3}}{2} = 1.5 - j2.60}$

(d) $\hat{A}_1 + \hat{A}_2 = 4e^{j45°} + 2e^{-j90°}$
$= 4(\cos 45° + j\sin 45°) + 2(\cos(-90°) + j\sin(-90°))$
$= 4(\sqrt{2}/2 + j\sqrt{2}/2) + 2(0 - j)$
$= (2\sqrt{2} + 0) + j(2\sqrt{2} - 2)$
$= \underline{2\sqrt{2} + j(2(\sqrt{2} - 1)) = 2.83 + j0.83}$

ES 332 HW 11 Soln

DE 4.6



mesh analysis

@ $\hat{I}_1$: $-5e^{-j30^\circ} - j50\hat{I}_1 + (j200+200)(\hat{I}_1 - \hat{I}_2) = 0$

$\Rightarrow (200+j150)\hat{I}_1 - (200+j200)\hat{I}_2 = 5e^{-j30^\circ}$ $\times 1/5$

$\Rightarrow (40+j30)\hat{I}_1 - (40+j48)\hat{I}_2 = 1e^{-j30^\circ}$

@ $\hat{I}_2$: $(200+j200)(\hat{I}_2 - \hat{I}_1) + 48\hat{I}_2 = 0$

$\Rightarrow -(200+j200)\hat{I}_1 + (248+j200)\hat{I}_2 = 0$

matrix-vector form

$$\begin{bmatrix} 40+j30 & -40-j40 \\ -200-j200 & 248+j200 \end{bmatrix}\begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} = \begin{bmatrix} 1e^{-j30^\circ} \\ 0 \end{bmatrix}$$

Cramer's Rule

$$\hat{I}_1 = \frac{\begin{vmatrix} 1e^{-j30^\circ} & -40-j40 \\ 0 & 248+j200 \end{vmatrix}}{\begin{vmatrix} 40+j30 & -40-j40 \\ -200-j200 & 248+j200 \end{vmatrix}} = \frac{314.8+j49.2}{3920-j560} = 0.0769+j0.0235$$

$$= 0.0805e^{j0.297} = 0.0805\angle 17^\circ$$

$$\hat{I}_2 = \frac{\begin{vmatrix} 40+j30 & 1e^{-j30^\circ} \\ -200-j200 & 0 \end{vmatrix}}{3920-j560} = \frac{273.2+j73.2}{3920-j560} = 0.0657+j0.0281$$

$$= 0.0714e^{j0.404} = 0.0714\angle 23.1^\circ$$

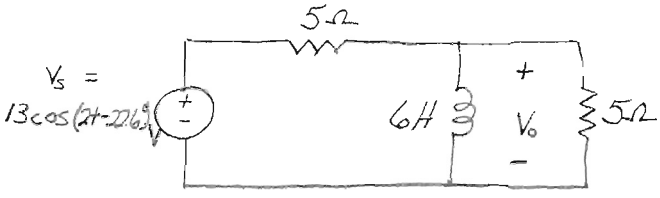
Back to time

$$i_1(t) = 0.0805\cos(4t+17.0^\circ)\,A = 0.0805\cos(4t+0.297)\,A$$

$$i_2(t) = 0.0714\cos(4t+23.1^\circ)\,A = 0.0714\cos(4t+0.404)\,A$$

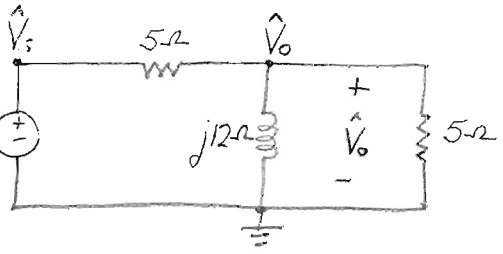
ES332 HW11 Soln

P4.8



to freq-domain →

$\hat{V}_s = 13e^{-j22.6°}$ V
@ $\omega = 2$ rad/sec



nodal analysis

@ $\hat{V}_s$: $\hat{V}_s = 13\angle -22.6°$ V known

@ $\hat{V}_o$: $\frac{\hat{V}_o - \hat{V}_s}{5} + \frac{\hat{V}_o}{j12} + \frac{\hat{V}_o}{5} = 0$) $\times j60$

$j12\hat{V}_o - j12\hat{V}_s + 5\hat{V}_o + j12\hat{V}_o = 0$

$\Rightarrow \hat{V}_o(j12 + 5 + j12) = j12\hat{V}_s$

$\Rightarrow \hat{V}_o(5 + j24) = 12e^{j90°}13e^{-j22.6°}$

$\Rightarrow \hat{V}_o = \frac{156e^{j67.4°}}{5 + j24}$

$= \frac{156e^{j67.4°}}{24.5e^{j78.2°}}$

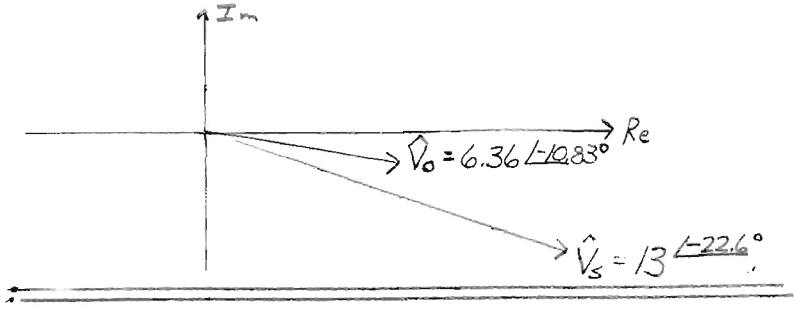
$= 6.25 - j1.196$ V

$= 6.36e^{-j0.189}$ V $= 6.36\angle -10.83°$ V

back to time

$$v_o(t) = 6.36\cos(2t - 10.83°)\text{ V} = 6.36\cos(2t - 0.189)\text{ V}$$

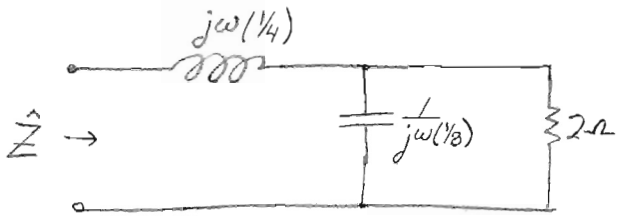
phasor diagram



V_o leads V_s

EE332 HW11 Soln

P4.13 in freq-domain:



$$\hat{Z} = j\omega(\tfrac{1}{4}) + \frac{\frac{1}{j\omega(1/8)} \cdot 2}{\frac{1}{j\omega(\frac{1}{8})} + 2}$$

$$= j\omega(1/4) + \frac{\frac{16}{j\omega}}{\frac{8}{j\omega} + 2} \quad \times \frac{j\omega}{j\omega}$$

$$= j\omega(1/4) + \frac{16}{8 + j\omega 2} \quad \times \frac{1/2}{1/2}$$

$$= j\omega(1/4) + \frac{8}{4 + j\omega} \quad \times \frac{4 - j\omega}{4 - j\omega}$$

$$= j\omega(1/4) + \frac{32 - j\omega 8}{16 + \omega^2}$$

$$= \frac{32}{16+\omega^2} + j\omega\left(\frac{1}{4} - \frac{8}{16+\omega^2}\right)$$

(a) $\omega = 2 \frac{rad}{sec} \Rightarrow \hat{Z} = \frac{32}{16+2^2} + j2\left(\frac{1}{4} - \frac{8}{16+2^2}\right) = 1.6 - j0.3\ \Omega$

$= 1.628 e^{-j0.185}\ \Omega$

$= 1.628 \angle -10.62\ \Omega$

(b) $\omega = 4 \frac{rad}{sec} \Rightarrow \hat{Z} = \frac{32}{16+4^2} + j4\left(\frac{1}{4} - \frac{8}{16+4^2}\right)$

$= 1\ \Omega$

(c) $\omega = 8 \frac{rad}{sec} \Rightarrow \hat{Z} = \frac{32}{16+8^2} + j8\left(\frac{1}{4} - \frac{8}{16+8^2}\right)$

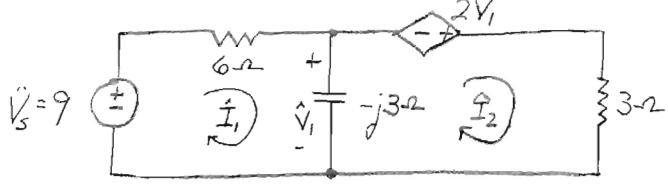
$= 0.4 + j1.2\ \Omega$

$= 1.265 e^{j1.249}\ \Omega$

$= 1.265 \angle 71.6°\ \Omega$

E5332 HW12 Soln

Example 4.1 (looking for $P, Q, \hat{S}$; can use $P=\frac{1}{2}VI\cos(\phi_1-\phi_2)$, $Q=\frac{1}{2}VI\sin(\phi_1-\phi_2)$ for $\hat{S}=\frac{1}{2}\hat{V}\hat{I}^*=P+jQ$)



$\hat{I}_1 = 1.30\angle 2.49^\circ$, $\hat{I}_2 = 1.24\angle -15.9^\circ$

indep source: $\hat{S}_s = -\frac{1}{2}9\hat{I}_1^* = -\frac{1}{2}9(1.30\angle 2.49^\circ) = \underline{-5.84 + j0.254 \text{ VA}}$

$\Rightarrow \underline{P_s = -5.84\text{ W},\ Q_s = 0.254\text{ VAR}}$

6Ω resistor: $\hat{S}_{6\Omega} = \frac{1}{2}(6\hat{I}_1)\hat{I}_1^* = \frac{1}{2}6I_1^2 = 3(1.3)^2 = \underline{5.07 + j0 \text{ VA}}$

$\underline{P_{6\Omega} = 5.07\text{W},\ Q_{6\Omega} = 0\text{VAR}}$

-j3 capacitor: $\hat{S}_c = \frac{1}{2}(-j3)(\hat{I}_1-\hat{I}_2)(\hat{I}_1-\hat{I}_2)^*$

$= -\frac{j3}{2}(1.30\angle 2.49^\circ - 1.24\angle -15.9^\circ)(1.30\angle 2.49 - 1.24\angle 15.9^\circ)^*$

$= \underline{0 - j0.252 \text{ VA}} \quad \Rightarrow \underline{P_c = 0\text{W},\ Q_c = -0.252\text{VAR}}$

dep source: $\hat{S}_d = \frac{1}{2}(2\hat{V}_1)(-\hat{I}_2)^* = \frac{1}{2}(2)(-j3)(\hat{I}_1-\hat{I}_2)(-\hat{I}_2)^*$

$= +j3(1.30\angle 2.49^\circ - 1.24\angle -15.9^\circ)(1.24\angle 15.9^\circ)^*$

$= \underline{-1.526 - j0.0238 \text{ VA}} \quad \Rightarrow \underline{P_d = -1.526\text{ W},\ Q = -0.0238\text{ VAR}}$

3Ω resistor: $\hat{S}_{3\Omega} = \frac{1}{2}(3\hat{I}_2)\hat{I}_2^* = \frac{3}{2}I_2^2$

$= \frac{3}{2}(1.24)^2 = \underline{2.31 + j0 \text{ VA}}$

$\Rightarrow \underline{P_3 = 2.31\text{W},\ Q_3 = 0\text{VAR}}$

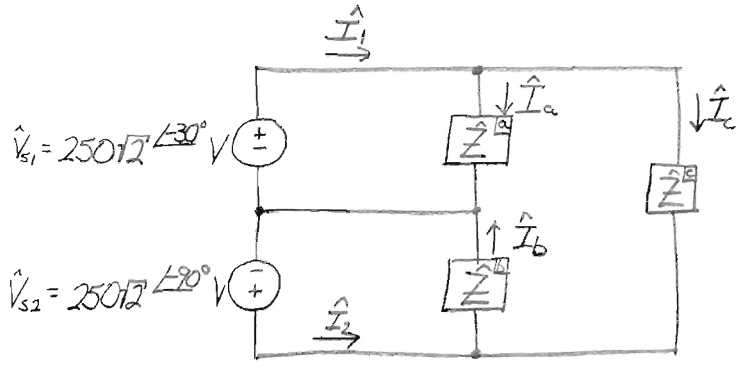
not required, but good check is $\Sigma\hat{S} = \Sigma P + j\Sigma Q = 0$

ΣP: $-5.84 + 5.07 + 0 - 1.526 + 2.31 = 0.006 \approx 0$ (round off from $\hat{I}_1, \hat{I}_2$)

ΣQ: $0.254 + 0 - 0.252 - 0.0238 + 0 = 0.022 \approx 0$

E5332 HW12 Soln

P4.24



$\hat{Z} = 78 - j45\ \Omega = 90.005\angle -30°$

$\hat{I}_a = \frac{\hat{V}_{s1}}{\hat{Z}} = \frac{250\sqrt{2}\angle -30°}{78 - j45} = 3.93\angle -0.184° = 3.93 e^{-j0.00d}$ A

$\hat{I}_b = \frac{\hat{V}_{s2}}{\hat{Z}} = \frac{250\sqrt{2}\angle -90°}{78 - j45} = 3.93\angle -60° = 3.93 e^{-j1.048}$ A

$\hat{I}_c = \frac{\hat{V}_{s1} - \hat{V}_{s2}}{\hat{Z}} = \frac{250\sqrt{2}\angle -30° - 250\sqrt{2}\angle -90°}{78 - j45} = 3.93\angle 60° = 3.93 e^{j1.047}$ A

$\hat{S}_a = \frac{1}{2}\hat{V}_{s1}\hat{I}_a^* = \frac{1}{2}(250\sqrt{2}\angle -30°)(3.93\angle -0.184°)^* = 601.2 - j346.8$ VA

$\Rightarrow P_a = 601.2$W, $Q_a = -346.8$ VAR

$\hat{S}_b = \frac{1}{2}\hat{V}_{s2}\hat{I}_b^* = \frac{1}{2}(250\sqrt{2}\angle -90°)(3.93\angle -60°)^* = 601.2 - j346.8$ VA

$\Rightarrow P_b = 601.2$W, $Q_b = -346.8$ VAR

$\hat{S}_c = \frac{1}{2}(\hat{V}_{s1} - \hat{V}_{s2})\hat{I}_c^* = \frac{1}{2}(250\sqrt{2}\angle -30° - 250\sqrt{2}\angle -90°)(3.93\angle 60°)^*$

$= 601.2 - j346.8$ VA $\Rightarrow P_c = 601.2$W, $Q = -346.8$VAR

$\hat{S}_{s1} = -\frac{1}{2}\hat{V}_{s1}\hat{I}_1^* = -\frac{1}{2}\hat{V}_{s1}(\hat{I}_a + \hat{I}_c)^* = -\frac{1}{2}(250\sqrt{2}\angle -30°)(3.93\angle -0.184° + 3.93\angle 60°)^*$

$= -601.4 + j1040.9$ VA

$\Rightarrow P_{s1} = -601.4$W, $Q_{s1} = 1041$VAR

$\hat{S}_{s2} = -\frac{1}{2}\hat{V}_{s2}\hat{I}_2^* = -\frac{1}{2}\hat{V}_{s2}(+\hat{I}_b - \hat{I}_c) = -\frac{1}{2}(250\sqrt{2}\angle -90°)(3.93\angle -60° - 3.93\angle 60°)^*$

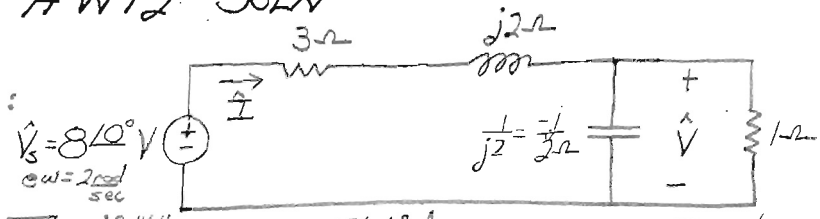
$= -1202.1 - j38.5$ VA

$\Rightarrow P_{s2} = -1202$W, $Q_{s2} = -38.5$VAR

~~not required~~, but good check is $\Sigma\hat{S} = \Sigma P + j\Sigma Q = 0$ ✓

ES 332 HW12 Soln

P4.29 in freq-domain:



$$\hat{I} = \frac{8}{3 + j2 + 0.2 - j0.4} = \sqrt{5}\,e^{j0.464} = 2.236\angle 26.6° \text{ A}$$

in ∥: $\frac{-j/2 \cdot 1}{1 - j/2} \cdot \frac{1+j/2}{1+j/2}$

$= \frac{-j/2 + 1/4}{1 + 1/4} = \frac{1 - j2}{5}$

$= 0.2 - j0.4\,\Omega$

$\hat{S}_s = \frac{-1}{2}\hat{V}_s\hat{I}^* = \frac{-1}{2}8(\sqrt{5}\angle 26.6°) = \underline{-8 - j4 \text{ VA}}$

$\Rightarrow \underline{P_s = -8 \text{ W}, \; Q_s = -4 \text{ VAR}}$

$\hat{S}_{3\Omega} = \frac{1}{2}(3\hat{I})\hat{I}^* = \frac{3}{2}(\sqrt{5})^2 = \underline{7.5 + j0 \text{ VA}}$

$\Rightarrow \underline{P_{3\Omega} = 7.5 \text{ W}, \; Q_{3\Omega} = 0 \text{ VAR}}$

$\hat{S}_L = \frac{1}{2}(j2\hat{I})\hat{I}^* = (\sqrt{5})^2 j = \underline{0 + j5 \text{ VA}}$

$\Rightarrow \underline{P_L = 0 \text{ W}, \; Q_L = 5 \text{ VAR}}$

$\hat{S}_c = \frac{1}{2}\hat{V}\left(\frac{\hat{V}}{\frac{1}{j2}}\right)^* = \frac{1}{2}\underbrace{(\hat{I}(0.2 - j0.4))}_{\hat{V}}\left(\frac{\hat{I}(.2 - j.4)}{\frac{1}{j2}}\right)^*$

$= \frac{1}{2}(-j2)(.2^2 + .4^2)\sqrt{5}^2$

$= \underline{0 - j1 \text{ VA}} \Rightarrow \underline{P_c = 0 \text{ W}, \; Q_c = -1 \text{ VAR}}$

$\hat{S}_{1\Omega} = \frac{1}{2}\hat{V}\left(\frac{\hat{V}}{1}\right)^* = \frac{1}{2}\hat{I}(.2 - j.4)\left(\frac{\hat{I}(.2 - j.4)}{1}\right)^*$

$= \frac{1}{2}(.2^2 + .4^2)\sqrt{5}^2$

$= \underline{0.5 + j0 \text{ VA}} \Rightarrow \underline{P_{1\Omega} = \frac{1}{2} \text{ W}, \; Q_{1\Omega} = 0 \text{ VAR}}$

not required, but as a check $\sum \hat{S} = \sum P + j\sum Q = 0$ ✓

E5332 HW 13 SOLN

P4.42 $V_e = 115\ V_{rms}$, $f = 60Hz$, $P = 500\ W$, $pf = 0.95$ lag

$$P = V_e I_e \cos\theta = V_e I_e\, pf$$

$$\Rightarrow \underline{I_e} = \frac{P}{V_e\, pf} = \frac{500}{(115)0.95} = \underline{4.58 A_{rms}}$$

P4.43 $V_e = 220 V_{rms}$, $I_e = 20 A_{rms}$, $f = 60Hz$, $P = 2200\ W$

(a) $$P = V_e I_e\, pf \Rightarrow pf = \frac{P}{V_e I_e} = \frac{2200}{(220)(20)} = \underline{0.5}$$

(b) $$Q_{m_{motor}} = V_e I_e \sin\theta = V_e I_e \sin(+\cos^{-1}(.5)) = (220)(20)\sin(\underbrace{1.047\,rad}_{60^\circ})$$
$$= 3810.5\ VAR$$

want total Q (C // motor) to be $0 = Q = Q_m + Q_{C_{cap}}$

$\Rightarrow$ want $Q_c = -Q_m = -3810.5\ VAR$

where $Q_c = -\omega C V_e^2$

$$\Rightarrow \underline{C} = \frac{Q_c}{\omega V_e^2} = \frac{-Q_m}{-\omega V_e^2} = \frac{+3810.5}{+(2\pi\cdot 60)(220)^2}$$

$$= \underline{2.09 \times 10^{-4} F = 209 \mu F}$$

ES332 HW13 Soln

P4.44 $V_e = 220 V_{rms}$, $f = 60Hz$, $I_e = 20 A_{rms}$, $pf = 0.75$ lag

(a) $P = V_e I_e \cos\theta = P_e I_e pf = (220)(20)0.75 = \underline{3300 W}$

(b) $Q_m = V_e I_e \sin\theta = V_e I_e \sin(\cos^{-1}(0.75))$

$= (220)(20)\sin(\underbrace{0.723 rad}_{41.4°})$

$= 2910$ VAR

want total Q (C+motor) $= 0 = Q_m + Q_c$

$\Rightarrow Q_c = -Q_m = -2910$ VAR

where $Q_c = -\omega C V_e^2$

$\Rightarrow \underline{C} = \frac{-Q_c}{\omega V_e^2} = \frac{Q_m}{\omega V_e^2} = \frac{2910}{(2\pi \cdot 60)(220)^2}$

$= \underline{1.595 \times 10^{-4} F = 159.5 \mu F}$

P4.48 $V_e = 220 V_{rms}$, $I_e = 5 A_{rms}$, $pf = 0.95$ lag $\Rightarrow \theta = \cos^{-1}(.95) = 0.318 rad = 18.2°$

(a) $\hat{S} = V_e I_e \angle\theta = (220)(5)\angle 18.2° = \underline{1100\angle 18.2° VA = 1100 e^{j0.318} VA = 1045 + j343 VA}$

(b) $P = Re\{\hat{S}\}$ or $= V_e I_e \cos\theta = \underline{1045 W}$

(c) $Q = Im\{\hat{S}\}$ or $= V_e I_e \sin\theta = \underline{343 VAR}$

(d) $|\hat{S}| = V_e I_e = \underline{1100 VA}$

(e) $\hat{Z} = \frac{\hat{V}_e}{\hat{I}_e} = \frac{V_e}{I_e}\angle\theta = \frac{220}{5}\angle 18.2° = \underline{44\angle 18.2° \Omega = 44 e^{j0.318} \Omega}$

$= \underline{41.8 + j13.74 \Omega}$