5.60 Find the Laplace transform of (a) $\sin(\beta t - \phi)u(t)$, (b) $\cos(\beta t - \phi)u(t)$, (c) $e^{-\alpha t} \sin(\beta t - \phi)u(t)$, and (d) $e^{-\alpha t} \cos(\beta t - \phi)u(t)$.

5.61 Find the inverse Laplace transform of each of the following functions:

(a) $\frac{600}{s(s + 10)(s + 30)}$
(b) $\frac{60(s + 4)}{s(s + 2)(s + 12)}$

5.62 Find the inverse Laplace transform of each of the following functions:

(a) $\frac{12s}{(s + 3)(s^2 + 9)}$
(b) $\frac{4(s^2 + 1)}{s(s^2 + 4)}$

5.63 Find the inverse Laplace transform of each of the following functions:

(a) $\frac{(s + 2)(s + 3)}{s(s + 1)^2}$
(b) $\frac{10s + 80}{s^2 + 8s + 20}$

5.64 Find the solution to the differential equation

$$\ddot{x}(t) + 7 \frac{dx(t)}{dt} + 6x(t) = 36u(t)$$

subject to the initial conditions $dx(0)/dt = 0$ and $x(0) = -4$.

5.65 Find the solution to the differential equation

$$\ddot{x}(t) + 3 \frac{dx(t)}{dt} + 2x(t) = 20 \cos 2t \ u(t)$$

subject to the zero initial conditions $dx(0)/dt = x(0) = 0$.

5.66 For the series $RC$ circuit shown in Fig. P5.66, suppose that $R = 5 \ \Omega$ and $C = 0.1 \ \text{F}$. Find the step responses $v(t)$ and $i(t)$ when $v_s(t) = 20u(t) \ \text{V}$.

5.67 For the series $RC$ circuit shown in Fig. P5.66, suppose that $R = 2 \ \Omega$ and $C = 2 \ \text{F}$. Find $v(t)$ and $i(t)$ when $v_s(t) = 12e^{-t}u(t) \ \text{V}$.

5.68 For the series $RC$ circuit shown in Fig. P5.66, suppose that $R = 2 \ \Omega$ and $C = 2 \ \text{F}$. Find $v(t)$ and $i(t)$ when $v_s(t) = 12e^{-t/4}u(t) \ \text{V}$.

5.69 For the series $RL$ circuit shown in Fig. P5.69, suppose that $R = 5 \ \Omega$ and $L = 5 \ \text{H}$. Find the step responses $i(t)$ and $v(t)$ when $v_s(t) = 20u(t) \ \text{V}$.
3.63 For the series RLC circuit shown in Fig. P3.63, suppose that $R = 7 \, \Omega$, $L = 1 \, \text{H}$, $C = 0.1 \, \text{F}$, $v_i(t) = 12 \, \text{V}$ for $t < 0 \, \text{s}$ and $v_i(t) = 0 \, \text{V}$ for $t \geq 0 \, \text{s}$. Find $v(t)$ and $i(t)$ for all time.

3.64 For the series RLC circuit shown in Fig. P3.63, suppose that $R = 2 \, \Omega$, $L = 0.25 \, \text{H}$, $C = 0.2 \, \text{F}$, $v_i(t) = 10 \, \text{V}$ for $t < 0 \, \text{s}$ and $v_i(t) = 0 \, \text{V}$ for $t \geq 0 \, \text{s}$. Find $v(t)$ and $i(t)$ for all time.

3.65 For the series RLC circuit shown in Fig. P3.63, suppose that $R = 2 \, \Omega$, $L = 1 \, \text{H}$, $C = 1 \, \text{F}$, $v_i(t) = 6 \, \text{V}$ for $t < 0 \, \text{s}$ and $v_i(t) = 0 \, \text{V}$ for $t \geq 0 \, \text{s}$. Find $v(t)$ and $i(t)$ for all time.

3.66 For the circuit shown in Fig. P3.66, suppose that $v_i(t) = 6 \, \text{V}$ for $t < 0 \, \text{s}$ and $v_i(t) = 0 \, \text{V}$ for $t \geq 0 \, \text{s}$. Find $v_2(t)$ and $v_1(t)$ for all time.

3.67 For the circuit shown in Fig. P3.67, suppose that $v_i(t) = 6 \, \text{V}$ for $t < 0 \, \text{s}$ and $v_i(t) = 0 \, \text{V}$ for $t \geq 0 \, \text{s}$. Find $i(t)$ and $v(t)$ for all time.

3.68 For the circuit shown in Fig. P3.67, interchange the inductor and the capacitor. Suppose that $v_i(t) = 6 \, \text{V}$ for $t < 0 \, \text{s}$ and $v_i(t) = 0 \, \text{V}$ for $t \geq 0 \, \text{s}$. Find the capacitor voltage $v(t)$ and the inductor current $i(t)$ for all time.

3.69 For the parallel RLC circuit shown in Fig. P3.69, suppose that $R = 0.5 \, \Omega$, $L = 0.2 \, \text{H}$, $C = 0.25 \, \text{F}$, and $i_i(t) = 2u(t) \, \text{A}$. Find the step responses $i(t)$ and $v(t)$.

3.70 For the parallel RLC circuit shown in Fig. P3.69, suppose that $R = 3 \, \Omega$, $L = 3 \, \text{H}$, $C = \frac{1}{6} \, \text{F}$, and $i_i(t) = 4u(t) \, \text{A}$. Find the step responses $i(t)$ and $v(t)$.

3.71 For the series RLC circuit shown in Fig. P3.63, suppose that $R = 7 \, \Omega$, $L = 1 \, \text{H}$, $C = 0.1 \, \text{F}$, and $v_i(t) = 12u(t) \, \text{V}$. Find the step responses $v(t)$ and $i(t)$.

3.72 For the series RLC circuit shown in Fig. P3.63, suppose that $R = 2 \, \Omega$, $L = 1 \, \text{H}$, $C = 1 \, \text{F}$, and $v_i(t) = 10 \, \text{V}$ for $t < 0 \, \text{s}$ and $v_i(t) = 0 \, \text{V}$ for $t \geq 0 \, \text{s}$. Find $v(t)$ and $i(t)$ for all time.
and $v_s(t) = 12u(t)$ V. Find the step responses $v(t)$ and $i(t)$.

3.73 For the RLC circuit shown in Fig. 3.43 on p. 172, suppose that $R = \frac{1}{2} \Omega$, $L = \frac{1}{3} \text{H}$, $C = \frac{1}{4} \text{F}$, and $V = 1$ V. Find the unit step responses $i(t)$ and $v(t)$.

3.74 For the RLC circuit shown in Fig. 3.43 on p. 172, suppose that $R = \frac{1}{2} \Omega$, $L = \frac{1}{4} \text{H}$, $C = \frac{1}{2} \text{F}$, and $V = 1$ V. Find the unit step responses $i(t)$ and $v(t)$.

3.75 For the circuit shown in Fig. P3.66, suppose that $v_s(t) = 9u(t)$ V. Find the step response $v_2(t)$.

3.76 For the circuit shown in Fig. P3.67, suppose that $v_s(t) = 6u(t)$ V. Find the step responses $i(t)$ and $v(t)$.

3.77 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.77 when $C = \frac{1}{3} \text{F}$ and $v_s(t) = 4u(t)$ V.

3.78 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.77 when $C = \frac{1}{8} \text{F}$ and $v_s(t) = 8u(t)$ V.

3.79 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.77 when $C = \frac{1}{4} \text{F}$ and $v_s(t) = 6u(t)$ V.

3.80 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.80 when $C = \frac{4}{1} \text{F}$ and $v_s(t) = 4u(t)$ V.

3.81 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.80 when $C = 1 \text{F}$ and $v_s(t) = 3u(t)$ V.

3.82 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.80 when $C = \frac{1}{3} \text{F}$ and $v_s(t) = 2u(t)$ V.

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**Fig. P3.77**

**Fig. P3.80**
Drill Exercise 3.17

Find the step responses $v(t)$ and $i(t)$ for the series $RLC$ circuit shown in Fig. 3.41 when $R = 16 \, \Omega$, $L = 2 \, H$, $C = 1/50 \, F$, and $V = 6$.

**ANSWER** \[6 - 10e^{-4t} \cos(3t - 0.927)]u(t) V, e^{-t} \sin 3t \, u(t) \, A

The Critically Damped Case

Consider the $RLC$ circuit shown in Fig. 3.43. By KCL,

\[\frac{V_u(t) - v}{R} = i + C \frac{dv}{dt}\] (3.59)

![Fig. 3.43 Another $RLC$ circuit.]

Substituting $v = L \frac{di}{dt}$ into Eq. 3.59 and simplifying results in

\[\frac{d^2i}{dt^2} + \frac{1}{RC} \frac{di}{dt} + \frac{1}{LC} i = \frac{V}{RLC} u(t)\] (3.60)

For $t < 0 \, s$, the right-hand side of this equation becomes zero, and for zero initial conditions the solution is $i(t) = 0 \, A$. However, for $t \geq 0 \, s$, Eq. 3.59 becomes

\[\frac{d^2i}{dt^2} + \frac{1}{RC} \frac{di}{dt} + \frac{1}{LC} i = \frac{V}{RLC}\] (3.61)

Thus we have that $\alpha = 1/2RC$ and $\omega_n = 1/\sqrt{LC}$.

We finally consider the critically damped case.