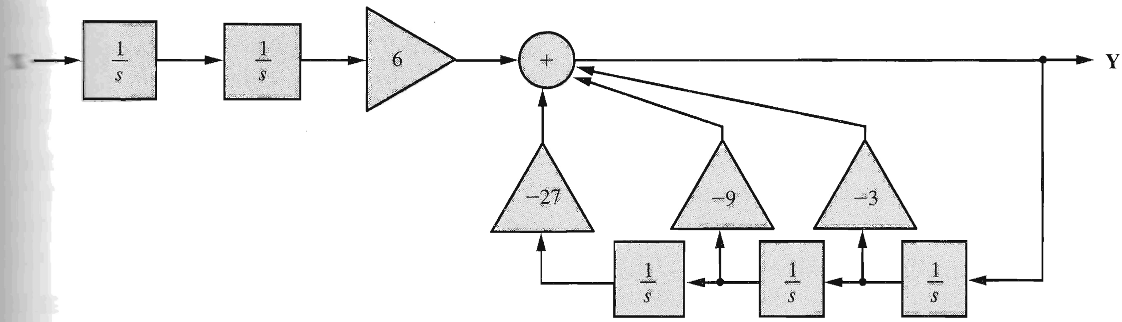

Fig. P5.54

5.60 Find the Laplace transform of (a) $\sin(\beta t - \phi)u(t)$, (b) $\cos(\beta t - \phi)u(t)$, (c) $e^{-\alpha t}\sin(\beta t - \phi)u(t)$, and (d) $e^{-\alpha t}\cos(\beta t - \phi)u(t)$.

5.61 Find the inverse Laplace transform of each of the following functions:

(a) $\dfrac{600}{s(s+10)(s+30)}$ (b) $\dfrac{60(s+4)}{s(s+2)(s+12)}$

5.62 Find the inverse Laplace transform of each of the following functions:

(a) $\dfrac{12s}{(s+3)(s^2+9)}$ (b) $\dfrac{4(s^2+1)}{s(s^2+4)}$

5.63 Find the inverse Laplace transform of each of the following functions:

(a) $\dfrac{(s+2)(s+3)}{s(s+1)^2}$ (b) $\dfrac{10s+80}{s^2+8s+20}$

5.64 Find the solution to the differential equation

$$\frac{d^2x(t)}{dt^2} + 7\frac{dx(t)}{dt} + 6x(t) = 36u(t)$$

subject to the initial conditions $dx(0)/dt = 0$ and $x(0) = -4$.

5.65 Find the solution to the differential equation

$$\frac{d^2x(t)}{dt^2} + 3\frac{dx(t)}{dt} + 2x(t) = 20\cos 2t\; u(t)$$

subject to the zero initial conditions $dx(0)/dt = x(0) = 0$.

5.66 For the series RC circuit shown in Fig. P5.66, suppose that $R = 5\ \Omega$ and $C = 0.1$ F. Find the step responses $v(t)$ and $i(t)$ when $v_s(t) = 20u(t)$ V.

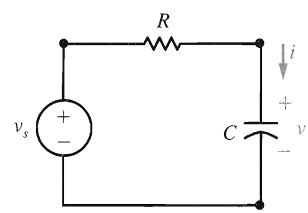

Fig. P5.66

5.67 For the series RC circuit shown in Fig. P5.66, suppose that $R = 2\ \Omega$ and $C = 2$ F. Find $v(t)$ and $i(t)$ when $v_s(t) = 12e^{-t/2}u(t)$ V.

5.68 For the series RC circuit shown in Fig. P5.66, suppose that $R = 2\ \Omega$ and $C = 2$ F. Find $v(t)$ and $i(t)$ when $v_s(t) = 12e^{-t/4}u(t)$ V.

5.69 For the series RL circuit shown in Fig. P5.69, suppose that $R = 5\ \Omega$ and $L = 5$ H. Find the step responses $i(t)$ and $v(t)$ when $v_s(t) = 20u(t)$ V.

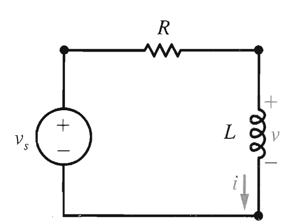

Fig. P5.69

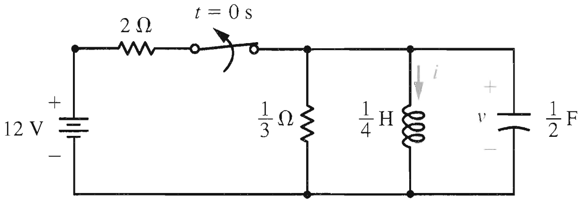

Fig. P3.60

3.63 For the series *RLC* circuit shown in Fig. P3.63, suppose that $R = 7\ \Omega$, $L = 1$ H, $C = 0.1$ F, $v_s(t) = 12$ V for $t < 0$ s and $v_s(t) = 0$ V for $t \geq 0$ s. Find $v(t)$ and $i(t)$ for all time.

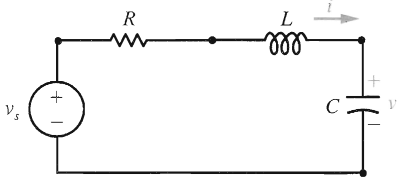

Fig. P3.63

3.64 For the series *RLC* circuit shown in Fig. P3.63, suppose that $R = 2\ \Omega$, $L = 0.25$ H, $C = 0.2$ F, $v_s(t) = 10$ V for $t < 0$ s and $v_s(t) = 0$ V for $t \geq 0$ s. Find $v(t)$ and $i(t)$ for all time.

3.65 For the series *RLC* circuit shown in Fig. P3.63, suppose that $R = 2\ \Omega$, L $= 1$ H, $C = 1$ F, $v_s(t) = 6$ V for $t < 0$ s and $v_s(t) = 0$ V for $t \geq 0$ s. Find $v(t)$ and $i(t)$ for all time.

3.66 For the circuit shown in Fig. P3.66, suppose that $v_s(t) = 6$ V for $t < 0$ s and $v_s(t) = 0$ V for $t \geq 0$ s. Find $v_2(t)$ and $v_1(t)$ for all time.

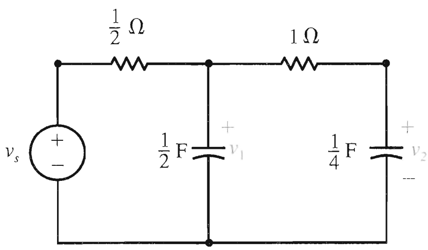

Fig. P3.66

3.67 For the circuit shown in Fig. P3.67, suppose that $v_s(t) = 6$ V for $t < 0$ s and $v_s(t) = 0$ V for $t \geq 0$ s. Find $i(t)$ and $v(t)$ for all time.

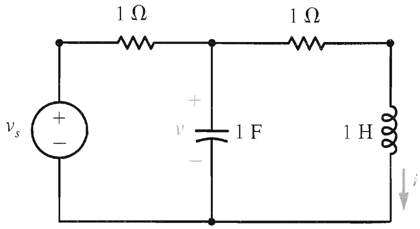

Fig. P3.67

3.68 For the circuit shown in Fig. P3.67, interchange the inductor and the capacitor. Suppose that $v_s(t) = 6$ V for $t < 0$ s and $v_s(t) = 0$ V for $t \geq 0$ s. Find the capacitor voltage $v(t)$ and the inductor current $i(t)$ for all time.

3.69 For the parallel RLC circuit shown in Fig. P3.69, suppose that $R = 0.5\ \Omega$, $L = 0.2$ H, $C = 0.25$ F, and $i_s(t) = 2u(t)$ A. Find the step responses $i(t)$ and $v(t)$.

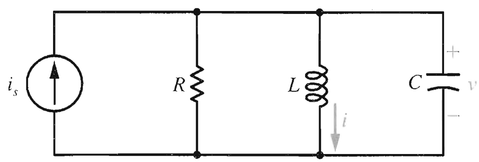

Fig. P3.69

3.70 For the parallel *RLC* circuit shown in Fig. P3.69, suppose that $R = 3\ \Omega$, L $= 3$ H, $C = \frac{1}{12}$ F, and $i_s(t) = 4u(t)$ A. Find the step responses $i(t)$ and $v(t)$.

3.71 For the series *RLC* circuit shown in Fig. P3.63, suppose that $R = 7\ \Omega$, $L = 1$ H, $C = 0.1$ F, and $v_s(t) = 12u(t)$ V. Find the step responses $v(t)$ and $i(t)$.

3.72 For the series *RLC* circuit shown in Fig. P3.63, suppose that $R = 2\ \Omega$, $L = 1$ H, $C = 1$ F, and $v_s(t) = 12u(t)$ V. Find the step responses $v(t)$ and $i(t)$.

3.73 For the *RLC* circuit shown in Fig. 3.43 on p. 172, suppose that $R = \frac{1}{2}\ \Omega$, $L = \frac{1}{3}$ H, $C = \frac{1}{4}$ F, and $V = 1$ V. Find the unit step responses $i(t)$ and $v(t)$.

3.74 For the *RLC* circuit shown in Fig. 3.43 on p. 172, suppose that $R = \frac{1}{2}\ \Omega$, $L = \frac{1}{4}$ H, $C = \frac{1}{2}$ F, and $V = 1$ V. Find the unit step responses $i(t)$ and $v(t)$.

3.75 For the circuit shown in Fig. P3.66, suppose that $v_s(t) = 9u(t)$ V. Find the step response $v_2(t)$.

3.76 For the circuit shown in Fig. P3.67, suppose that $v_s(t) = 6u(t)$ V. Find the step responses $i(t)$ and $v(t)$.

3.77 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.77 when $C = \frac{1}{3}$ F and $v_s(t) = 4u(t)$ V.

3.78 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.77 when $C = \frac{1}{8}$ F and $v_s(t) = 8u(t)$ V.

3.79 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.77 when $C = \frac{1}{4}$ F and $v_s(t) = 6u(t)$ V.

3.80 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.80 when $C = \frac{4}{3}$ F and $v_s(t) = 4u(t)$ V.

3.81 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.80 when $C = 1$ F and $v_s(t) = 3u(t)$ V.

3.82 Find the step response $v_o(t)$ for the op-amp circuit shown in Fig. P3.80 when $C = \frac{1}{5}$ F and $v_s(t) = 2u(t)$ V.

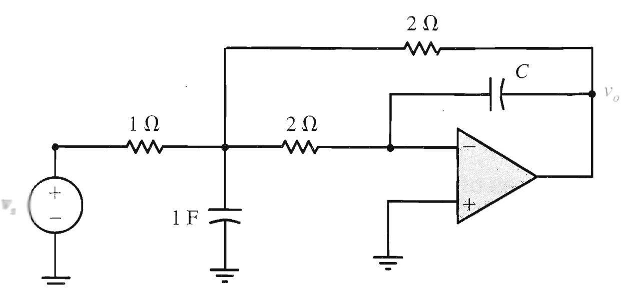

Fig. P3.77

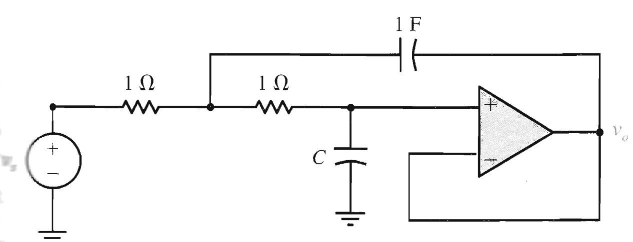

Fig. P3.80

Drill Exercise 3.17

Find the step responses $v(t)$ and $i(t)$ for the series RLC circuit shown in Fig. 3.41 when $R = 16\ \Omega$, $L = 2$ H, $C = 1/50$ F, and $V = 6$.

ANSWER $[6 - 10e^{-4t}\cos(3t - 0.927)]u(t)$ V, $e^{-t}\sin 3t\, u(t)$ A

The Critically Damped Case

Consider the RLC circuit shown in Fig. 3.43. By KCL,

$$\frac{Vu(t) - v}{R} = i + C\frac{dv}{dt} \tag{3.59}$$

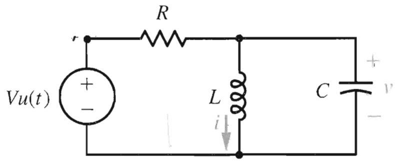

Fig. 3.43 Another RLC circuit.

Substituting $v = L\,di/dt$ into Eq. 3.59 and simplifying results in

$$\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC}i = \frac{V}{RLC}u(t) \tag{3.60}$$

For $t < 0$ s, the right-hand side of this equation becomes zero, and for zero initial conditions the solution is $i(t) = 0$ A. However, for $t \geq 0$ s, Eq. 3.59 becomes

$$\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC}i = \frac{V}{RLC} \tag{3.61}$$

Thus we have that $\alpha = 1/2RC$ and $\omega_n = 1/\sqrt{LC}$.

We finally consider the critically damped case.