Important circuit concepts such as the principle of superposition and Thévenin's theorem are also applicable in the frequency domain.

6. The instantaneous power absorbed by an element is equal to the product of the voltage across it and the current through it.

7. The average power absorbed by a resistance R having a sinusoidal current of amplitude I and voltage of amplitude V is

$$P_R = \frac{1}{2}VI = \frac{1}{2}RI^2 = \frac{1}{2}\frac{V^2}{R}$$

 The average power absorbed by a capacitance or an inductance is zero.

9. A circuit whose Thévenin-equivalent (output) impedance is  $\mathbf{Z}_o$  transfers maximum power to a load  $\mathbf{Z}_L$  when  $\mathbf{Z}_L$  is equal to the complex conjugate of  $\mathbf{Z}_o$ .

10. For the case in which  $\mathbf{Z}_L$  is restricted to be purely resistive, maximum power is transferred when  $\mathbf{Z}_L$  equals the magnitude of  $\mathbf{Z}_o$ .

11. The effective or rms value of a sinusoid of amplitude A is  $A/\sqrt{2}$ .

## **Problems**

**4.1** Find the exponential form of the following complex numbers given in rectangular form: (a) 4 + j7, (b) 3 - j5, (c) -2 + j3, (d) -1 - j6, (e) **4.** (f) -5, (g) j7, (h) -j2.

**4.2** Find the rectangular form of the following complex numbers given in exponential form:

(a)  $3e^{j70^\circ}$ , (b)  $2e^{j120^\circ}$ , (c)  $5e^{-j60^\circ}$ , (d)  $4e^{-j150^\circ}$ , (e)  $\delta e^{i^{i90^\circ}}$ , (f)  $e^{-j90^\circ}$ , (g)  $2e^{j180^\circ}$ , (h)  $2e^{-j180^\circ}$ .

**4.3** Find the rectangular form of the product  $A_1A_2$ iven that: (a)  $A_1 = 3e^{j30^\circ}$ ,  $A_2 = 4e^{j60^\circ}$ ; (b)  $A_1 = 3e^{j30^\circ}$ ,  $A_2 = 4e^{-j30^\circ}$ ; (c)  $A_1 = 5e^{-j60^\circ}$ ,  $A_2 = 2e^{j120^\circ}$ ; (d)  $A_1 = 4e^{i45^\circ}$ ,  $A_2 = 2e^{-j90^\circ}$ .

**4.4** Find the rectangular form of the quotient  $\mathbf{A}_1$   $\mathbf{A}_2$  for  $\mathbf{A}_1$  and  $\mathbf{A}_2$  given in Problem 4.3.

12. The average power absorbed by a resistance R having a current whose effective value is  $I_e$  and a voltage whose effective value is  $V_e$  is

$$P_R = V_e I_e = R I_e^2 = \frac{V_e^2}{R}$$

13. The power factor (pf) is the ratio of average power to apparent power.

14. If current lags voltage, the pf is lagging. If current leads voltage, the pf is leading.

15. Average or real power can be generalized with the notion of complex power.

16. The ordinary household uses a single-phase, three-wire electrical system.

17. The most common polyphase electrical system is the balanced three-phase system.

18. Three-phase sources are generally Y connected, and three-phase loads are generally  $\Delta$  connected.

19. The device commonly used to measure power is the wattmeter.

20. Three-phase load power measurements can be taken with the two-wattmeter method.

**4.5** Find the rectangular form of the sum  $A_1 + A_2$  for  $A_1$  and  $A_2$  given in Problem 4.3.

**4.6** For the ac circuit shown in Fig. P4.6, suppose that  $v_s(t) = 13 \cos(2t - 22.6^\circ)$  V. Find  $v_o(t)$  by using voltage division. Draw a phasor diagram. Is this circuit a lag network or a lead network?



**4.7** Connect a 5- $\Omega$  resistor in parallel with the inductor in the circuit shown in Fig. P4.6. Suppose that  $v_s(t) = 13 \cos(2t - 22.6^\circ)$  V. Find the voltage  $v_o(t)$  across the inductor by using voltage division. Draw a phasor diagram. Is this circuit a lag network or a lead network?

**4.8** Connect a 5- $\Omega$  resistor in parallel with the inductor in the circuit shown in Fig. P4.6. Suppose that  $v_s(t) = 13 \cos(2t - 22.6^\circ)$  V. Find the voltage  $v_o(t)$  across the inductor by using nodal analysis. Draw a phasor diagram. Is this circuit a lag network or a lead network?

**4.9** For the circuit given in Fig. P4.9, suppose that  $i_s(t) = 5 \cos 3t$  A. Find  $v_o(t)$  and  $v_s(t)$  by using current division.



Fig. P4.9

**4.10** For the circuit given in Fig. P4.9, suppose that  $i_s(t) = 5 \cos 3t$  A. Find  $v_o(t)$  and  $v_s(t)$  by using nodal analysis.

**4.11** A voltage of  $v_s(t) = 10 \cos \omega t$  V is applied to a series *RLC* circuit. If  $R = 5 \Omega$ ,  $L = \frac{1}{5}$  H, and  $C = \frac{1}{5}$  F, by how many degrees does  $v_c(t)$  lead or lag  $v_s(t)$  when (a)  $\omega = 1$  rad/s, (b)  $\omega = 5$  rad/s, and (c)  $\omega = 10$  rad/s?

**4.12** A voltage of  $v_s(t) = 10 \cos \omega t$  V is applied to a series *RLC* circuit. If  $R = 5 \Omega$ ,  $L = \frac{1}{5}$  H, and  $C = \frac{1}{5}$  F, by how many degrees does  $v_R(t)$  lead or lag  $v_s(t)$  when (a)  $\omega = 1$  rad/s, (b)  $\omega = 5$  rad/s, and (c)  $\omega = 10$  rad/s?

**4.13** For the *RLC* connection given in Fig. P4.13, find the impedance **Z** when  $\omega$  is (a) 2, (b) 4, and (c) 8 rad/s.





**4.14** For the *RLC* connection shown in Fig. P4.14, find the admittance **Y** when  $\omega$  is: (a) 1, (b) 3, and (c) 7 rad/s.



Fig. P4.14

**4.15** Show that a general expression for the impedance **Z** depicted in Fig. P4.13 is

$$\mathbf{Z} = \frac{32}{\omega^2 + 16} + j\frac{\omega(\omega^2 - 16)}{4(\omega^2 + 16)}$$

**4.16** Show that a general expression for the admittance Y depicted in Fig. P4.14 is

$$\mathbf{Y} = \frac{1}{2(\omega^2 + 1)} + j\frac{\omega(\omega^2 - 9)}{20(\omega^2 + 1)}$$

**4.17** For the circuit shown in Fig. P4.17, find the Thévenin equivalent of the circuit in the shaded box when  $v_s(t) = 4 \cos(4t - 60^\circ)$  V. Use this to determine  $v_o(t)$ .



Fig. P4.17