CONTROL OF UNCERTAIN NONHOLONOMIC MECHANICAL SYSTEMS  
WITH CHAINED FORM KINEMATICS: CASE STUDY

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1. Introduction

In this paper we employ two adaptive strategies for motion control of uncertain nonholonomic mechanical systems with chained form kinematics. Chained form systems were first introduced in [1], and recent work has demonstrated that a great many systems of importance in applications can be converted to this form. Additionally, chained form systems possess sufficient structure to permit very effective control strategies to be developed for them. For example, these systems are differentially flat, which can be shown to be quite useful for motion control, and they are also easier to stabilize than some other classes of nonholonomic systems; both of these properties will be exploited in this paper. As a consequence of the adaptive nature of the control laws and the properties of chained form systems, the employed schemes are simple and modular, are easily implemented with a wide range of mechanical systems, and provide a reasonable starting point for addressing such practical problems as collision avoidance and sensor-based planning and control. The first controller combines an efficient trajectory generation algorithm with an adaptive tracking strategy to obtain the desired motion control, while the second scheme achieves this control objective through the use of a stabilizing control law developed utilizing adaptation and homogeneous system theory. Each of the control strategies is computationally efficient, ensures accurate motion control, and is implementable in the presence of considerable uncertainty concerning the mechanical system model. The efficacy of the approach is illustrated through computer simulations with a single chain nonholonomic system.

2. Redundant Robot Simulation Results

The performance of two approaches to nonholonomic system control is examined through computer simulations with a redundant robot whose generalized inverse of the Jacobian is nonholonomic. The redundant robot chosen is the PPR planar robot shown in Figure 1 and described in [2]. The PPR planar robot possesses two prismatic joints and one revolute joint connected in series. All joints are actuated making the robot redundant for the task of positioning the end-effector in the plane with the end-effector orientation left unspecified. The robot arm has links of mass \( m \), assumed for simplicity to be point masses, where the last link has length \( L \). Let \( \mathbf{p} = [p_x \ p_y]^T \in \mathbb{R}^2 \) be the position of the robot end-effector and \( \mathbf{q} = [q_1 \ q_2 \ q_3]^T \in \mathbb{R}^3 \) be the joint configuration vector where \( q_1, q_2 \) denote the linear position of the first two (prismatic) joints, and \( q_3 \) represents the angle of the third (revolute) joint.

A common method for specifying joint motion from desired task-space motion is through a generalized inverse of the Jacobian. One such technique is to use the weighted pseudo inverse kinematic relationship

\[
T^{\#}(q) = W^{-1} J^T(q) [J(q) W^{-1} J^T(q)]^{-1}
\]

where \( T^{\#} \in \mathbb{R}^{2 \times 2} \) is the weighted pseudo inverse of the Jacobian \( J(q) \in \mathbb{R}^{2 \times 3} \), \( W = \text{diag}[1 \ w^2] \in \mathbb{R}^{3 \times 3} \) is a positive definite weighting matrix with \( w \) a scalar constant, \( q_d \in \mathbb{R}^3 \) is the vector of desired joint velocities, and \( v_d \in \mathbb{R}^2 \) is the vector of desired end-effector velocities. The weighted pseudo inverse (1) was found in [2] to have a nonholonomic differential constraint, and in addition, was found feedback equivalent to the single chain representation

\[
\dot{q}_1 = u_1, \quad \dot{q}_2 = u_2, \quad \dot{q}_3 = z_2 u_1
\]

where \( q \) is the joint configuration vector. The coordinate and input transformations which permit the weighted inverse kinematic relationship to be written in chained form are given in [2]. The implication of the nonholonomic constraint is that the desired end-effector velocities \( v_d \) need to be specified in such a way that the corresponding desired joint velocities \( \dot{q}_d \) from (1) satisfy the differential constraint.

We first use a tracking control method to accomplish the desired motion control. More specifically, we utilize the following algorithm to drive the system from some initial configuration \( q(0) \) to a specified goal configuration \( q(T) \) (and correspondingly \( p(0) \) to \( p(T) \)) during the time interval \([0, T]\) with no model information or rate measurements.

Algorithm 1:

1. Given initial and goal configurations \( q(0) \) and \( q(T) \) corresponding to \( p(0) \) and \( p(T) \), respectively, determine the corresponding sets of chained coordinates \( z(0) \) and \( z(T) \). Observing that systems with chained form kinematics (2) are differentially flat with flat outputs \( y = [y_1 \ y_2]^T \) where

\[
z_1 = y_1, \quad z_2 = y_2, \quad z_2 = \frac{dy_2}{dy_1}
\]
map these endpoint conditions to conditions on the flat outputs and derivatives of outputs \((y(0), \dot{y}(0))\) and \((y(T), \dot{y}(T))\). Fit a third order polynomial \(y_{\approx}(t)\) in the flat output space connecting these initial and final output values.

2. Use the relations (2), (3) together with the chained form input transformations from \([2]\) to determine the desired trajectory \(v_d(t)\) from \(y_{\approx}(t)\). Observe that this trajectory completely specifies the velocity of the nonholonomic system at each instant and therefore ensures that the full system state evolves as desired.

3. Map desired task-space velocities \(v_d\) to desired joint-space velocities \(q_d\) using (1) and track the corresponding desired joint trajectory \(q_d(t)\) using the adaptive tracking strategy found in \([3]\).

The above control algorithm is applied to the mathematical model of the PPR robot through computer simulation with a sampling period of two milliseconds. All integrations required by the controller are implemented using a simple trapezoidal integration rule with a time-step of two milliseconds. The system model parameters are chosen as \(L = w = 1\) meter and \(m = 10\) kilograms. The controller parameters \(k_1, k_2, \gamma\) are chosen as \(k_1 = 100, k_2 = 1000, \gamma = 5\). The controller term \(w\) and the adaptive gain \(f\) are set to zero initially, and the adaptation parameters are set as follows: \(\alpha_i = 0.01, \beta_i = 1000\) for \(i = 1, 2, 3\). Note that no attempt was made to "tune" the gains to obtain the best possible performance. The control strategy given in Algorithm 1 was tested using a wide range of initial conditions; sample results are given in Figure 2 and indicate that the motion of the system is accurately controlled.

Next we examine the effectiveness of a stabilization approach to nonholonomic system motion control through computer simulations with the redundant robot. We wish to stabilize the system to the goal \(q_1 = q_2 = 2\) meters, \(q_3 = 2\pi/3\) radians in the presence of dynamic model uncertainty. This goal configuration does not correspond to a zero goal configuration for the chained form coordinates, thus a change of variables needs to be performed before the \(\rho\)-exponentially stabilizing controller

\[
u_1 = -z_1 - \frac{z_2^2}{\rho(z)} \sin t, \quad u_2 = -z_2 + \frac{z_3}{\rho(z)} \cos t \quad (4)
\]

where the homogeneous norm is defined as \(\rho(z) = (z_1^2 + z_2^2 + z_3^2)^{1/4}\) can be applied. One such change of variables is given in \([4]\) where the new coordinates are also in chained form with a goal configuration at the origin. The kinematic stabilizer (4) along with the input transformation found in \([2]\) can now be used to generate the desired velocity \(v_d\) to allow stabilization to a goal configuration. This velocity trajectory is mapped to a desired joint trajectory through (1) which is tracked with the adaptive tracking scheme found in \([3]\). The system model parameters are set to the values used in the previous simulation, and the adaptive tracking scheme is implemented exactly as described in the above simulation. The control strategy was tested using a wide range of initial conditions; sample results are given in Figure 3 and indicate that accurate stabilization of the system is achieved.

3. References