4.43. For each of the circuits in Fig. P4.43, find the labeled node voltages. For all transistors, $k_n' \frac{W}{L} = 0.4 \text{ mA/V}^2$, $V_t = 1 \text{ V}$, $\lambda = 0$.

(a) Figure P3.43a

Since $v_{DS} = v_{GS} + V$, $(V = 5 \text{ V})$ we are in the triode region. Insert expression for $v_{DS}$ to get

$$i_D = k_n' \frac{W}{L} \left[ (v_{GS} - V_i) (v_{GS} - V_i) - \frac{(v_{GS} - V_i)^2}{2} \right] = \frac{k_n' W}{2 L} (v_{GS} - V_i)^2$$

which gives

$$V_1 = -v_{GS} = -\sqrt{2i_D \left( \frac{k_n' W}{L} \right)^{-1}} - V_i$$

$$= -\sqrt{2 \times \frac{10 \times 10^{-6}}{0.4 \times 10^{-3}}} - 1$$

$$= -1.22 \text{ V}$$

(b) Figure P3.43b

Same as (a), except larger current, so we get
\[ V_2 = -\sqrt{\frac{2 \times 100 \times 10^{-6}}{0.4 \times 10^{-3}}} - 1 = -1.71 \text{ V} \]

(c) Figure P3.43c

Same as (a), except larger current still, so we get

\[ V_3 = -\sqrt{\frac{2 \times 1 \times 10^{-3}}{0.4 \times 10^{-3}}} - 1 = -3.24 \text{ V} \]

(d) Figure P3.43d

In this case we are in the saturation region because \( v_{DS} > v_{GS} - V_t \),

\[ i_D = \frac{k_n'}{2} \frac{W}{L} (v_{GS} - V_t)^2 \]
\[ V_4 = v_{GS} = \sqrt{2i_D \left( \frac{k_n W}{L} \right)^{-1}} + V_t \]
\[ = \sqrt{\frac{2 \times 10 \times 10^{-6}}{0.4 \times 10^{-3}}} + 1 \]
\[ = 1.22 \text{ V} \]

(e) Figure P3.43e

Same as previous case except larger current.

\[ V_5 = v_{GS} = \sqrt{2 \times 10^{-3}} + 1 \]
\[ = 3.24 \text{ V} \]

(f) Figure P3.43f

Saturation mode, so

\[ i_D = \frac{k_n W}{2L} (v_{GS} - V_t)^2 \]

and
\[ i_D = \frac{V - v_{GS}}{R} \]

\[ V - v_{GS} = R \frac{k'_n W}{2} \left[ v_{GS}^2 + V_t^2 - 2v_{GS}V_t \right] \]

\[ V - v_{GS} = Av_{GS}^2 + AV_t^2 - 2Av_{GS}V_t \]

\[ Av_{GS}^2 + v_{GS} (1 - 2AV_t) + AV_t^2 - V = 0 \]

\[ Av_{GS}^2 + Bv_{GS} + C = 0 \]

\[ v_{GS} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \]

where

\[ A = \frac{R \frac{k'_n W}{L}}{2} = 20 \text{ V}^{-1} \]

\[ B = 1 - 2AV_t = 1 - 2 \times 20 \times 1 = -39 \]

\[ C = AV_t^2 - V = 20 \times 1^2 - 5 = 15 \text{ V} \]

so that

\[ v_{GS} = \frac{39 \pm \sqrt{39^2 - 4 \times 20 \times 15}}{2 \times 20} \]

\[ v_{GS} = 1.43 \text{ V} \text{ or } v_{GS} = 0.53 \text{ V} \]

Only one of these solutions is correct. We assume non-zero current and thus conducting mode, which is only true for \( v_{GS} > V_t \). Therefore the correct solution is

\[ v_{GS} = 1.43 \text{ V} \]

(g) Figure P3.43g
This is the same problem except a different resistance. In this case we have

\[
A = \frac{R}{2}k'_nW = 0.2\text{V}^{-1} \quad B = 1 - 2AV_t = 1 - 2 \times 0.2 \times 1 = 0.6
\]

\[
C = AV_t^2 - V = 0.2 \times 1^2 - 5 = -4.8\text{V}
\]

\[
v_{GS} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
\]

\[
v_{GS} = 3.62\text{V} \text{ or } v_{GS} = -6.6\text{V}
\]

The first potential solution is the correct one as we assumed \( v_{GS} > V_t \).

(h) Figure P3.43h

In this case we have

\[
v_{DS} + i_D R_D = 2\text{V} \quad V_8 = i_D R - V = -v_{GS}
\]

since \( v_{DS} = v_{GS} + V > v_{GS} - V_t \), it is operating in the saturation region

\[
i_D = \frac{k'_nW}{2L}(v_{GS} - V_t)^2
\]
Inserting the expression for \( i_D \) in terms of \( v_{GS} \) above,

\[
i_D = \frac{V - v_{GS}}{R}
\]

we get

\[
\frac{V - v_{GS}}{R} = \frac{k'_n W}{2 L} (v_{GS} - V_t)^2
\]

This is identical to the expression in problem (f) above, so the solution is the same, \( v_{GS} = 1.43 \text{ V} \), and

\[
V_8 = -v_{GS} = -1.43 \text{ V}
\]

4.44. For each of the circuits shown in Fig P4.44, find the labeled node voltages. The NMOS transistors have \( V_t = 1 \text{ V} \), and \( k'_n \frac{W}{L} = 2 \text{ mA/V}^2 \). Assume \( \lambda = 0 \).

![Figure P4.44](image)

(a) For Figure P4.44a

We have \( v_{DS1} = 2V - V_1 \), and \( v_{GS1} = V - V_1 \), where \( V = 5 \text{ V} \). Thus, transistor 1 is operating in saturation. Also notice that \( V_2 = i_D R - V \). If we assume that transistor 2 is also operating in saturation then \( v_{GS1} = v_{GS2} \). And since \( v_{GS2} = -V_2 \), we have

\[
-v_{GS1} = i_D R - V = R \frac{k'_n W}{2 L} (v_{GS1} - V_t)^2 - V
\]

Choose \( A = R \frac{k'_n W}{2 L} \), we get
\[-v_{GS1} = A v_{GS1}^2 + AV_t^2 - 2A v_{GS1} V_t - V\]

\[A v_{GS1}^2 + AV_t^2 - 2A v_{GS1} V_t + v_{GS1} - V = 0\]

or

\[A v_{GS1}^2 + B v_{GS} + C = 0\]

where

\[A = R \frac{k_n' W}{2 L} = \frac{1 \times 10^3}{2} \times 2 \times 10^{-3} = 1 \frac{1}{V}\]

\[B = 1 - 2AV_t = 1 - 2 = -1\]

\[C = AV_t^2 - V = -4 V\]

\[v_{GS1} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{1 \pm \sqrt{1 + 4 \times 4}}{2}\]

\[v_{GS1} = 2.56 V \text{ or } v_{GS1} = -1.56 V\]

The second cannot be a solution, because we assumed \(v_{GS1} > V_t\) to be conducting. In the first case we get

\[V_1 = V - v_{GS1} = 5 - 2.56 = 2.44 V\]

which puts the second transistor in saturation mode, so that assumption is OK. Thus,

\[V_2 = -v_{GS2} = -v_{GS1} = -2.56 V\]

(b) For Figure P4.44b

Both transistors are operating in saturation, and are identical. We have

\[v_{DS1} = v_{DS2} = v_{DS} = v_{GS1} = v_{GS2} = v_{GS}\]

Thus,

\[2v_{GS} + 2i_D R = V\]

\((V = 10 V)\). We also have

\[i_D = \frac{k_n' W}{2 L} (v_{GS} - V_t)^2\]
Thus

\[
\frac{V - 2v_{GS}}{2R} = i_D = \frac{k_n'W}{2L} (v_{GS} - V_t)^2
\]

Choosing \( A = \frac{k_n'W}{2L} \), we get

\[
\frac{V - 2v_{GS}}{2R} = Av_{GS}^2 + AV_t^2 - 2Av_{GS}V_t
\]

\[
Av_{GS}^2 + AV_t^2 - 2Av_{GS}V_t + \frac{v_{GS}}{R} - \frac{V}{2R} = 0
\]

\[
Av_{GS}^2 + Bv_{GS} + C = 0
\]

where

\[
A = \frac{k_n'W}{2L} = 1 \times 10^{-3} \frac{A}{V^2} \quad B = -2AV_t + \frac{1}{R} = -1 \times 10^{-3} \Omega^{-1}
\]

\[
C = AV_t^2 - \frac{V}{2R} = -4 \times 10^{-3} A
\]

\[
v_{GS} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{1 \times 10^{-3} \pm \sqrt{1 \times 10^{-6} + 4 \times 10^{-3} \times 4 \times 10^{-3}}}{2 \times 10^{-3}}
\]

\[
v_{GS} = 2.56 \text{ V or } v_{GS} = -1.56 \text{ V}
\]

The first solution is the correct one because we assumed \( v_{GS} > V_t \). In that case,

\[
i_D = \frac{k_n'W}{2L} (v_{GS} - V_t)^2 = 1 \times 10^{-3} (2.56 - 1)^2 = 2.43 \times 10^{-3} \text{ A}
\]

and

\[
V_3 = V - i_D R = 10 - 2.43 = 7.57 \text{ V}
\]

\[
V_4 = V_3 - v_{GS} = 7.57 - 2.56 = 5.01 \text{ V}
\]

\[
V_5 = V_4 - v_{GS} = 5.01 - 2.56 = 2.45 \text{ V}
\]

\( V_5 \) should also be \( V_5 = 0 + i_D R = 2.43 \text{ V} \), which is close to within a few rounding errors.
4.45. For the PMOS transistor in the circuit shown in Fig. P4.45, $k'_p = 8 \mu A/V^2$, $\frac{W}{L} = 25$, and $|V_{tp}| = 1$ V. For $I = 100 \mu A$, find the voltages $V_{SD}$ and $V_{SG}$ for $R = 0, 10$ kΩ, 30 kΩ, and 100 kΩ. For what value of $R$ is $V_{SD} = V_{SG}$? $V_{SD} = \frac{V_{SG}}{10}$?

![Circuit Diagram](image)

We are being asked about both saturation and triode mode operation. Choose $V_t = 1$ V. First we determine the operating mode of the device. That depends on the relative values of $V_{SD}$ and $V_{SG}$. When $V_{SD} < V_{SG} - V_t$, or $V_{SG} - V_{SD} > V_t$, then we are operating in triode mode. Otherwise saturation mode. But $V_{SG} - V_{SD} = IR$, so the criterion is $IR > V_t$ for triode mode. Here is a table

<table>
<thead>
<tr>
<th>$R$ (kΩ)</th>
<th>0</th>
<th>10</th>
<th>30</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$ (μA)</td>
<td>100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_t$ (V)</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$IR$ (V)</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>Mode</td>
<td>Saturation</td>
<td>Saturation</td>
<td>Triode</td>
<td>Triode</td>
</tr>
</tbody>
</table>

For the two saturation mode cases we have

$$i_D = \frac{k'_p W}{2L} (v_{SG} - V_t)$$

which gives us

$$v_{SG} = \sqrt{\frac{2i_D L}{Wk'_p}} + V_t = \sqrt{\frac{2 \times 100 \times 10^{-6}}{25 \times 8 \times 10^{-6}}} + 1 = 2 \text{ V}$$

And $v_{SD} = v_{SG} - IR$. For $R = 0$ we get $v_{SD} = v_{SG} = 2$ V, whereas for $R = 10$ kΩ we get $v_{SD} = 2 - 100 \times 10^{-6} \times 10 \times 10^3 = 1$ V.

For the triode region we have

$$i_D = \frac{W}{L} k'_p \left[(v_{SG} - V_t) v_{SD} - \frac{v_{SD}^2}{2}\right]$$
In this case we can substitute $V_{SD} = V_{SG} - IR$ and get
\[
i_D = \frac{W}{L} k'_p \left[ (V_{SG} - V_t) (V_{SG} - IR) - \frac{(V_{SG} - IR)^2}{2} \right]
\]
\[
\frac{i_D L}{W k'_p} = \left[ \frac{V_{SG}^2}{2} - V_{SG} IR - V_{SG} V_t + V_t IR - \frac{V_{SG}^2}{2} - \frac{I^2 R^2}{2} + \frac{V_{SG} IR}{2} \right] \]
\[
\frac{V_{SG}^2}{2} + V_{SG} \left( -IR - V_t + \frac{IR}{2} \right) + V_t IR - \frac{I^2 R^2}{2} - \frac{i_D L}{W k'_p} = 0
\]
\[
\frac{V_{SG}^2}{2} + V_{SG} \left( -V_t - \frac{IR}{2} \right) + V_t IR - \frac{I^2 R^2}{2} - \frac{i_D L}{W k'_p} = 0
\]
Making
\[
A = \frac{1}{2} \quad B = -V_t - \frac{IR}{2} \quad C = V_t IR - \frac{I^2 R^2}{2} - \frac{i_D L}{W k'_p}
\]
We can solve the quadratic equation as
\[
V_{SG} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}
\]

<table>
<thead>
<tr>
<th>( R ) (kΩ)</th>
<th>30</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( B ) (V)</td>
<td>-2.5</td>
<td>-6</td>
</tr>
<tr>
<td>( C ) (V²)</td>
<td>-2</td>
<td>-40.5</td>
</tr>
<tr>
<td>( V_{SG} ) (V)</td>
<td>5.7/-0.7</td>
<td>11.2/-7.2</td>
</tr>
<tr>
<td>( V_{SD} ) (V)</td>
<td>2.7</td>
<td>1.25</td>
</tr>
</tbody>
</table>

Only the positive solutions are physical and those are the answer. As a final check we should verify that these voltage do indeed correspond to triode region operation. That is the case if $V_{SD} - V_{SG} < V_t$. That is indeed the case.

Next we are asked the values of \( R \) for which there is a particular relationship between \( V_{SD} \) and \( V_{SG} \). First, for the case where \( V_{SD} = V_{SG} \), that can only be the case when \( R = 0 \). Secondly, the value for which \( V_{SD} = \frac{V_{SG}}{2} \). We already found that is the case for \( R = 10 \) kΩ. Third, for \( v_{SD} = \frac{v_{SG}}{10} \), we must be operating in the triode mode.

\[
i_D = \frac{W}{L} k'_p \left[ (v_{SG} - V_t) v_{SD} - \frac{v_{SG}^2}{2} \right]
\]

and inserting \( v_{SD} = \frac{v_{SG}}{10} \) (and \( i_D = I \)),

\[
I = \frac{W}{L} k'_p \left[ (v_{SG} - V_t) \frac{v_{SG}}{10} - \frac{v_{SG}^2}{200} \right]
\]
\[
= \frac{W}{L} k'_p \left[ \frac{v_{SG}^2}{10} - \frac{v_{SG} V_t}{10} - \frac{v_{SG}^2}{200} \right]
\]
\[
= \frac{W}{L} k'_p \left[ \frac{19}{200} v_{SG}^2 - \frac{1}{10} v_{SG} V_t \right]
\]

\[10\]
\[
\frac{19}{200}v_{SG}^2 - \frac{1}{10}V_t v_{SG} - \frac{IL}{Wk_p'} = 0
\]

\[
A = \frac{19}{100} \quad B = -\frac{V_t}{10} = -0.1 \text{ V} \quad C = -\frac{IL}{Wk_p'} = \frac{-100 \times 10^{-6}}{25 \times 8 \times 10^{-6}} = -0.5 \text{ V}^2
\]

\[
v_{SG} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = 2.880 \text{ or } -1.827
\]

The second solution is not valid, so we continue with the first solution,

\[
R = \frac{v_{SG} - v_{SD}}{I} = \frac{9v_{SG}}{10I} = \frac{9 \times 2.880}{10 \times 100 \times 10^{-6}} = 25.92 \text{ k}\Omega
\]

4.53. The expression for the incremental voltage gain \( A_v \) given in Eq (4.4) can be written as

\[
A_v = -\frac{2(V_{DD} - V_{DS})}{V_{OV}}
\]

where \( V_{DS} \) is the bias voltage at the drain (called \( V_{OQ} \) in the text). This expression indicates that for given values of \( V_{DD} \) and \( V_{OV} \), the gain magnitude can be increased by biasing the transistor at a lower \( V_{DS} \). This, however, reduces the allowable output signal swing in the negative direction. Assuming linear operation around the bias point, show that the largest possible negative output signal peak \( \hat{v}_o \) that is achievable while the transistor remains saturated is

\[
\hat{v}_o = \frac{V_{DS} - V_{OV}}{1 + \frac{1}{A_v}}
\]

For \( V_{DD} = 5 \text{ V} \) and \( V_{OV} = 0.5 \text{ V} \), provide a table of values for \( A_v, \hat{v}_o \), and the corresponding \( \hat{v}_i \) for \( V_{DS} = 1 \text{ V}, 1.5 \text{ V}, 2 \text{ V}, \) and \( 2.5 \text{ V} \). If \( k_{nL}' = 1 \text{ mA/V}^2 \), find \( I_D \) and \( R_D \) for the design for which \( V_{DS} = 1 \text{ V} \).

Here is an illustration of the problem in which we are to determine \( \hat{v}_o \):
We are looking for the point, as we travel up the straight line, where \( v_{DSSat} = v_{GS} - V_t \), which is the edge of the saturation region. Then \( \hat{v}_o = V_{DS} - v_{DSSat} \). Let’s model linearly

\[
v_{DS} = V_{DS} + v_{ds} \quad v_{GS} = V_{GS} + \frac{v_{ds}}{A_v}
\]

At the maximum excursion we can write

\[
v_{DSSat} = V_{DS} - \hat{v}_o \quad v_{GSSat} = V_{GS} - \frac{\hat{v}_o}{A_v}
\]

Now we just set \( v_{DSSat} = v_{GSSat} - V_t \) and solve for \( \hat{v}_o \).

\[
V_{DS} - \hat{v}_o = V_{GS} - \frac{\hat{v}_o}{A_v} - V_t = V_{OV} - \frac{\hat{v}_o}{A_v}
\]

\[
\hat{v}_o = \frac{V_{DS} - V_{OV}}{1 - \frac{1}{A_v}}
\]

This is not the same expression as is given in the book. Nevertheless, I will continue with this expression, and tabulate for \( V_{DD} = 5 \text{ V} \), and \( V_{OV} = 0.5 \text{ V} \)

<table>
<thead>
<tr>
<th>( V_{DS} ) (V)</th>
<th>( A_v )</th>
<th>( \hat{v}_o ) (V)</th>
<th>( \hat{v}_i ) (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-16</td>
<td>0.47</td>
<td>0.029</td>
</tr>
<tr>
<td>1.5</td>
<td>-14</td>
<td>0.93</td>
<td>0.066</td>
</tr>
<tr>
<td>2</td>
<td>-12</td>
<td>1.38</td>
<td>0.12</td>
</tr>
<tr>
<td>2.5</td>
<td>-10</td>
<td>1.82</td>
<td>0.18</td>
</tr>
</tbody>
</table>

4.54. Figure P4.54 shows a CS amplifier in which the load resistor \( R_D \) has been replaced with another NMOS transistor \( Q_2 \) connected as a two-terminal device. Note that because \( v_{DG} \) of \( Q_2 \) is zero, it will be operating in saturation at all times, even when \( v_I = 0 \) and \( i_D2 = i_D1 = 0 \). Noe also that the two transistors conduct equal drain currents. Using \( i_{D1} = i_{D2} \), show that for the range of \( v_I \) over which \( Q_1 \) is operating in saturation, that is for

\[
V_{t1} \leq v_I \leq v_O + V_{t1}
\]

the output voltage will be given by

\[
v_o = V_{DD} - V_t + \sqrt{\frac{W/L_1}{W/L_2}}V_t - \sqrt{\frac{W/L_1}{W/L_2}}v_I
\]

where we have assumed \( V_{t1} = V_{t2} = V_t \). Thus the circuit functions as a linear amplifier, even for large input signals. For \( (W/L)_1 = (50 \mu m/0.5 \mu m) \) and \( (W/L)_2 = (5 \mu m/0.5 \mu m) \), find the voltage gain.
For $Q_2$ we have

$$i_D = \frac{1}{2} \frac{W_2}{L_2} k'_n (V_{DD} - v_o - V_t)^2$$

and for $Q_1$ we have

$$i_D = \frac{W_1}{L_1} k'_n \left[ (v_I - V_t) v_o - \frac{v_o^2}{2} \right]$$

We want to find $v_o$, so we eliminate $i_D$ between the two equations.

$$\frac{1}{2} \frac{W_2}{L_2} (V_{DD} - v_o - V_t)^2 = \frac{W_1}{L_1} \left[ (v_I - V_t) v_o - \frac{v_o^2}{2} \right]$$

This is a quadratic equation in $v_o$. Solving it is straightforward but tedious. You will get two solutions of which one is not physical and the other one is the one shown.

The voltage gain is the factor in front of $v_I$, so

$$A_v = -\sqrt{\frac{W_1}{L_1}} \sqrt{\frac{W_2}{L_2}} = -\sqrt{\frac{50}{5/0.5}} = -3.2$$