SS 13.34. Figure P13.34 shows a monostable multivibrator circuit. In the stable state, \( v_o = L_+ \), \( v_A = 0 \), and \( v_B = -V_{\text{ref}} \). the circuit can be triggered by applying a positive input pulse of height greater than \( V_{\text{ref}} \). For normal operation, \( C_1 R_1 \ll CR \). Show the resulting waveforms of \( v_o \) and \( v_A \). Also, show that the pulse generated at the output will have a width \( T \) given by

\[
T = CR \ln \left( \frac{L_+ - L_-}{V_{\text{ref}}} \right)
\]

\( v_A \) begins at ground because there is no current through \( R \). As the positive pulse is applied to trigger, \( v_B \) is pulled low which causes \( v_o \) to go low. The voltage across the capacitor is still \( L_+ \), so the voltage \( v_A = L_- - L_+ \). The capacitor begins to charge from that voltage to ground. However, once it reaches \( -V_{\text{ref}} \) the output flips. The waveforms are here
and here is the expression for determining $T$

$$(L_+ - L_-) e^{-T RC} = -V_{\text{ref}}$$

which can be re-written as

$$\ln\left(\frac{-V_{\text{ref}}}{L_+ - L_-}\right) = -\frac{T}{RC}$$

$$T = RC \ln\left(\frac{L_+ - L_-}{V_{\text{ref}}}\right)$$

SS 13.39. Using a 680 pF capacitor, design the astable circuit of Fig. 13.29(a) to obtain a square wave with a 50 kHz frequency and a 75% duty cycle. Specify the values of $R_A$ and $R_B$.

This is a straightforward application of formulas given in the textbook,

$$T = \frac{1}{f} = 0.69C (R_A + 2R_B) \quad \text{duty} = \frac{R_A + R_B}{R_A + 2R_B}$$

Begin by finding $R_A + 2R_B$,

$$R_A + 2R_B = \frac{1}{0.69 f C} = \frac{1}{0.69 \times 50 \times 10^3 \times 680 \times 10^{-12}} = 42.63 \, \text{k}\Omega$$

Next, find $R_A + R_B$ from the duty cycle formula,

$$R_A + R_B = \text{duty} \times (R_A + 2R_B) = 0.75 \times 42.63 = 31.97 \, \text{k}\Omega$$

Next,
\[ R_B = (R_A + 2R_B) - (R_A + R_B) = 42.63 - 31.97 = 10.66 \, k\Omega \]

and

\[ R_A = (R_A + R_B) - R_B = 31.97 - 10.66 = 21.31 \, k\Omega \]

SS 13.40 The node in the 555 timer at which the voltage is \( V_{TH} \) (i.e. the inverting input terminal for the comparator 1) is usually connected to an external terminal. This allows the user to change \( V_{TH} \) externally (i.e., \( V_{TH} \) no longer remains at \( \frac{2}{3}V_{CC} \)). Note, however, that whatever the value of \( V_{TH} \) becomes, \( V_{TL} \) always remains \( \frac{1}{2}V_{TH} \).

(a) For the astable circuit of Fig. 13.29, rederive the expression for \( T_H \) and \( T_L \), expressing them in terms of \( V_{TH} \) and \( V_{TL} \).

(b) For the case \( C = 1 \, \text{nF}, R_A = 7.2 \, \text{k}\Omega, R_B = 3.6 \, \text{k}\Omega, \) and \( V_{CC} = 5 \, \text{V}, \) find the frequency of oscillation and the duty cycle of the resulting square wave when no external voltage is applied to the terminal \( V_{TH} \).

(c) For the design in (b), let a sine-wave signal of a much lower frequency than that found in (b) and of 1-V peak amplitude be capacitively coupled to the circuit node \( V_{TH} \). This signal will cause \( V_{TH} \) to change around its quiescent value of \( \frac{2}{3}V_{CC} \), and thus \( T_H \) will change correspondingly - a modulation process. Find \( T_H \), and find the frequency of oscillation and the duty cycle at the two extreme values of \( V_{TH} \).

(a) \( T_H \) is the time when the output is high and the capacitor is charging up. It is charging from \( V_{TL} \) towards \( V_{CC} \) with time-constant \( (R_A + R_B)C \) and gets interrupted at \( V_{TH} \). Thus we have

\[
\left(1 - e^{-\frac{T_H}{(R_A+R_B)C}}\right)(V_{CC} - V_{TL}) - V_{TL} = V_{TH}
\]
\[1 - e^{-\frac{V_{TH}}{(R_A + R_B)C}} = \frac{V_{TH} - V_{TL}}{V_{CC} - V_{TL}}\]

\[e^{-\frac{V_{TH}}{(R_A + R_B)C}} = 1 - \frac{V_{TH} - V_{TL}}{V_{CC} - V_{TL}} = \frac{V_{CC} - V_{TH} - V_{TL} + V_{TL}}{V_{CC} - V_{TL}} = \frac{V_{CC} - V_{TH}}{V_{CC} - V_{TL}}\]

\[T_H = (R_A + R_B)C \ln \frac{V_{CC} - V_{TL}}{V_{CC} - V_{TH}}\]

\[T_L = R_B C \ln \frac{V_{TH}}{V_{TL}} = R_B C \ln 2\]

\(T_L\) is the time when the output is low and the capacitor is discharging. It discharges from \(V_{TH}\) towards ground with time constant \(R_B C\) and gets interrupted at \(V_{TL}\). Thus we have

\[e^{-\frac{T_H}{R_B C}} V_{TH} = V_{TL}\]

\[T_L = R_B C \ln \frac{V_{TH}}{V_{TL}} = R_B C \ln 2\]

(not it is independent of \(V_{TH}\))

(b) In this case we have \(V_{TH} = \frac{2}{3} V_{CC}\) and \(V_{TL} = \frac{1}{3} V_{CC}\). Then \(T_H\) is

\[T_H = (7.2 + 3.6) \times 10^3 \times 1 \times 10^{-9} \ln \left(\frac{1 - \frac{1}{3}}{1 - \frac{2}{3}}\right) = 7.5 \times 10^{-6} \text{s} = 7.5 \mu\text{s}\]

and \(T_L\) is

\[T_L = 3.6 \times 10^3 \times 1 \times 10^{-9} \times \ln 2 = 2.5 \times 10^{-6} = 2.5 \mu\text{s}\]

The period of the signal is then \(T = T_H + T_L = 10 \mu\text{s}\) and the frequency is \(f = 100 \text{kHz}\). The duty cycle for high output is 75%.

(c) We always have \(V_{TL} = \frac{V_{TH}}{2}\). At the two extremese we have \(V_{TH} = \frac{2}{3} V_{TH} \pm A\), where \(A\) is the amplitude of the sine wave. When the the voltage is increased we have

\[V_{TH} = \frac{2}{3} V_{CC} + A = \frac{2}{3} \times 5 + 1 = 4.33 \text{ V}\]

and

\[V_{TL} = \frac{V_{TH}}{2} = 2.17 \text{ V}\]

so that
\[ T_H = (7.2 + 3.6) \times 10^3 \times 1 \times 10^{-9} \ln \frac{5 - 2.17}{5 - 4.33} = 1.55 \times 10^{-5} \text{s} = 15.5 \mu\text{s} \]

and the frequency is

\[ f = \frac{1}{T_H + T_L} = \frac{10^6}{15.5 + 2.5} = 55.5 \text{kHz} \]

and the duty cycle is

\[ \frac{T_H}{T_H + T_L} = \frac{15.5}{15.5 + 2.5} = 86.1\% \]

when the voltage is one volt lower we get

\[ V_{TH} = 2.33 \text{V} \quad V_{TL} = \frac{2.33}{2} = 1.17 \text{V} \]

and

\[ T_H = (7.2 + 3.6) \times 10^3 \times 1 \times 10^{-9} \ln \frac{5 - 1.17}{5 - 2.33} = 3.90 \mu\text{s} \]

Then the frequency is

\[ f = \frac{1}{T_H + T_L} = \frac{10^6}{3.90 + 2.5} = 156 \text{kHz} \]

and the duty cycle is

\[ \frac{T_H}{T_H + T_L} = 60.9\% \]