EE 570: Location and Navigation: Theory & Practice

Navigation Sensors and INS Mechanization
The Fundamental Inertial Navigation Problem:

- Using inertial sensors (accels & gyros) and an initial position and orientation, determine the vehicle’s (i.e. body frame) current position, velocity, and attitude (PVA)

Assumptions:

1. Know where we started (initial PVA: $\mathbf{r}_b^i$, $\mathbf{v}_b^i$, & $\mathbf{C}_b^i$)
2. Inertial sensors ($\mathbf{\omega}_{ib}$ and $\mathbf{f}_{ib}$) are error free (relax later)
3. Have a gravity ($\mathbf{g}_b$) and/or gravitational ($\mathbf{\gamma}_b$) model

Where am I? – Current PVA?

- With respect to which frame?
Inertial Navigation

- The process of “integrating” angular velocity & acceleration to determine one’s position, velocity, and attitude (PVA)
  - Effectively “dead reckoning”
- To measure the acceleration and angular velocity vectors we need at least 3-gyros and 3-acceals
  - Typically configured in an orthogonal triad
- The “mechanization” can be performed wrt:
  - The ECI frame,
  - The ECEF frame, or
  - The Nav frame.
• An Inertial Navigation System (INS)
  
  - ISA – Inertial Sensor Assembly
    - Typically, 3-gyros + 3-acccels + basic electronics (power, ...)
  
  - IMU – Inertial Measurement Unit
    - ISA + Compensation algorithms (i.e. basic processing)
  
  - INS – Inertial Navigation System
    - IMU + gravity model + “mechanization” algorithms
Navigation Sensors and INS Mechanization

Navigation Equations – Mechanization Process

IMU Measurements

\[ \omega_{ib}^b \]

\[ \ddot{f}_{ib}^b \]

Prior PVA

Prior Attitude

Prior Velocity

Prior Position

Updated Attitude

Updated Velocity

Updated Position

Updated PVA

Gravity / Gravitational Model
Navigation Sensors and INS Mechanization
Navigation Equations – A Four Step Mechanization

• Can be generically performed in four steps:
  1. Attitude Update
     - Update the prior attitude (rotation matrix) using the current angular velocity measurement ($\dot{C}^0_1 = C^0_1 \Omega^1_{01} = \Omega^0_{01} C^0_1$)
  2. Transform the specific force measurement ($\vec{f}_{ib}^? = C^?_b \vec{f}_{ib}^b$)
     - Typically, using the attitude computed in step 1.
  3. Update the velocity
     - Essentially integrate the result from step 2. with the use of a gravity/gravitation model ($\vec{f}_{ib} = \vec{a}_{ib} - \vec{y}_{ib}$)
  4. Update the Position
     - Essentially integrate the result from step 3.
Navigation Sensors and INS Mechanization

Navigation Equations – A Four Step Mechanization

1. Attitude Update
2. SF Transform
3. Velocity Update
3. Position Update

Prior PVA

Prior Velocity

Prior Position

Prior Attitude

IMU Measurements

Prior PVA

Updated Attitude

Updated Velocity

Updated Position

Updated PVA

f_{ib}^b
\omega_{ib}^b
Grav Model
CASE 1: ECI Frame Mechanization

- Determine the Position, Velocity, and Attitude of the Body frame with respect to the Inertial Frame

Determine our PVA wrt the ECI frame

- **Position**: Vector from the origin of the inertial frame to the origin of the body frame resolved in the inertial frame: $\mathbf{r}_{ib}$

- **Velocity**: Velocity of the body frame wrt the inertial frame resolved in the inertial frame: $\mathbf{v}_{ib}$

- **Attitude**: Orientation of the body frame wrt the inertial frame $C_b^i$
1. Attitude Update: Method A

- Body orientation frame at time “k” wrt time “k-1”
  - $\Delta t = \text{Time}_k - \text{Time}_{k-1}$

\[
\dot{C}_b^i = C_b^i \Omega_{ib}^b \\
\dot{C}_b^i = \lim_{\Delta t \to 0} \left( \frac{C_b^i (k) - C_b^i (k - 1)}{\Delta t} \right) = C_b^i (k - 1) \Omega_{ib}^b \\
C_b^i (+) - C_b^i (-) \approx C_b^i (-) \Omega_{ib}^b \Delta t \\
C_b^i (+) \approx C_b^i (-) \left( I + \Omega_{ib}^b \Delta t \right)
\]
1. Attitude Update: Method B

- Body orientation frame at time “k” wrt time “k-1”
  - $\Delta t = \text{Time}_k - \text{Time}_{k-1}$

\[
C_{b(k)}^i = C_{b(k-1)}^i C_{b(k)}^{b(k-1)}
\]

\[
C_{b(k)}^{b(k-1)} = e^{\Omega_{ib}^b \Delta t} = e^{K \Delta \theta}
\]

\[
\omega_{ib}^b \Delta t = \hat{k} \Delta \theta
\]

\[
C_{b(k)}^i = e^{\Omega_{ib}^b \Delta t} = e^{K \Delta \theta}
\]

\[
= I + K \Delta \theta + \frac{K^2 \Delta \theta^2}{2!} + \frac{K^3 \Delta \theta^3}{3!} + \ldots
\]

\[
= I + \sin(\Delta \theta) K + [1 - \cos(\Delta \theta)] K^2
\]

\[
C_{b(+)i} = C_{b(-)i} e^{\Omega_{ib}^b \Delta t}
\]

\[
\simeq C_{b(-)i} \left( I + \Omega_{ib}^b \Delta t \right)
\]
1. Attitude Update:
   - High Fidelity
     \[ C^i_b (+) = C^i_b (-) \left[ I + \sin(\Delta \theta) \mathcal{K} + \left[ 1 - \cos(\Delta \theta) \right] \mathcal{K}^2 \right] \]
   - Lower Fidelity
     \[ C^i_b (+) \approx C^i_b (-) \left( I + \Omega^b_{ib} \Delta t \right) \]
2. Specific Force Transformation
   - Simply coordinatize the specific force
     \[ \vec{f}_{ib}^i = C_b^i (+) \vec{f}_{ib}^b \]

3. Velocity Update
   - Assuming that we are in space (i.e. no centrifugal component)
     \[ \vec{a}_{ib}^i = \vec{f}_{ib}^i + \vec{\gamma}_{ib}^i \]
   - Thus, by simple numerical integration
     \[ \vec{v}_{ib}^i (+) = \vec{v}_{ib}^i (-) + \vec{a}_{ib}^i \Delta t \]

4. Position Update
   - By simple numerical integration
     \[ \vec{r}_{ib}^i (+) = \vec{r}_{ib}^i (-) + \vec{v}_{ib}^i (-) \Delta t + \vec{a}_{ib}^i \frac{\Delta t^2}{2} \]
Navigation Sensors and INS Mechanization

Navigation Equations – Case 1: ECI Mechanization

\[ C_b^i (+) = C_b^i (-) \left( I + \Omega_{ib}^b \Delta t \right) \]

\[ \vec{f}_{ib}^i = C_b^i (+) \vec{f}_{ib}^b \]

\[ \vec{a}_{ib}^i = \vec{f}_{ib}^i + \vec{\gamma}_{ib}^i \]

\[ \vec{v}_{ib}^i (+) = \vec{v}_{ib}^i (-) + \vec{a}_{ib}^i \Delta t \]

\[ \vec{r}_{ib}^i (+) = \vec{r}_{ib}^i (-) + \vec{v}_{ib}^i (-) \Delta t + \frac{\vec{a}_{ib}^i \Delta t^2}{2} \]

1. Attitude Update

2. SF Transform

3. Velocity Update

Grav Model \( \vec{\gamma}_{ib}^i \)

3. Position Update
• In continuous time notation:
  - Attitude: \( \dot{C}_b^i = C_b^i \Omega_{ib} \)
  - Velocity: \( \dot{v}_{ib}^i = C_b^i \dot{f}_{ib} + \gamma_{ib} \)
  - Position: \( \dot{r}_{ib}^i = \dot{v}_{ib}^i \)

• Combining into a state-space equation: